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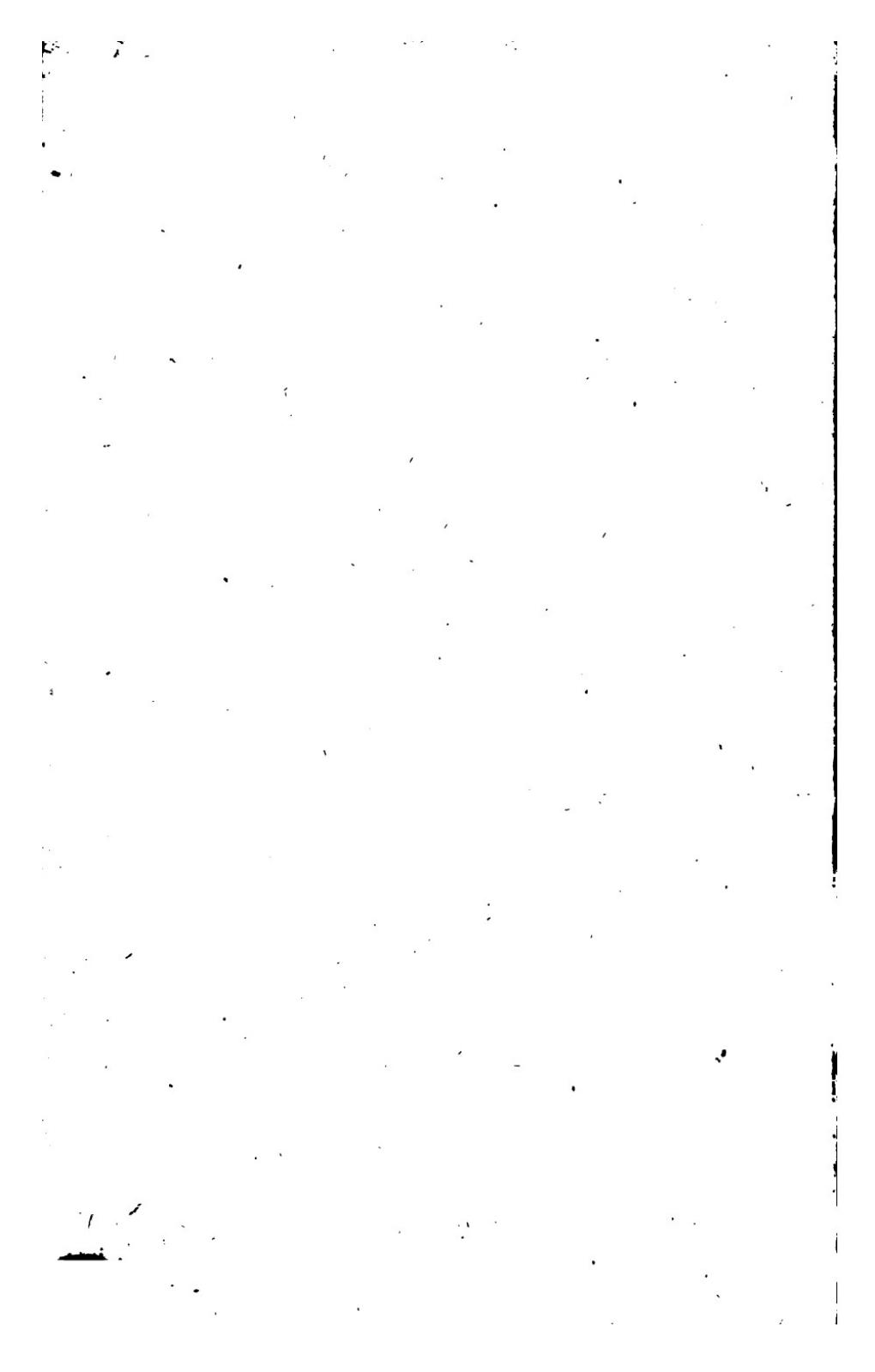
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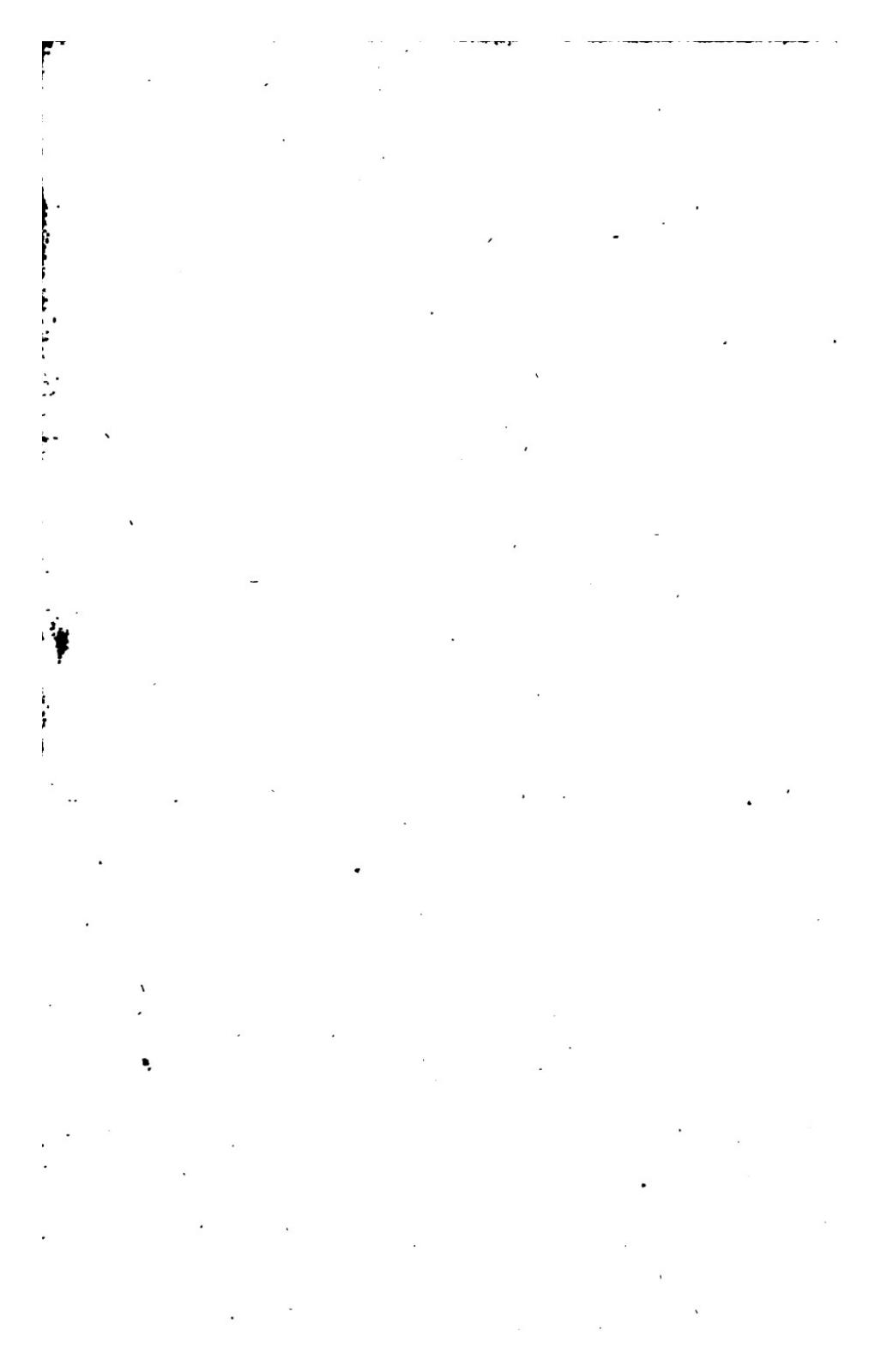
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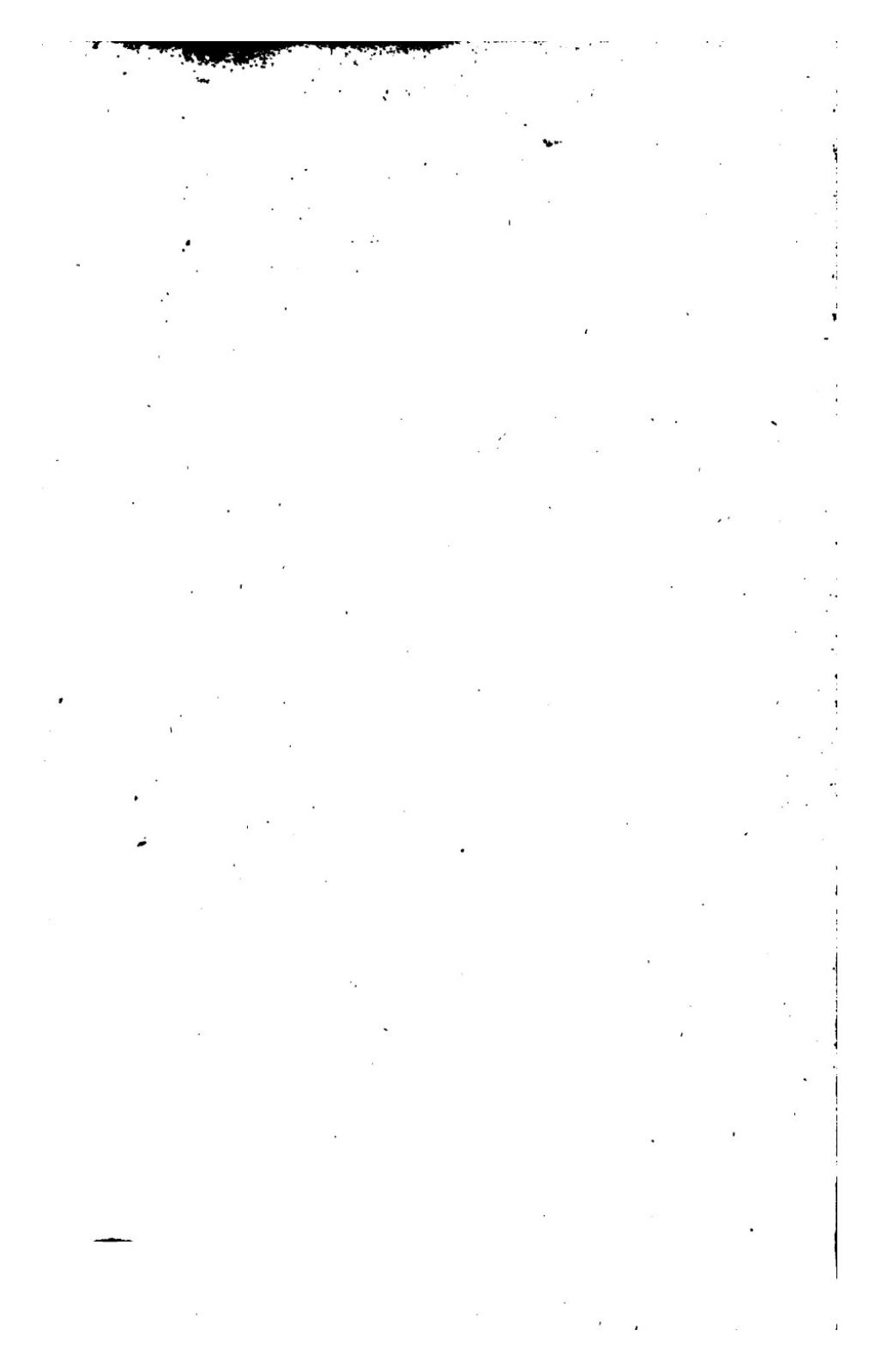


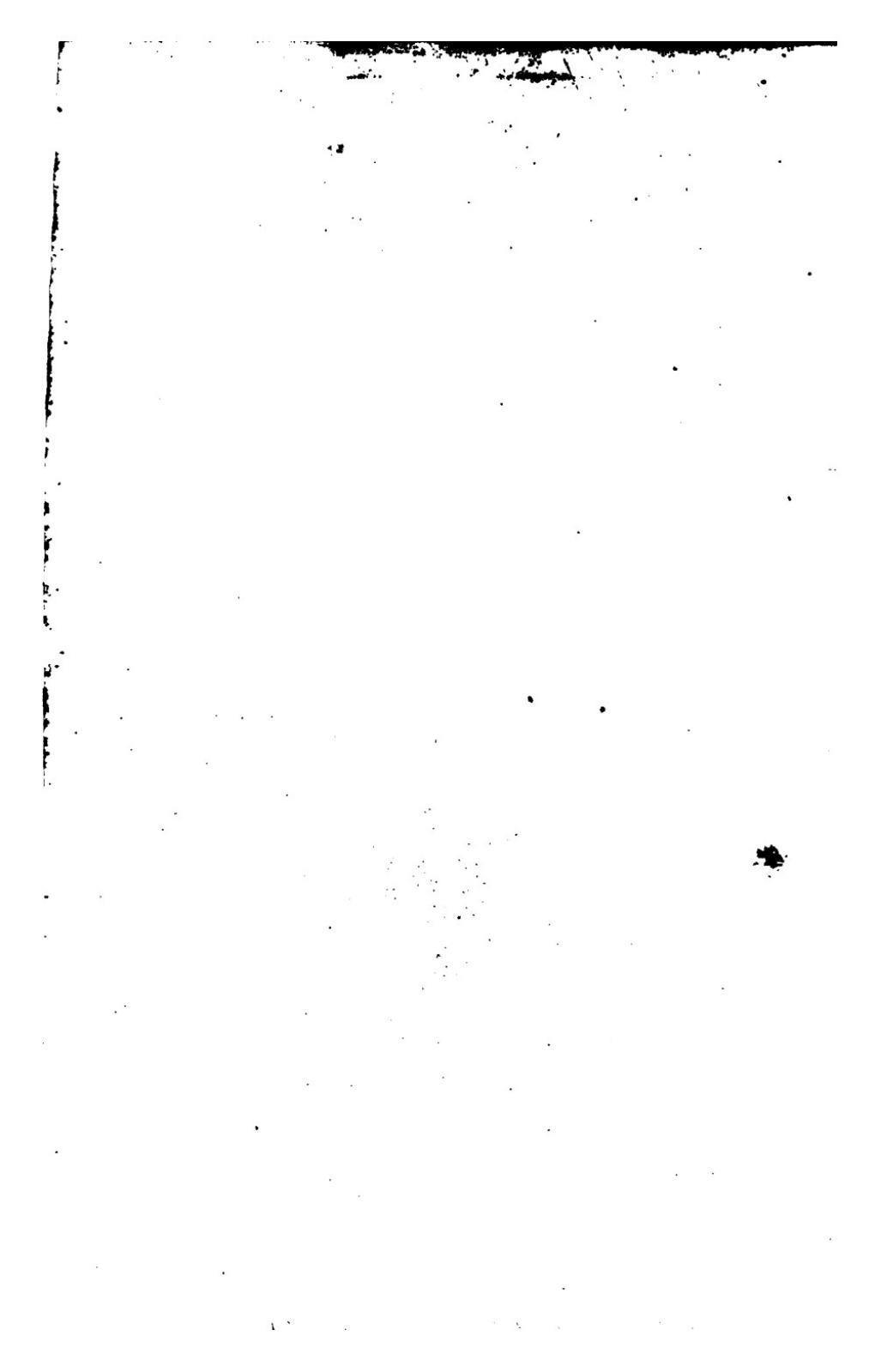
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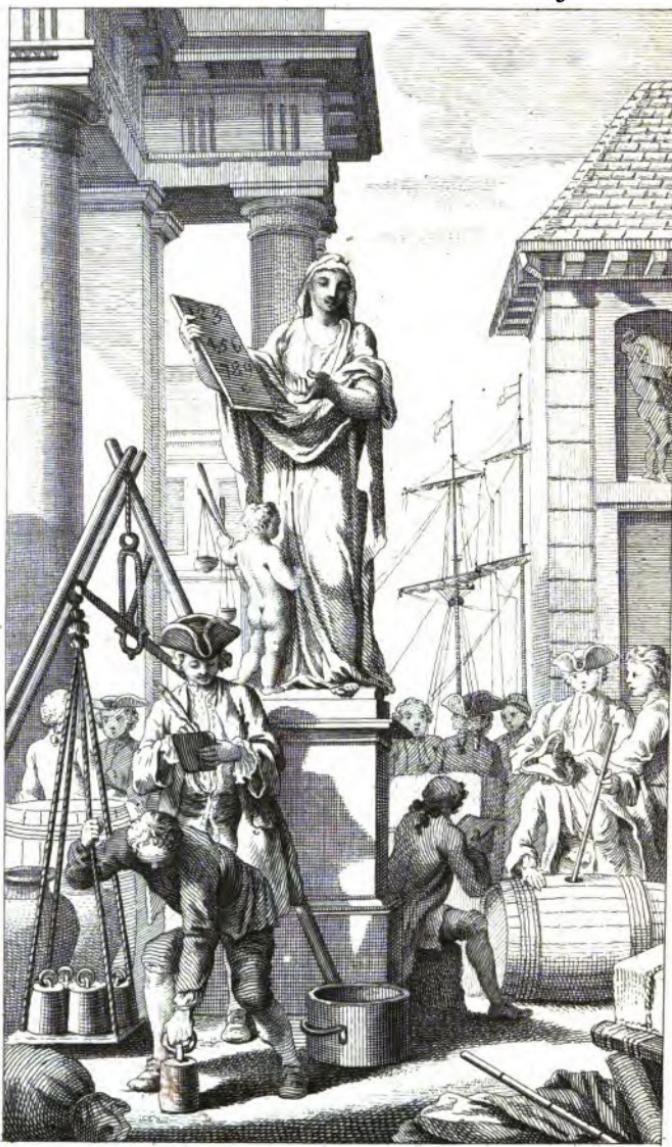








To face the Title



C. Morley Sculp.

A PLAIN and FAMILIAR
M E T H O D
For Attaining the
Knowledge and Practice
O F
COMMON ARITHMETIC.

C O N T A I N I N G

All the USEFUL RULES both in Whole
Numbers, and Fractions, Vulgar and Decimal,
Extraction of the Square and Cube-Roots,
Simple and Compound Interest, Annuities, &c.
delivered in a more practical and correct Manner,
than in any Work hitherto extant.

By EDMUND WINGATE, of *Gray's-Inn*, Esq;

THE EIGHTEENTH EDITION.

Wherein the Additions and Emendations made by Mr. *John Kersey*, in his Appendix; and Mr. *George Shelley*, in his Supplement, are introduced in their proper Places; and all the Improvements in this Science that have appeared in other Writers since their Time, are carefully inserted. Also sundry others, that are entirely new, are added,

By JAMES DODSON, Accomptant, and
Teacher of the Mathematics.

L O N D O N :

Printed for J. and P. K N A P T O N, in *Ludgate-street*; and
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M.DCC.LI.



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TO THE
RIGHT HONOURABLE
T H O M A S,
Earl of *Arundel and Surrey*,
Earl Marshal of
ENGLAND, &c.

Right Honourable,

THE good Affection You bear to all Kind of Learning, and in particular to the Mathematics, makes me adventure to present your Lordship with this Tractate of Arithmetic; because that Art, compared with other Mathematical Sciences, is as the *Primum Mobile*, in respect of the other inferior Orbs: For as the Poets used in Times past to say of *Venus, Sine Cerere & Baccho, friget Venus*; so may I also confidently aver.

A of

of them, without Arithmetic, they are poor, and without Motion. Presuming therefore that your Lordship, loving the Art, cannot disaffect the Artist, nor his Intention to do Good in that Kind ; I am bold to shelter this TREATISE under your Lordship's Protection, humbly entreating your gracious Acceptation, and earnestly desiring for ever to remain,

Your Honour's, in all Service,

Affectionately Devoted,

EDM. WINGATE.



THE



T H E P R E F A C E.

TH E many Editions this Work has gone through, and the Reputation it has deservedly maintained for upwards of 120 Years, would undoubtedly have been Authority sufficient for the Publication of this Edition, without any Alteration from the former: But as several Arithmetical Improvements, both in Theory and Practice, have appeared since this Treatise received the last Hand, it has been thought convenient to insert them in this Edition, together with some which have not been published before.

The first Edition of Wingate's Arithmetic was printed about the Year 1629, by himself; in which his principal Design was to obviate the Difficulties which ordinarily occur in the using of Logarithms: To perform this, he divided his Work into two Books; the first he called Natural, and the second Artificial Arithmetic. The Basis on which the present Structure has been, at different Times, reared is the first of those Books.

For after the first Impression of the above two Books were disposed of, Mr. Wingate (not having Leisure to revise the same, and to supply some Defects which too strict an Attention to his Design, viz. that of explaining the Use of Logarithms, had occasioned) requested Mr. John Kersey, an able Mathematician, to undertake the same. Accordingly Mr. Kersey (in several Editions) made many

Improvements,

Improvements, which take in his own Words, as they stood in his Preface to the former Editions.

“ First, For the Ease and Benefit of those Learners, that desire only so much Skill in Arithmetic, as is useful in Accompts, Trade, and such like ordinary Employments ; the Doctrine of whole Numbers, (which, in the first Edition, was intermingled with Definitions and Rules concerning broken Numbers, commonly called Fractions,) is now entirely handled a-part. And to the end the full Knowledge of Practical Arithmetic in whole Numbers might more clearly appear, I have explained divers of the old Rules in the first five Chapters, and framed a-new the Rules of Division, Reduction, and the Golden-Rule, in the sixth, seventh, eighth, and ninth Chapters : So that now, Arithmetic in whole Numbers is plainly and fully handled before any Entrance be made into the craggy Paths of Fractions, at the Sight of which some Learners are so discouraged, that they make a Stand, and cry out, *Non plus ultra*, There's no Progress farther.

“ Secondly, To assist such young Students as would lay a good Foundation for the attaining of a general Knowledge in the Mathematics, I have in a familiar Method delivered the entire Doctrine of Fractions, both Vulgar and Decimal, which was omitted in the first Edition ; and have also newly framed the Extraction of the Square and Cube-roots, in a Method which by Experience is found to be much easier than that commonly used heretofore, and is exactly suitable to the Construction or Composition of Square and Cube Numbers.

“ Lastly, I have added an Appendix, furnished with Variety of choice and delightful Knowledge in Numbers, both Practical and Theoretical.”

But as Mr. Kersey has omitted to enumerate the Particulars of which his Appendix consisted, the Editor has here

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bere given them, as printed in a Table of Contents pre-fix'd thereto.

Chap. 1. Of Contractions in the Rule of Three.

Chap. 2. Of Rules of Practice by Aliquot Parts.

Chap. 3. Of Exchanges of Coins, Weights, and Measures.

Chap. 4. Practical Questions about Tare, Tret, Loss, Gain, Barter, Factorship, and measuring of Tapestry.

Chap. 5. Of Interest of Money, and the Construction of Tables to value Annuities, &c.

Chap. 6. A Demonstration of the Rule of Three.

Chap. 7. A Demonstration of the Double Rule of Fellowship.

Chap. 8. A Demonstration of the Rule of Alligation : Where also of the Composition of Medicines.

Chap. 9. A Demonstration of the Rule of False:

Chap. 10. A Collection of choice Questions to exercise all the Parts of Vulgar Arithmetic ; to which are added various Practical Questions, about the Mensuration of Superficial Figures and Solids, with the Gauging of Vessels.

Chap. 11. Sports and Pastimes.

The Work, thus enlarged and amended, passed through divers Editions, till about the Year 1700, when Mr. George Shelley, Writing-Master of Christ's-Hospital, wrote a Supplement to it, containing divers practical, compendious, and easy Methods for the Performance of particular Cases in most of the Rules of Arithmetic ; together with Decimal Tables useful in the Computation of Interest and Exchanges, and some useful Rules and Observations relating to Practical Measuring.

Such was the State of this Work when it came into the present Editor's Hands, under whose Care it has met with the following Alterations and Additions :

1. *All the different Materials relating to any one Subject, which in the former Editions lay separated from each other*

The P R E F A C E.

either in the different Parts of the Work, the Appendix, and the Supplement, are here collected together into their proper Places ; so far as the same could be done consistently with the keeping the Doctrine of whole Numbers separate from that of Fractions, before-mentioned by Mr. Kersey.

2. Many useful Properties of Numbers, practical Observations, and Compendiums in Operations (not mentioned in the former Editions) are here inserted in their proper Places.

3. The Demonstrations given by Mr. Kersey, which were founded on Geometrical and Algebraical Principles, are supplied by others purely Arithmetical.

4. The Properties of Numbers considered as Prime and Composite are delivered, as a necessary Help to the Management of Fractions.

5. The Operations of Vulgar Fractions are rendered much easier, by an Artifice in the Management and Abbreviation of them.

6. The Doctrine of Repeating or Circulating Decimals is introduced ; and the Management of them, in a more general and easy Manner than hitherto taught, is scientifically explained.

7. An universal Rule of Proportion, which answers the Purpose of the several Rules of Three, single, double, direct or inverse, in whole Numbers or Fractions, is delivered and illustrated by many Examples ; in which the great Usefulness of the above-mentioned Method of managing and abbreviating Vulgar Fractions, will abundantly appear.

8. The Rule of Alligation Alternate, as hitherto delivered, will give but few Answers to Questions propounded therein ; most of which are capable of many, and some of innumerable Answers : To remedy this Defect, a Method is laid down to find more Answers than the common Rules will give, in those Cases which admit of more ; and as many Answers as the Computer pleases, in those Cases where the Number of Answers is unlimited.

9. The Properties of Numbers considered as Roots and Powers, and the Nature and Properties of Logarithms are compendiously explained.

10. A more practical Method of extracting the Cube-Root is inserted, instead of that delivered by Mr. Kersey; to which is annexed a Table of the Square and Cube-Roots of all Integers less than 145.

11. The Properties of Numbers in Arithmetical Proportion continued, are more fully explained and illustrated.

12. The Properties of Numbers in Geometrical Progression are also more particularly handled, with a View to the Application of such Numbers, to the Computations relating to Compound Interest.

13. Mr. Shelley's Tables of Simple Interest are brought into a narrower Compass, and adapted to those Rates which are now commonly wanted.

14. The Tables for Compound Interest and Annuities, are enlarged from 30 to 60 Years, and computed to the same Rates as those of Simple Interest; the manner of making and using them is more particularly shewn; easy Methods are given to find Numbers beyond the Extent of the Tables; and some new Tables are added, relating to Half-yearly and Quarterly Payments.

15. A great Variety of Examples, with their Answers, are inserted; which will not only be a good Exercise for Learners, but will also serve to ease Teachers of the Burthen of writing out Questions.

16. A copious Index is annexed, to which the Reader may turn for more particular Information of the Contents of the whole.

Thus altered and enlarged, the Editor conceives that this Work contains, methodically, all that is necessary to be known, or performed in Common Arithmetic; and by consequence, that the Purchasers hereof need not be at the Trouble, or Charge of looking into any other of the, almost, numberless Writers on that Subject.

The P R E F A C E.

If the above Improvement in the Operations of the Rule of Alligation Alternate, should prove of Service in real Business; the Knowledge thereof may induce the Editor to publish some farther Thoughts upon that Subject.

Bell-Dock, Wapping,
April 4th, 1751.

The Explanation of certain Marks and Characters, which, for the Sake of Brevity and Perspicuity, are frequently used in the ensuing Work.

- + is the Mark of Addition; and, when it stands between two Numbers, denotes that they are to be added together.
- is the Mark of Subtraction; and, when it stands between two Numbers, denotes that the latter is to be taken from the former.
- × is the Mark of Multiplication; and, when it stands between two Numbers, it denotes that they are to be multiplied together.
- ÷ is the Mark of Division; and, when two Numbers are placed in the same manner as the two Points are here, it denotes that the Number above is to be divided by that below.
- = is the Mark of Equality; which, being set between two numerical Expressions, denotes that they are equal between themselves.
- ::: are the Marks of Proportionality; and denote that the Numbers, between which they are placed, are proportional Numbers.

E X A M P L E S.

For $4+3=7$; read, the sum of 4 and 3 is equal to 7.

For $4-3=1$; read, when three is taken from 4, the Remainder is equal to 1.

For $4\times 3=12$; read, the Product of 4 and 3 is equal to 12.

For $\frac{12}{3}=4$; read, if 12 be divided by 3, the Quotient is equal to 4.

For $1:4::3:12$; read, as 1 is to 4, so is 3 to 12.



A
T R E A T I S E
O F
Common Arithmetic.

C H A P. I.

Concerning NOTATION of Numbers.

1. ARITHMETIC teaches the properties of Numbers ; and by them deduces the methods of calculating, or computing from certain *data*, the values, weights, measures, distances, proportions, &c. of things.

2. Number is that by which every thing is counted ; or that which answers this question, How many ? (unless it be answer'd by nothing :) So if it be asked, how many days are in a week ? the answer is seven, which is therefore called the Number of days in a week.

3. The Notes or Characters, by which Number is ordinarily expressed, are these, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 nothing.



4. These Notes or Characters are either significant Figures, or a Cypher.

5. The significant Figures are the first nine, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, usually called Digits. The first of these is more particularly called an Unit or Unity, and the rest are said to be composed of Units: So 2 is composed of two Units, 3 of three Units, &c. that is, 1 more 1 is equal to 2; 2 more 1 is equal to 3, &c.

Note. The Characters prefixed to the several Articles of this Treatise may serve for an Example of the natural rank or series of Numbers, so increasing by the continual addition of 1.

These Characters were first used in *England*, about the year 1130.

Note also. That in the natural series of Numbers, 1, 2, 3, 4, 5, 6, 7, 8, &c. the first, third, fifth, &c. Numbers, *viz.* 1, 3, 5, 7, 9, 11, &c. are called odd Numbers; and the second, fourth, sixth, &c. Numbers, *viz.* 2, 4, 6, 8, 10, 12, &c. are called even Numbers.

6. The Cypher is the last, which tho' of itself it signifies nothing, yet, being annexed after any of the rest, it increases their value; as will appear in the following Rules.

7. Arithmetic has two Parts, Notation and Numeration.

8. Notation teaches how to express, read, or declare the signification or value of any number written; and also to write down any number proposed, with proper characters, in their due places.

9. A Number is said to have so many places, as there are characters in the number, *viz.* when divers figures, whether they be intermixed with a cypher or cyphers or not, are placed together, like letters in a word, without any point, comma, line, or other note of distinction interposed; all those characters make but one number, which consists of so many places as there are characters so placed together; so this number 205 consists of 3 places, and this 30,600 of 5 places, &c.

10. Notation consists in the knowledge of two things, *viz.* the order of places, and their values.

11. The order of the places is from the right-hand towards the left: So in this number 465, the figure 5 stands in the first place, 6 in the second, and 4 in the third; likewise in this number 7560, a cypher stands in the first place, 6 in the second, 5 in the third, and 7 in the fourth.

12. The first place of a number, which, as before, is the outermost towards the Right-hand is called the place of Units; in which place any figure signifies its own simple value: So in

this number 465; the figure 5 standing in the first place signifies five units, or five.

13. The second place of a number is called the place of Tens, in which place any figure signifies so many tens as the figure contains units: So in this number 465, the figure 5 in the first place signifies simply 5, but the figure 6 in the second place denotes six tens, or sixty.

14. The third place of a number is called the place of Hundreds; in which place any figure signifies so many hundreds as there are units contained in the figure: So in this number 465, the figure 4 in the third place expresses four hundreds: Wherefore if it be required to read or pronounce this number 465, you are to begin on the left-hand; and, according to the aforesaid rules, to pronounce it thus, four hundred sixty-five; likewise this number 315, is to be pronounced thus, three hundred and fifteen; and this number 205, two hundred and five; also this number 500, five hundred. Whence it is manifest, that although a cypher of itself signifies nothing, yet being placed on the right-hand of a figure it increases the value of it, by advancing such figure to an higher place, than that wherein it would be seated, if the cypher were absent.

The true reading or pronouncing the value of any number written, as also the writing down any number proposed, depends principally upon a right understanding of the three first places before-mentioned, and therefore the learner should be well exercised therein, before he proceeds to the following Rules.

15. The fourth place of a number is called the place of Thousands (that is, any number of thousands under ten thousand); the fifth place Tens of Thousands; the sixth place Hundreds of Thousands; the seventh place Millions; (a million being ten hundred thousand;) the eighth place Tens of Millions; the ninth place Hundreds of Millions; the tenth place Thousands of Millions; the eleventh place Tens of Thousands of Millions; the twelfth place Hundreds of Thousands of Millions: And, in that order, you may conceive places to be continued infinitely from the right-hand towards the left, each following place being ten times the value of the next preceding.

16. *From the Rules aforesaid, an easy way may be collected to read or express the value of a number propounded, viz. Let it be required to read or pronounce this number, 521426341. First, distinguish by a comma, or point, every three places, beginning at the right-hand, and proceeding towards the left, so will the aforesaid number be distinguish'd into parts, which may be called periods, and stands thus, 521,426,341. Where you may note the first period towards the right-hand to consist of*

these figures, 341, the second of these 426, and the third of these 521. Secondly, read or pronounce the figures in every period, as if they stood apart from the rest; so will the first period be pronounced three hundred forty-one, the second four hundred twenty-six, and the third five hundred twenty-one. Thirdly, to every period, except the first towards the right-hand, a peculiar denomination or surname is to be applied, viz. the surname of the second period, is thousands; of the third, millions; of the fourth, thousands of millions, &c. Therefore beginning to pronounce at the highest period, which in this example is the third, and giving every period its due surname, the said number will be pronounced thus, five hundred twenty-one millions, four hundred twenty-six thousand, three hundred and forty-one.

17. And, when 'tis required to write down or read more places than twelve, let the fifth period be called billions; the sixth, thousands of billions; the seventh, trillions; the eighth, thousands of trillions; the ninth, quadrillions, &c.

Note, When a number is distinguished into periods, as before, the highest period will not always compleatly consist of three places, but sometimes of one place, and sometimes of two; nevertheless after such period is pronounced as if it stood apart, the due surname is to be annexed; so this number 3204689, after it is divided into periods will stand thus, 3,204,689, and is to be pronounced thus, three millions, two hundred and four thousand, six hundred eighty-nine; and this number 17,213, is to be read, seventeen thousand two hundred thirteen.

18. The aforesaid Rules for the right pronouncing or reading of a number which is written down, being well understood, will sufficiently inform the reader how to write down any number propounded to be written.

The TABLE of NOTATION.

*The Order of Places.**The Value of Places.*

		&c.	&c.
Fourth Period	{ Twelfth Place	3 Hundreds of Thousands of Millions.	
	{ Eleventh Place	2 Tens of Thousands of Millions.	
	{ Tenth Place	1 Thousands of Millions.	
Third Period	{ Ninth Place	9 Hundreds of Millions.	
	{ Eighth Place	8 Tens of Millions.	
	{ Seventh Place	7 Millions.	
Second Period	{ Sixth Place	6 Hundreds of Thousands.	
	{ Fifth Place	5 Tens of Thousands.	
	{ Fourth Place	4 Thousands.	
First Period	{ Third Place	3 Hundreds.	
	{ Second Place	2 Tens.	
	{ First Place	1 Units.	

19. NOTATION of Numbers, by Latin Letters.

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	III. or thus IV,	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIII. or thus LIX,
7	VII.	60	LX.
8	VIII. or thus IIX.	89	LXXXIX.
9	VIII. or thus IX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII. or thus XIIIX.	500	D. or thus IC.
19	XVIII. or thus XIX.	600	DC. or thus ICI.
20	XX.	700	DCC. or thus ICCC.
		1000	CI. or thus M.
		2000	CI. CI. or MM.
		3000	CI. CI. CI. or MMM.
		5000	CCI. CCI.
		10,000	CCCI.
		50,000	CCCI.
		100,000	CCCI. CCC. or thus CM.
		500,000	CCCCI.
		1,000,000	CCCCCI.
		1750	CI. CI. CCC. L, or MDCCCL.

C H A P. II.

Concerning English Money, Weights, Measures, &c.

20. THE things expressed by numbers, are *Money, Weight, Measure, Time, &c.* Of the three first of these, there are infinite kinds and varieties, according to the diversity of the several *Commonwealths* wherein they are used; all which here to produce were both endless and needless: Wherefore only such *Money, Weights, Measures, &c.* as are used in this nation, will be here treated of.

21. The least piece of money used in *England* is a Farthing, from whence this following Table is produced.

4 Farthings	}	1 Penny.
12 Pence		1 Shilling.
20 Shillings		1 Pound.

English (or Sterling) Money, is ordinarily written down with Figures, after this manner :

l.	s.	d.	f.
34	13	05	2
09	05	10	1
09	00	06	3
00	12	11	0
00	00	07	2

The first Rank of the said Numbers signifies thirty-four pounds thirteen shillings five-pence two farthings; the second Rank expresses nine pounds five shillings ten-pence one farthing; the third Rank, sixty-nine pounds no shillings six-pence three farthings, &c.

22. The smallest weight used in *England* is a Grain, that is, the weight of a grain of wheat, well dried, and gathered out of the middle of the ear, of which thirty-two make another weight, called a Penny-weight, and twenty penny-weights make an Ounce *Troy* *

Here observe, that by the *Statutes* quoted in the margin †, the weight of two and thirty grains of wheat, make a penny-weight, which weight being once discovered by thirty-two such grains, the said penny-weight (being the twentieth part of an ounce *Troy*) is usually subdivided into twenty-four parts only, called also *Grains*, as appears by the ensuing Table.

B 4

* Vide Stat. de Compositione Ponderum, 51 Hen. III.

† 31 Ed. I. v. Raft. Weights. 7, & 8. 12 Hen. VII. 5.

A TABLE of TROY-WEIGHTS.

32 Grains of Wheat	make	24 Artificial Grains.
24 Grains		1 Penny-weight.
20 Penny-weights		1 Ounce.
12 Ounces		1 Pound.

Troy-weight is ordinarily set down with Figures, after this manner :

lb.	oz.	p.w.	gr.
17	. 05	. 13	. 13
00	. 11	. 07	. 06
00	. 00	. 05	. 20

The first Rank of the said Numbers expresseth seventeen pounds five ounces thirteen penny-weights, thirteen grains of *Troy-weight*: The second Rank, no pounds eleven ounces seven penny-weights six grains; and the third, no pounds no ounces five penny-weights and twenty grains.

Now this *Troy-weight* serves only to weigh Gold, Silver, and Electuaries. And here observe also, that *Troy-weight* regulates and prescribes a form how to keep the money of *England* at a certain Standard: For about two hundred years before the conquest, *Osbright a Saxon*, being then King of *England*, caused an ounce *Troy* of silver to be divided into twenty pieces, at the same time called pence; and so an ounce of silver at that time was worth no more than twenty-pence, or one shilling eight-pence; which continued at the same value until the time of *Henry the Sixth*, who (in regard of the enhancing of money in foreign parts) valued the same at thirty-pence; so that then there were accordingly thirty pieces made out of the ounce, and the old pieces went then for three half-pence, 'till the time of *Edward the Fourth*, who valued the ounce at forty-pence, and then the old pieces went at two-pence apiece. After this *Henry the Eighth* valued the ounce of Sterling-silver at forty-five pence, which value continued 'till Queen *Elizabeth's* time, who valued the same old pence at three-pence the piece; so that all three-pences coined by the same Queen weigh'd but a penny-weight, and every six-pence two penny-weights; and the shilling and other pieces accordingly; which made the ounce *Troy* of silver to be valued at sixty-pence, or five shillings, as it remains at this day without alteration.

Note

Note, Jewels, Pearls, Diamonds, &c. are also weigh'd by *Troy-weight*, the Ounce being subdivided into 150 parts, called *Carats*, and each Carat into 4 *Grains*.

23. The weights used by Apothecaries are derived from a pound *Troy*, which is subdivided, as in the following Table.

A T A B L E of APOTHECARIES WEIGHTS.

$\frac{1}{16}$	<i>A Pound Troy</i>	{ is equal to }	12	<i>Ounces.</i>
$\frac{3}{16}$	<i>An Ounce</i>		8	<i>Drams.</i>
$\frac{3}{16}$	<i>A Dram</i>		3	<i>Scuples.</i>
$\frac{3}{16}$	<i>A Scruple</i>		20	<i>Grains.</i>

So that if you were to express, in Figures, twelve pounds ten ounces five drams two scruples and sixteen grains; also three pounds five ounces seven drams one scruple and two grains, the ordinary way to write them down is thus :

$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	gr.
12	10	5	2	.16
3	.05	.7	.1	.02

By these weights Apothecaries compound their medicines; but they buy and sell their drugs by *Avoirdupoise-weight*.

24. Besides *Troy-weight* before-mentioned, there is another kind of weight used in *England*, called *Avoirdupoise-weight*, a pound whereof is equal to fourteen ounces eleven penny-weights sixteen grains *Troy*. *Avoirdupoise-weight* serves to weigh all kinds of *Grocery-ware*, as also Butter, Cheese, Flesh, Tallow, Wax, Tobacco, Iron, Bras, Lead, Tin, Soap, Pitch, Tar, Rosin, Salt, Flax, Hemp, Drugs, and every other thing that bears the name of *Garbel*, and from which issues a refuse or waste.

25. *Avoirdupoise-weight* is either greater or less.

26. The greater is, when one hundred and twelve pounds *Avoirdupoise* are considered as one entire weight, commonly called an *Hundred-weight*, and then such *Hundred-weight* is subdivided first into four quarters, and each quarter into twenty-eight pounds : Again, each pound into four quarters, or (if you will be more exact) into sixteen ounces, and if you please each ounce into four quarters. But ordinarily a pound is the least quantity that is taken notice of, in *Avoirdupoise gross weights*.

A TABLE of AVOIRDUPOISE Greater WEIGHT.

28 Pounds	{ make {	1 Quarter of 112 lb.
4 Quarters		An Hundred-weight, or 112 lb.
20 Hundred weights		One Ton.

So that if you were to express, by Figures, eight hundreds three quarters and five pounds ; likewise, seven hundreds one quarter and seventeen pounds ; the ordinary way to set them down is thus :

C.	q.	lb.
8	3	.05
7	1	.17

27. The lesser Avoirdupoise-weight is, when a Pound is the highest name or integer, each pound being subdivided into sixteen ounces, and each ounce again into sixteen drams, and if you please each dram into four quarters, as by the subsequent Table is manifest.

A TABLE of AVOIRDUPOISE Lesser WEIGHT.

4 Quarters of a Dram	{ make {	1 Dram.
16 Drams		1 Ounce.
16 Ounces		1 Pound.

So that if you were to express, by Figures, eighteen pounds twelve ounces five drams and three quarters of a dram ; likewise five pounds no ounces twelve drams and one quarter of a dram ; the ordinary way to write them down is thus :

lb.	oz.	dr.	q.
18	.12	.05	.3
5	.00	.12	.1

N.B. A Stone of Meat in London, is 8 lb. Avoirdupoise ; but in some other places 14 lb.

A Stone of Wool, is 14 lb. and two Stones make a Todd.

A Stone Horseman's weight, is 14 lb.

A Stone of Hemp, is 32 lb.

A Fother of Lead is 19*1*/₂ C. wt.

28. The Measures used in England are either of Capacity, or Length.

29. The Measures of Capacity are those which are produced from weight, and they are either liquid or dry.

30. All Measures of Capacity, both liquid and dry, were at first made from *Troy-weight*: Vide *Statutes, 9 Henry III. 51 Henry III. 12 Henry VII. &c.* wherein it is enacted, that eight Pounds *Troy-weight* of wheat, gathered out of the middle of the ear, and well dried, should make one *Gallon of Wine-measure*: And that there should be but one *Measure for Wine, Ale and Corn*, throughout this Realm. (*Vid. Stat. 14 Edw. III. 15 Richard II.*) But time and custom hath altered Measures, as they have done Weights (and perhaps for one and the same reason); for now we have three different Measures, *viz.* one for *Wine*, one for *Ale or Beer*, and one for *Corn*.

31. The common *Wine-gallon*, seal'd at *Guild-hall* in *London*, by which all *Wines, Brandies, Spirits, Strong-waters, Mead, Perry, Cyder, Vinegar, Oil and Honey, &c.* are measured and sold, is supposed to contain 231 cubic Inches; and from thence the rest are computed, as in this Table.

A TABLE of WINE-MEASURE.

231 Cubic Inches	make	1 Gallon.
42 Gallons		1 Tierce.
63 Ditto		1 Hogshead.
84 Ditto		1 Puncheon.
2 Hogsheads		1 Pipe, or Butt.
2 Pipes		1 Tun.

Note, $31\frac{1}{2}$ Gallons is a *Wine or Vinegar Barrel*, and 236 Gallons is a *Tun of sweet Oil*.

32. The *Beer or Ale Gallon* (which are both one) is much larger than the *Wine Gallon*; it being probably made at first to correspond with *Avoirdupois-weight*, as the *Wine-gallon* did with *Troy-weight*: For one pound *Avoirdupois* being nearly equal to 14 ounces 12 penny-weights *Troy*: And as one pound *Troy* is in-proportion, to the cubic inches in a *Wine-gallon*, so is one pound *Avoirdupois* to the cubic inches in an *Ale-gallon*. That is, $12 : 14\frac{1}{2} :: 231 : 282$ nearly.

A TABLE of ALE-MEASURE.

282 Cubic Inches	make	1 Gallon.
8 Gallons		1 Firkin.
2 Firkins		1 Kilderkin.
2 Kilderkins		1 Barrel.
3 Ditto		1 Hogshead.

A TABLE of BEER-MEASURE.

282	Cubic Inches				I	Gallon.
9	Gallons				I	Firkin.
2	Firkins				I	Kilderkin.
2	Kilderkins				I	Barrel.
3	Ditto				I	Hoghead.
3	Barrels				I	But.

Note, A Firkin of Soap, and of Herrings, are the same with that of Ale.

33. This distinction or difference betwixt *Ale* and *Beer* Measure, is now only used in *London*. But in all other places of *England*, the following Table of *Beer* or *Ale*, whether it be strong or small, is to be observed, according to a Statute of Excise made in the year 1689.

282	Cubic Inches				I	Gallon.
8½	Gallons				I	Firkin.
2	Firkins				I	Kilderkin.
2	Kilderkins				I	Barrel.

Note, In all Measures liquid or dry.

2	Pints				I	Quart.
2	Quarts				I	Pottle.
2	Pottles				I	Gallon.

34. Dry measure is different both from *Wine* and *Ale* measure, being as it were a mean betwixt both, tho' not exactly so; which, upon examination, will be found to be in proportion to the aforesaid old standard *Wine-gallon*, as *Avoirdupoise-weight* is to *Troy-weight*; that is, as one pound *Troy* is to one pound *Avoirdupoise*, so is the cubic inches contained in the old *Wine-gallon*, to the cubic inches contained in the *dry* or *Corn-gallon*, viz. $12 : 14 \frac{1}{2} :: 224 : 272 \frac{1}{4}$, the common received content of a *Corn-gallon* nearly. Altho' now it is otherwise settled by an act of parliament, made in 1697, the words of that act are these: *Every round Bushel with a plain and even bottom, being made eighteen Inches and a half wide throughout, and eight Inches deep, should be esteemed a legal Winchester Bushel, according to the standard in his Majesty's Exchequer.* Now a vessel thus made, will contain 2150,42 cubic inches, consequently the *Corn-gallon* doth contain but 268 $\frac{1}{3}$ cubic Inches.

A TABLE of DRY or CORN-MEASURE.

268½	Cubic Inches	make	I	Gallon.
2	Gallons		I	Peck.
4	Pecks		I	Bushel.
8	Bushels		I	Quarter.

Note, When Salt and Sea-coal are measured by the Corn-bushel, they are heaped; 36 Bushels is a Chalder of Coals, and 21 Chalders a Score.

35. As the least part of Weight was originally a *Wheat-corn*, so the least part of Long-measure was a *Barley-corn*, taken out of the middle of the ear, and being well dried, three of them in length were to make one inch; and thence the rest, as in the following Table.

3	Barley-corns in length	make	I	Inch.
12	Inches		I	Foot.
3	Feet, or 16 Nails		I	Yard.
3	Feet 9 Inches		I	Ell.
6	Feet, or two Yards		I	Fathom.
5	Yards and an half		I	Pole, or Perch.
40	Poles, or Perches		I	Furlong.
8	Furlongs		I	English Mile.

A L S O,

16	2	4	10	1	Yard.
20	5	5	5	1	Ell English.
12	9	3	3	1	Flemish Ell, or Auln.

Note, That a Yard is usually subdivided into four Quarters, and each Quarter into four Nails.

And each Ell into four Quarters; but each Quarter of an Ell contains five Nails.

36. Superficial, or square Measures of Land, are such as are express'd in the following Table:

40	Square Poles, or	make	I	Rood, or Quarter of an Acre.
	Perches		I	Acre.
4	Roods		I	Mile.

For 40 Poles or Perches in length, and 4 in breadth do make a Statute Acre of Land; that is 220 yards multiplied by 22 yards,

yards, which is equal to 4840 square Yards are a Statute Acre.

Note. Land is best measured by a Chain of 4 Poles long, divided into 100 parts, called Links.

And if you would express, by Figures, these quantities of Land, viz. thirty-six Acres three Rods twenty Perches; also seven Acres no Rods thirty-two Perches, the ordinary way to set them down, is thus:

A.	R.	P.
36	3	20
7	0	32

37. A TABLE of TIME is this that follows:

60 Seconds	make	I Minute.
60 Minutes		I Hour.
24 Hours		I Day natural.
7 Days		I Week.
4 Weeks		I Month of 28 Days.
13 Months 1 Day and 6 Hours		I Year very near.

But in ordinary computations of time, the whole year, consisting of three hundred sixty-five days, is divided either into twelve equal parts or months; every month then containing thirty days and ten hours; or else into twelve unequal Kalendár-months, according to the ancient Verse:

*Thirty Days hath September, April, June, and November;
February hath twenty-eight alone, and each of the rest, thirty-one.*

Note. That every Leap-year (which happens once in four years) contains three hundred sixty-six days; and, in such year, February contains twenty-nine days.

38: Of Things accounted by the dozen, a Gross is the Integer, consisting of twelve dozens, each dozen containing twelve particulars. So that if you would express, in Figures, seven gross four dozens and five particulars; also four dozens and eight particulars, they may be written thus:

G.	D.	P.
7	4	5
0	4	8

C H A P. III.

ADDITION of Whole Numbers.

39. Concerning Notation of Numbers, and how thereby the quantities of things are usually expressed, a full declaration has been made in the preceding Chapters: Numeration follows, which comprehends all manner of operations by Numbers.

40. In Numeration, the four primary or fundamental operations are these, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

41. Addition is that, by which divers numbers are collected together, to the end that their sum, aggregate or total, may be discovered.

42. In Addition, place the numbers given one above another, in such sort, that like places or degrees in every number, may stand in the same rank; that is, Units above Units, Tens above Tens, Hundreds above Hundreds, &c. So these numbers 1213 and 462, being given to be added together, you 1213
462 are to order them as appears in the margin.

43. Having thus placed the numbers, and drawn a line under them, add them together, beginning with the units first, and saying thus, 2 and 3 make 5, which write under the rank of units; then proceed to the second rank, and say, 6 and 1 make 7, which write under the second rank (being the place of tens); again 4 and 2 make 6, which write under the third rank. Lastly, write down 1, being all that stands in the fourth rank; so the sum of the said given numbers is found to be 1675, and the operation will appear 1675 as in the margin.

In like manner, the numbers 2315, 7423, and 141, being given to be added together, their sum will be 9879, and the operation will stand as in the example.

$$\begin{array}{r} 2315 \\ 7423 \\ 141 \\ \hline 9879 \end{array}$$

44. When the sum of the figures of any of the ranks amounts to ten, or any number of tens without any excess, write down a cypher under that rank; but when the sum of any rank exceeds ten, or any number of tens, set down the excess under such rank; and for every ten contained in the sum of any rank, reserve an unit or 1 in your mind, and add such

such unit or units to the figures of the next rank towards the left-hand; so the numbers 4937, 9878, and 394, being given to be added together, the operation will be thus, *viz.* Beginning with the rank of units, 4, 8 and 7 make 19, wherefore write down 9, the excess above ten, and carry 1 in mind instead of the ten contained in the said 19: Then 1 and 9 (9 being the lowermost figure of the second rank) make 10, which added to 7 and 3, the other figures of the same rank, the whole sum of them is 20; therefore set down a cypher under the line in that rank, because the excess above the two tens is nothing; next, carry 2 to the third rank: 2 and 3 (3 being the lowermost figure of the third rank) make 5, which being added to 8 and 9 (the other figures of the same rank) the sum of them is 22; therefore writing down 2 (being the excess above the two tens) under the line in the third rank, carry 2 in mind (because there were two tens in 22) to the fourth rank: Lastly, 2 and 9 make 11, which added to 4 make 15; this 15, because it is the sum of the last rank, write totally down under the line, on the left-hand of the figures before subscribed; so the sum of the three numbers given, is found to be 15209, as in the example.

45. The reason of the above operations will be very evident from this undeniable maxim, *viz.* that the whole is equal to all its parts; and the method of setting down the total, may easily be accounted for, from the nature of numeration, which explains the different value of places as they proceed from the right, to the left-hand: For, as 9 is the greatest simple character or figure, so every number exceeding 9, being compound, must require more places than one to express it. Thus, the number 10 can no otherwise be expressed in figures, but by removing the figure into the place of tens, which is done by supplying the unit's place with a cypher: And as it is the same with every other column (ten being still the proportion of increase) consequently, when the sum of any column amounts to 10 or more, the units exceeding, if there be any, or a cypher, if none, must be set under such column, and the ten or tens of the amount carried on, as so many units, to the next column on the left.

What is here observed, as to carrying the tens (the proportion of increase) from one column to another in integers, may be as justly applied to the proper numbers in adding sums of different denominations.

This demonstration may be applied to the example work'd in Art. 44, as follows:

The Sum of	4,	8,	and	7	is	<u>19</u>
The Sum of	90,	70,	and	30	is	<u>190</u>
The Sum of	300,	800,	and	900	is	<u>2000</u>
The Sum of	9000,	and	4000	is	<u>13000</u>	

Therefore the Sums of 394, 9878, and 4937 is 15209

46. From the same maxim is deduced a method of proving the truth of any operation in Addition, *viz.* by parting or separating the given Numbers into two parcels (or more, according to the largeness of it), and then adding up each parcel by itself: For if those particular sums, so found, be added into one sum, and that sum prove equal, or the same with the total sum first found, then all is right; if not, care must be taken to discover and correct the error.

E X A M P L E.

Add	$\begin{array}{r} \left. \begin{array}{r} 5647 \\ 3289 \\ \hline 4916 \end{array} \right\} \\ \hline \begin{array}{r} 2900 \\ 5007 \\ \hline 1606 \end{array} \right\} \end{array}$	The Sum of these Parts is <u>12952</u>
		The Sum of these is <u>9513</u>

The total Sum of all these Parts 22465 The Sum of each Parcel put together 22465

Note. This last method may be usefully applied in the addition of long columns, in order to ease the memory; and the same may be proved by dividing them into different parcels.

47. Another method of proof is, to perform the operation again, beginning with the uppermost figure of each rank, and adding downwards.

Note. 1. The sum of two or more even numbers will be an even number.

2. The sum of two odd numbers will be an even number.

3. The sum of two numbers; one even, the other odd, will be an odd number.

4. If more than two odd numbers be added together; the sum will be even, when the number of its parts, so to be added together, is even; but odd, if the number of parts be odd.

5. If even numbers be intermixed with odd, the sum will be even or odd, according as the number of odd parts be even or odd.

48. When numbers given to be added, express things of diverse denominations ; first write them down orderly (according to the Examples in *Chap. II.*) then after a line is drawn under them all, begin to add the numbers of the least denomination, and if the sum of them amounts to one integer, or many integers of the next greater denomination, with some excess of the less denomination, set down that excess, or a cypher when there is no excess, under the line, so as it may stand under the least denomination, and keep the said integer or integers in mind, to be added to those of the next greater denomination on the left-hand ; but when the sum of the numbers of the least denomination does not amount to one integer of the next greater denomination, set down the sum itself under the line : Then add the integer or integers kept in mind (when any happens) to the numbers of the next greater denomination on the left-hand, and proceed to add them, as also those of every greater denomination, in like manner as above is directed ; until you come to the numbers of the greatest (or highest) denomination, which are to be added according to *Art. 43.* and *44.* So these several sums 24*l.* 13*s.* 5*d.* 3*f.* also 12*l.* 0*s.* 8*d.* and 5*l.* 18*s.* 0*d.* 2*f.* being propos'd to be added, their total sum is 42*l.* 12*s.* 2*d.* 1*f.* For having written them down orderly according to *Art. 21.* and drawn a line underneath, begin with the farthings first, and say, two farthings and three farthings make five farthings, that is one penny with a farthing over and above ; wherefore setting down 1 under the denomination of farthings, carry one penny to the denomination of pence ; then say, 1, 8, and 5 pence make 14 pence, which contain one shilling and two-pence ; wherefore writing two under the denomination of pence, carry one shilling to the denomination of shillings ; then adding the said 1 shilling to 18 shillings and 13 shillings, the sum will found 1 pound and 12 shillings ; wherefore setting down 12 under the denomination of shillings, carry 1 pound in mind to the denomination of pounds, saying, 1 pound in mind, together with 5, 2, and 4 pounds, which stand in the first rank of pounds, make 12 pounds ; wherefore (according to *Art. 44.*) write 2, the excess above 10, under the said first rank of pounds, and carry 1 in mind for the said 10 to the second rank of pounds ; then saying, in like manner, 1 in mind, together with 1 and 2, which stand in the second rank of pounds make 4, which write under the line ; that done, the total of the three given sums appears to be 42*l.* 12*s.* and 2*d.* 1*f.*

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
24	13	05	3
12	00	08	0
05	18	00	2
42	12	02	1

Chap. III. Of Whole Numbers.

19

In like manner 3 lb. 05 oz. 19 p.w. 15 gr. Also 2 lb. 0 oz. 3 p.w. 7 gr. Also 0 lb. 10 oz. 6 p.w. 0 gr. And 0 lb. 9 oz. 0 p.w. 17 gr. being given to be added together, their sum will be found 7 lb. 01 oz. 9 p.w. 15 gr. and the work will stand thus :

$$\begin{array}{r}
 3 \cdot 05 \cdot 19 \cdot 15 \\
 2 \cdot 00 \cdot 03 \cdot 07 \\
 0 \cdot 10 \cdot 06 \cdot 00 \\
 0 \cdot 09 \cdot 00 \cdot 17 \\
 \hline
 \end{array}$$

$$7 \cdot 01 \cdot 09 \cdot 15$$

Note. In adding together the numbers in the last example, it must be remember'd, that twenty-four grains make one penny-weight; twenty penny-weights one ounce; and twelve ounces one pound *Troy* (as before is declared in Art. 22.); and then you are to proceed according to Art. 48.

More Examples are these following, which presuppose the learner to be well exercised in the Tables of Chap. II. that he may readily know what integers are to be carried from every lesser denomination to the next greater.

ADDITION OF ENGLISH MONEY.

lb.	s.	d.	f.	lb.	s.	d.
230	17	10	1	0	13	05
175	12	11	3	0	17	08
052	05	06	0	0	00	10
009	00	08	1	0	10	03
596	13	07	2	0	15	06
<hr/>				<hr/>		
974	10	07	3	2	17	08

ADDITION OF TROY-WEIGHT.

lb.	oz.	p.w.	gr.	oz.	p.w.	gr.
23	07	16	13	536	13	16
17	10	15	07	206	11	10
325	06	19	20	68	00	05
49	11	07	12	96	00	12
<hr/>				<hr/>		
417	00	19	04	907	05	19

ADDITION of AVOIRDUPOISE-WEIGHT.

C.	q.	lb.	lb.	oz.	dr.
235	3	13	14	13	12
576	1	17	5	10	13
628	2	15	12	00	06
412	0	10	06	09	05
<hr/>			<hr/>		
1852	3	27	39	02	04

ADDITION of MEASURES of LENGTH.

Yards.	q.	nails.	Ells.	q.	n.
26	3	2	15	3	2
13	1	3	16	1	3
12	0	1	09	0	1
29	1	1	12	2	1
<hr/>			<hr/>		
81	2	3	53	2	3

ADDITION of SUPERFICIAL MEASURES of LAND.

Acres.	Roods.	Per.	A.	R.	P.
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	10
<hr/>			<hr/>		
1379	3	09	996	3	36

49. The following Questions are designed to put the learner upon reflection, which will enable him the better to see the nature and use of this Rule.

1. A man was born in the year 1721; when will he be 45 years of age? *Answe.* in the year 1766.
2. If after having paid 12*l.* 10*s.* in part of a debt, there still remains due 17*l.* 10*s.* what was that debt? *Answe.* 30*l.*
3. There are two numbers, whose difference is 17, and the lesser number is 44; what is the greater number? *Answe.* 61.
4. A man has 6 bags of Hops; the first weighs 2*qu.* 14*lb.* and each of the others weighs 1*4 lb.* what quantity has he in all? *Answe.* 1*C.* 1*qu.*

5. Re-

5. Received of *A* three half-pence, of *B* seven farthings, of *C* fifteen-pence, of *D* five groats, of *E* half a crown, and of *F* thirty shillings; how much was received? *Answ.* 1*l.* 15*s.*
8*1**d.*

6. Bought a parcel of goods for 40*l.* 10*s.* paid for packing them 13*s.* for carriage 1*l.* 6*s.* 8*d.* and expended at making the bargain 15*s.* 6*d.* what do the goods stand me in? *Answ.* 43*l.* 5*s.* 2*d.*

7. Between *London* and *Royton* are 33 miles; from thence to *Cambridge* 10; thence to *Newmarket* 10; thence to *Bury* 10; and from thence to *Norwich* 32 miles: How many miles on this road from *London* to *Norwich*? *Answ.* 95 miles.

8. How many days are there in the 12 Calendar months, *January* having 31 days, *February* 28, *March* 31, *April* 30, *May* 31, *June* 30, *July* 31, *August* 31, *September* 30, *October* 31, *November* 30, and *December* 31 days? *Answ.* 365.

9. How many days are there from the 28th day of *March* to the 17th day of *December* in the same year, both days inclusive? *Answ.* 265.

10. How many days are there from the 19th day of *April*, 1748, to the 27th day of *November*, 1750, both days exclusive? *Answ.* 951.

11. A merchant, in the year 1750, imported, by one ship 8 tuns of Claret, by another 12 tuns 3 hogsheads 42 gallons of Red Port, by another 14 tuns 2 hogsheads 11 gallons of Sherry, and by another 5 tuns 3 hogsheads 10 gallons of Canary; what quantity of wines did he import in all? *Answ.* 41 tuns 1 hogshead.

12. From the creation of the world to the beginning of the deluge is accounted 1656 years; and from the beginning of the deluge to the birth of *Arphaxad*, *Helvicus* reckons 2 years; and thence, to *Terah* 220 years; thence to the birth of *Abraham* 70 years (*Gen. 11.*); thence to the promise given (mentioned *Gen. 12.*) he reckons 75 years; and thence to the going out of *Egypt* 430 years; and from that going out, to the Temple of *Solomon* 480 years; and thence to the birth of *Cbrift* 1015 years; and he supposes, that from thence, to the beginning of the common Christian or *Dionysian* *Æra* 2 years elapsed; and thence, to the present year wherein this was writ, we reckon 1750 years: According to the Chronology of *Helvicus*, therefore, how long is it since the creation of the world? *Answ.* 5700 years.

13. A surveyor, having measured 5 several pieces of land, finds one of them to contain 7 acres 3 roods 24 perches, another to contain 18 acres 1 rood 16 perches, another 12 acres

10 perches, and the last 15 acres 2 roods; how many acres were surveyed in all? *Ans*w. 53 acres, 3 roods, 10 perches.

14. In the year 1563, 20000 persons died of the plague in London; in 1593, 10635 died of the same distemper; in 1603, 30578; in 1625, 54265, of the plague but 35417; in 1665, 97351 died, and of them 68586 of the plague; how many died of the plague between the years 1563 and 1665, both years included? *Ans*w. 165216.

C H A P. IV.

S U B T R A C T I O N of Whole Numbers.

50. **S**UBTRACTION is that by which one number is taken out of another, to the end that the remainder or difference between the two numbers given may be known.

51. The number out of which the Subtraction is to be made, must be greater, or at least, equal with the other. For you may subtract 4347 out of 9478, but you cannot subtract 9478 out of 4347.

52. In Subtraction rank the two given numbers one under the other, as in Addition, with this caution, that the number placed uppermost may exceed, or at least be equal to the other: So if the number 4347 be given to be subtracted from 9478, place them as in the margin: Then proceeding to the Subtraction, say, 7 taken out of 8, there remains 1, 9478 which place in the same rank under the line. In like manner, 4 being taken out of 7, the remainder is 3, — which likewise set under the line in the next rank; 5131 again, taking 3 from 4, the remainder is 1, which place under the third rank: Lastly, subtracting 4 from 9, there will remain 5, which subscribe under the fourth rank; now the whole operation being finished, it appears that if 4347 be taken out of 9478, the remainder is 5131, or (which is the same) the difference between the numbers 9478 and 4347 is 5131, as in the Example.

In like manner, if 106 be subtracted from 2856, the remainder will be found 2750; for after the numbers are orderly ranked, begin at the place of units, and say 6 from 6, there remains nothing; therefore subscribe o (or nothing); 2856 then proceeding to the second rank, say, if o be taken 106 from 5, there will remain 5, which also subscribe under the line; again, 1 from 8, there remains 7: Lastly, o 2750 from 2, there remains 2. See the Work in the margin.

53. When any of the figures of the number given to be subtracted is greater than the upper figure out of which it is to be subtracted,

subtracted, you must borrow 10 of the next rank towards the left-hand, and add the said 10 to the said upper figure; then from the sum of such Addition subtract the lower figure and set down the remainder: In this case the figure of the next rank which is to be subtracted, must be esteemed an unit greater than it is; therefore keeping 1 in your mind, add it to the next figure of the number given to be subtracted, and deducting all out of the figure above it, proceed in like sort 'till you have finished the whole operation. *Example,* Let it be required to subtract 374 out of 8023. Having ranked them as before, say, 4 out of 3, that cannot be, wherefore borrowing 10 of the next rank, and adding the same to the said 3, say, 4 out of 13, there remains 9; then writing 9 under the line, and carrying 1 in mind, say, 1 and 7 make 8, 8 out of 2 that cannot be, but 8 out of 12 (12 because 10 being borrowed, and added to 2, makes 12) there remains 4, which subscribe under the line; again 1 in mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6, which likewise subscribe under the line; lastly, 1 in mind being taken out of 8, there remains 7. Thus you see that the remainder after 374 is subtracted from 8023, is 7649. Note diligently, that as often as 10 is borrowed, 1 must be kept in mind to be added to the figure standing in the next place of the lower number, and the sum of such Addition must be subtracted from the upper place; but if it happen that there is no figure in the next place of the lower number, then the 1 in mind must be subtracted from the upper place; (as in the last rank of the last Example.) *Another Example,* Let it be required to subtract 92 from 62801. Having placed the greater number uppermost, and the lesser orderly underneath, begin at the place of units, and say, 2 from 1 that cannot be, but borrowing 10, and adding it to the said 1, say, 2 from 11 there remains 9, which subscribe under the line; then proceed and say, 1 in mind with 9, makes 10, 10 out of 0 that cannot be, but borrowing 10, say, 10 out of 10 and there remains 0, wherefore subscribe 0 under the line; again, 1 in mind out of 8, there remains 7; then because there are no figures in the lower number, say 0 out of 2 there remains 2; lastly, 0 out of 6 there remains 6; therefore it appears that 62801 exceeds 92 by 62709.

54. Those who have considered Art. 45. and are thence satisfied of the reason of the operations used in Addition, will easily perceive that the same maxim may be applied to the work of

Subtraction, as directed in Art. 52. And the truth of the operations in Art. 53. will appear from the following

Proposition: *If to each of two numbers, the same or an equal number be added, the difference between the numbers resulting will be the same with the difference of the first two numbers.* Example:

If to 14 and 6 whose difference is 8,
the number 3 . . 3 be added; then,

The Results 17 and 9 also differ by 8.

Now, in Art. 53. by taking 4 out of 13 (instead of 3) 10 is added to the upper number, and by carrying in mind 1, and adding it to the figure which stands in the place of tens in the lower number, ten is added to that lower number, therefore the true difference will thence arise by the proposition.

55. If the numbers proposed have diverse denominations, place them as before, and beginning with the least denomination first, subtract the lower number from the upper when it may be subtracted, and place the remainder underneath; but if it happens that the lower number cannot be taken out of the upper, you must borrow an integer of the next greater denomination on the left-hand; which integer, after it is converted into the same denomination with the said upper number, must be added to it: Then from the sum of such Addition, you are to subtract the lower number, and write down the remainder, keeping 1 (that is the integer borrowed) in your mind, to be added to the next place of the number given to be subtracted, as before: So 90*l.* 14*s.* 10*d.* 3*f.* being subtracted from 124*l.* 11*s.* 7*d.* 1*f.* the remainder is 33*l.* 16*s.* 8*d.* 2*f.* For beginning with the farthings, say, 3 farthings out of 1 farthing cannot be taken, therefore borrowing 1 penny (that is an integer of the next greater denomination) and having converted this penny into 4 farthings, add them to the aforesaid 1 farthing; so the sum is 5 farthings, out of which subtracting 3 farthings, there remain 2 farthings, which place under the denomination of farthings; then proceed to the next denomination, and say, 1 penny borrowed and 10*d.* make 11*d.* this 11*d.* out of 7*d.* cannot be taken; therefore borrowing 1 shilling, or 12*d.* and adding 12*d.* to the said 7*d.* the sum is 19*d.* from which subtract the said 11*d.* so there remains 8*d.* which subscribe under the denomination of pence: Again, 1 shilling borrowed, being added to 14*s.* makes 15*s.* which cannot be subtracted out of 11*s.* and therefore borrow 1 pound or 20*s.* which being added to the said 11*s.* make 31*s.* from

$$\begin{array}{r}
 l. \quad s. \quad d. \quad f. \\
 124. \quad 11. \quad 07. \quad 1 \\
 - 90. \quad 14. \quad 10. \quad 3 \\
 \hline
 33. \quad 16. \quad 08. \quad 2
 \end{array}$$

from which subtracting 15*s.* there remains 16*s.* which subscribe under the denomination of shillings; then carrying 1 pound, borrowed, to the lower place of pounds, say, 1 in mind with 0 make 1, which taken out of 4, there remains 3; again, 9 out of 2 cannot be taken, but 9 out of 12 (10 being borrowed and added to the said 2, according to Art. 53.) and there remains 3. Lastly, 1 (for the 10 that was borrowed) being taken out of 1, there remains nothing; whence if *A* being indebted to *B*, in 124*l.* 11*s.* 7*d.* 1*f.* has paid in part thereof 90*l.* 14*s.* 10*d.* 3*f.* there remains yet undischarged 33*l.* 16*s.* 8*d.* 2*f.*

56. When many numbers are given to be subtracted from a number propounded, you must first add those numbers together, according to the rules of the third chapter, and then the sum found is to be subtracted from the number first given. Example, *A* being indebted to *B*, in 3240*l.* paid thereof at one time 700*l.* at a second payment 1236*l.* and at a third 305*l.* the question is, how much of the debt remain'd undischarged? First, add together the several sums paid, and find the total 2241*l.* this subtract from 3240*l.* there remains 999*l.* undischarged. See the operation in the margin.

Another Example of the like nature. *A* being indebted to *B* in 500*l.* paid in part thereof at one payment 340*l.* 12*s.* 6*d.* at a second payment 13*l.* 18*s.* 3*d.* at a third 17*l.* 16*s.* 10*d.* the question is, how much was in arrear? Here, if the operation be prosecuted as before, it will appear that there was 127*l.* 12*s.* 5*d.* unpaid. See the Work in the margin.

I.	s.	d.
<i>The Debt</i>	500 . 00 . 00	
<i>Payments</i>	340 . 12 . 06	
	13 . 18 . 03	
	17 . 16 . 10	
<i>Paid in all</i>	372 . 07 . 07	

57. Addition may be proved by Subtraction, and Subtraction by Addition. For having added divers numbers together, if you subtract one of them out of the sum, the remainder must be equal to all the rest, as you may observe by the Example following, viz. supposing these 4 numbers are given to be added, viz. 236, 452, 29, 217, and that their sum is found to be 934 (by the rules of the third chapter); it is required to prove

prove whether the said sum be true or not; to perform this, draw a line under the uppermost number 236, to separate it from the rest, and seek the sum of all the numbers given, except that uppermost, which sum is 698. Then subtract the said uppermost number 236, from 934, (the total sum of all the numbers first found) and because the remainder 698 is the same with the sum of all the numbers, excluding the uppermost, it may be concluded, that the sum of all the numbers, first found, was truly computed.

236	
452	934
29	236
217	698
934	
698	

In like manner Subtraction is proved by Addition; for if you add the remainder and the number given to be subtracted together, the sum must be equal to the number out of which the Subtraction is made; so if 4347 be subtracted from 9478, the remainder is 5131; for if 5131 be added to 4347, the sum is 9478, which is the same with the number out of which the Subtraction was made.

Example 1.		Example 2.	
l.	s. d.	l.	s. d.
out of	9478	24 . 13 . 07	
subtract.	4347	19 . 15 . 08	
Rest	5131	04 . 17 . 11	
Proof	9478	24 . 13 . 07	

Again, if a servant receive 24*l.* 13*s.* 7*d.* and lay out or disburse 19*l.* 15*s.* 8*d.* there must remain in his hands 4*l.* 17*s.* 11*d.* for this being added to 19*l.* 15*s.* 8*d.* which was the money he expended, the sum will be equal to 24*l.* 13*s.* 7*d.* (being the money with which he was first charged.)

Note 1. The difference of two even numbers, will be an even number.

2. The difference of two odd numbers, will be an even number.

3. The difference of two numbers, the one even, and the other odd, will be an odd number.

58. More Examples of Subtraction are these that follow:

S U B T R A C T I O N o f E N G L I S H M O N E Y.

	l.	s.	d.	f.		l.	s.	d.	f.
Rec.	3090	. 10	. 07	. 1		24	. 00	. 00	. 0
Paid.	0099	. 14	. 08	. 3		05	. 17	. 11	. 3
Rest	2990	. 15	. 10	. 2		18	. 02	. 90	. 1
Proof	3090	. 10	. 07	. 1		24	. 00	. 00	. 0

S U B T R A C T I O N of T R O Y - W E I G H T .

	lb.	oz.	p.w.	gr.	lb.	oz.	p.w.	gr.
Bought	352	10	13	19	205	13	13	19
Sold	019	41	16	18	118	16	20	
<hr/>								
Rest	332	10	10	71	86	16	23	
<hr/>								
Proof	352	10	13	19	205	13	13	19

S U B T R A C T I O N of A V O I R D U P O I S E - W E I G H T .

	C.	q.	lb.	lb.	oz.	dr.
Bought	256	2	23	25	13	12
Sold	079	3	26	00	14	13
<hr/>						
Rest	176	2	25	24	14	15
<hr/>						
Proof	256	2	23	25	13	12

S U B T R A C T I O N of S U P E R F I C I A L M E A S U R E S o f L A N D .

	Acre,	Rods,	P.	A.	R.	P.
Bought	780	2	35	2040	1	20
Sold	090	3	36	919	3	30
<hr/>						
Rest	689	2	39	1120	1	30
<hr/>						
Proof	780	2	35	2040	1	20

59. Questions to exercise Addition and Subtraction.

Quest. 1. Two persons *A* and *B* are of several ages, the age of the elder, being that of *A* is 70, the difference of their ages is 19, what is the age of *B*? Answ. 51.

Quest. 2. What number is that which being added to 168, makes the sum to be 205? Answ. 37.

Quest. 3. The sum of two numbers is 517, the lesser is 40, what is the greater? Answ. 477.

Quest. 4. A certain person born in the year of our Lord 1616, desired to know his age in the year 1676, what was his age? Answ. 60.

Quest. 5. The greater of two numbers is 130, their difference is 40, what is the lesser number? Answ. 81.

Quest. 6. What number of pounds, shillings, and pence, added to 34 l. 16 s. 9 d. will make 100 l.? Answ. 65 l. 3 s. 3 d.

Quest. 7. How many years since the Spanish Invasion, it being in the year 1588, and the present year being 1750? Answ. 162 years.

Quest. 8. From 100 pounds borrow'd, take 72 paid;
'Twas a Virgin that lent it, what's due to the Maid?

Answ. 28 l.

Quest. 9. A Miser hath three bags of money, containing in all 2984 l. 6 s. of which the first contains 324 l. 10 s. and the second 913 l. 0 s. 6 d. what doth the third contain? *Answ.* 1746 l. 15 s. 6 d.

Quest. 10. A Merchant had 5 debtors, *A*, *B*, *C*, *D*, and *E*, which, together, owe him 1156 l. now *B*, *C*, *D*, and *E*, together, owe him 737 l. what is *A*'s debt? *Answ.* 419 l.

Quest. 11. The three Towns of *London*, *Huntingdon*, and *York*, lie in the same road; the distance between the farthest of these Towns, *viz.* *London* and *York*, is 192 miles; now, if from *London* to *Huntingdon* be 57 miles, how far is it from *Huntingdon* to *York*? *Answ.* 135 miles.

CHAP. V.

MULTIPLICATION of Whole Numbers.

60. **M**ULTIPLICATION teaches how by two Numbers given to find a third, which shall contain either of the Numbers given so many times, as the other contains 1 or unity: Or Multiplication may be considered as a manifold addition, or the repeating of a given Number as often as required.

61. Of the two Numbers given in Multiplication, one (which you will) is called the **Multiplicand**, and the other the **Multiplier**, or both are called **Factors**.

62. The Number sought, or arising by the Multiplication of the two Numbers given, is called the **Product**, **the Fact**, or the **Rectangle**: So if 5 be given to be multiplied by 3, or 3 by 5, the Product is 15; that is, 3 times 5, or 5 times 3 makes 15; and here 5 may be called the **Multiplicand**, and 3 the **Multiplier**, or 3 may be called the **Multiplicand**, and 5 the **Multiplier**; and as 3 (one of the two numbers given) contains 1 or unity thrice, so 15 the Product contains 5 (the other given number) thrice; likewise as 5 (one of the given numbers) contains unity 5 times, so 15 (the Product) contains 3 (the other given Number) 5 times: This same Product may be found by Addition two ways, *viz.* either by writing down the number 5, three

three times ; or the number 3, five times ; and adding them together, as below.

$$\begin{array}{r}
 5 \\
 5 \\
 5 \\
 \hline
 25 \\
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 3 \\
 3 \\
 3 \\
 3 \\
 \hline
 15
 \end{array}$$

63. Multiplication is either single or compound.

64. Single Multiplication is, when the Multiplicand and Multiplier consist each of them of one figure only, as in the last example. In like manner if you multiply 9 by 5, the product is 45 ; this is likewise single Multiplication : Now the several Varieties of single Multiplication are well expressed in the following Table, usually called *Pythagoras's Table*. The truth of which may be proved by Addition, as above.

The TABLE of MULTIPLICATION.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the Table is this : Having one figure given to be multiplied by another, to know the product of them, find the Multiplicand in the top of the Table, and the Multiplier in the first column thereof towards the left-hand ; then the product will be found on the same line with the latter, and under the former. So 9 being given to be multiplied by 5, I find 9 in the top of the Table, and 5 in the first column towards the left-hand ; then carrying my eye from 5 in a right line equidistant to the upper side or top line of the Table, until I come to that square which is directly under 9, I find there 45, which is the product required. The particular varieties of this Table ought to be learned by heart, (that is, a man must be able to give the product of any single Multiplication, without the least pause

or stay) before he can readily work compound Multiplication, as will appear hereafter.

- Note 1.* The product of any number by an even number, will be an even number.
 2. The product of an odd number by an odd number, will be an odd number.
 3. The product of any two numbers can have, at most, but as many places of figures as are in both Factors; and, at least, but one place fewer. *Examp.* $9+9=81$, and $1+1=1$.
65. Compound Multiplication, is when the Multiplier and Multiplicand, either one or both, consist of more figures than one.

66. In Compound Multiplication, when the numbers given end with significant figures, place them as in *Addition* and *Subtraction*. So 134 being given to be multiplied by 2, place them thus: Then proceeding to the Multiplication, say thus, 2 times 4 is 8, which set under the line in the rank of your multiplying figure; again say 2 times 3 is 6, which likewise set under the line in the next rank: Lastly, 2 times 1 is 2, which being likewise set down under the line in the next rank, the product is discovered to be 268, and the work will stand as in the margin.

67. The truth of this process may be made evident: Thus,
- | | |
|------------------------------------|-----|
| Since 100 multiplied by 2 produces | 200 |
| 30 multiplied by 2 produces | 60 |
| And 4 multiplied by 2 produces | 8 |

Therefore 134 multiplied by 2 produces 268

For since the parts 100, 30, and 4, added together, make the whole 134; therefore the products of each of those parts, being added together, will be the product of the whole.

68. When the Multiplier consists of more figures than one, as many figures as it has, so many several products must be set down under the line, which at last being added into one sum, will give you the total product of all. So 1232 being given to be multiplied by 23, the operation will stand thus; 1232 being multiplied by 3 (according to the last rule) the product is 3696. Again, 1232 being multiplied by 2, the product is 2464, which several products after they are placed in their due order, (that is, the first figure arising in every product under its respective multiplying figure) and added together, produce 28336, the product required: In like manner 1321 being given to be multiplied by 123, the pro-
- | | |
|-------|--------|
| 1232 | 123 |
| 3696 | 3963 |
| 2464 | 2642 |
| 28336 | 1321 |
| | 162483 |

duct

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duct is 162483, and the operation will appear as in the margin.

69. The product of 1232 by 23, is equal to the product of 1232 by 20 and the product of 1232 by 3, added together. See *Examp. I. Art. 68.*

But 1232 multiplied by 20 produces	24640
And 1232 multiplied by 3 produces	<u>3696</u>

Therefore 1232 multiplied by 23 produces 28336

70. When the product of any of the particular figures exceeds ten, place the excess under the line, as before, and for every ten that it so exceeds, keep one in mind to be added to the next rank; as was taught in Addition.

Example, 3084 being given to be multiplied by 36, the work will stand thus; 6 times 4 being 24, set 4 under the line, and reserve 2 in mind for the two tens; then say 6 times 8 is 48, to which add 2 kept in mind, the whole is 50; therefore set down 0 in the next rank under the line (0 because there is no excess of 50 above 5 tens) and keep 5 in mind for the 5 tens; again, say 6 times nothing is nothing, to which adding 5 that was kept in mind, the whole will be but 5, which set down under the line in the next rank; again, 6 times 3 is 18, which (in regard 3 is the last figure of the Multiplicand) set wholly down; so that the particular product arising from the multiplying by the figure 6 is 18504: In like manner proceeding with the multiplying figure 3, the particular product arising will be 9252. Lastly, these several products being placed in due order, and added together (after the manner of the last Article) will give 111024, which is the total product arising from the Multiplication of 3084 by 36, and the operation will stand as in the margin. After the same manner, if 5073 be given to be multiplied by 256, the product will be found to be 1298688, and the operation will stand as you see in the example.

$$\begin{array}{r}
 5073 \\
 \times 256 \\
 \hline
 30438 \\
 25365 \\
 \hline
 1298688
 \end{array}$$

71. When the two numbers given to be multiplied, do, one or both of them, end with a cypher or cyphers towards the right-hand, multiply the significant figures in both numbers, one by the other, neglecting such cyphers; and when the Multiplication of the significant figures is finished, annex, on the right-hand of the number produced by the Multiplication, the cypher or cyphers, with which one or both of the numbers first given did end, so will the whole give you the true product demanded. *Example*, 43100 being given to be multiplied by 15000, the product will be found 646500000; for, omitting the

the cyphers which stand in the last places towards the right-hand, as well in the Multiplicand as the Multiplier, multiply the significant figures 431, by the figures 15 (according to the former rules) so there will arise 6465, to which annexing on the right-hand all the cyphers before omitted, the true product will be 646500000. More examples hereof are these following :

$$\begin{array}{r}
 43100 \\
 1500 \\
 \hline
 2155 \\
 431 \\
 \hline
 646500000
 \end{array}$$

$$\begin{array}{r}
 43125 \\
 1500 \\
 \hline
 215625 \\
 43125 \\
 \hline
 64687500
 \end{array}
 \quad
 \begin{array}{r}
 5108000 \\
 125 \\
 \hline
 25540 \\
 10216 \\
 5108 \\
 \hline
 638500000
 \end{array}$$

72. When, in the Multiplier, cyphers are included between significant figures, multiply by the said significant figures, neglecting such cyphers or cypher, but observe diligently to set the particular products of the significant figures in their due places, according to Art. 68. So if 56324 be given to be multiplied by 20006, first multiply the whole Multiplicand 56324 by 6, and place the product orderly under the line ; then passing over the three cyphers, multiply 56324 by 2, and place 8 (which is the first figure of this particular product) directly under the multiplying figure 2, and the rest in their order ; so at last the true product will be found to be 1126817944, and the Work will stand as in the example.

$$\begin{array}{r}
 56324 \\
 20006 \\
 \hline
 337944 \\
 112648 \\
 \hline
 1126817944
 \end{array}$$

More Examples hereof are these that follow :

$$\begin{array}{r}
 3094 \\
 104 \\
 \hline
 12376 \\
 3094 \\
 \hline
 321776
 \end{array}
 \quad
 \begin{array}{r}
 23765 \\
 10302 \\
 \hline
 47530 \\
 71295 \\
 23765 \\
 \hline
 244827030
 \end{array}$$

Note. That one of the principal cautions, to be observed in Multiplication, is the due placing of the particular products arising by each multiplying figure ; and that may be performed, either, by taking care to place the first figure or cypher which arises

arises in each product under the respective multiplying figure ; or at least the first place arising in the second product must stand under the second place of the first product, and the first place of the third particular product, under the third place of the first, &c.

73. When a number is given to be multiplied by a number that consists of 1 (or an unit) in the first place towards the left-hand, and a cypher or cyphers on the right-hand of such unit (as are 10, 100, 1000, 10000, &c.) the Multiplication is performed by annexing the cypher or cyphers of the Multiplier at the end (to wit, on the right-hand) of the Multiplicand ; so if 326 be given to be multiplied by 10, the product is 3260 ; if by 100, the product is 32600 ; if by 1000, the product is 326000 ; in like manner if 170 be multiplied by 10, the product is 1700 ; if by 100, 17000, &c.

74. Hence if it be required to multiply by 5, add a cypher to the Multiplicand and take $\frac{1}{2}$ the result, so $326 \times 5 = \frac{3260}{2} = 1630$.

75. How to multiply by any number under 20, so as to bring the whole operation into one line.

Rule. Multiply each figure in the Multiplicand by the unit figure in the Multiplier, adding to each its back figure ; as in the following Examples.

$$\begin{array}{r} 47947 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 623311 \\ \hline \end{array}$$

$$\begin{array}{r} 27942 \\ \times 17 \\ \hline \end{array}$$

$$\begin{array}{r} 475014 \\ \hline \end{array}$$

Which are done thus ; viz. 3 times 7 is 21, set down 1 and carry 2 ; then 3 times 4 is 12, and 2 is 14, and 7 (which is the back figure to 4) is 21, 1 and carry 2 ; 3 times 9 is 27, and 2 is 29, and 4 (the back figure) is 33, 3 and carry 3 ; 3 times 7 is 21, and 3 is 24, and 9 (the back figure) is 33, 3 and carry 3 ; 3 times 4 is 12, and 3 is 15, and 7 is 22, 2 and carry 2, which added to 4 makes 6.

76. In like manner the whole operation of the Multiplication of Numbers between 20 and 30, are brought into one line, by taking in the double of the back figure, and between 30 and 40, the treble of it, &c.

By which Rule in multiplying by Numbers compounded of those under 20, the product is readily found thus :

By 112, either multiply by 2 and 11, or by 12 and 1 ; as,

$$\begin{array}{r}
 94267 \\
 112 \\
 \hline
 188534 \\
 1036937 \\
 \hline
 10557904
 \end{array}
 \qquad
 \begin{array}{r}
 94267 \\
 112 \\
 \hline
 1131204 \\
 94267 \\
 \hline
 10557904
 \end{array}$$

Also in multiplying by 1614, and 1715; in the first multiply by 14 in one line, and by 16 in the other: And in the latter by 15 in one line, and 17 in the other, as above directed; observing always to place the first figure of a line under its own Multiplier; as

$$\begin{array}{r}
 479479 \\
 1614 \\
 \hline
 6712706 \\
 7671664 \\
 \hline
 773879106
 \end{array}
 \qquad
 \begin{array}{r}
 479479 \\
 1715 \\
 \hline
 7192785 \\
 8151143 \\
 \hline
 822306485
 \end{array}$$

77. Otherwise for any number under 20, a method not so troublesome to the memory.

Rule. Multiply by the unit figure, setting that whole product one figure back on the right-hand, which added to the Multiplicand, gives the product, viz.

$$\begin{array}{r}
 47947 \\
 13 \\
 \hline
 143841 \\
 \hline
 623311
 \end{array}
 \qquad
 \begin{array}{r}
 27642 \\
 17 \\
 \hline
 193494 \\
 \hline
 469924
 \end{array}
 \qquad
 \begin{array}{r}
 479479 \text{ by } 14 \\
 1917916 \\
 \hline
 6712706
 \end{array}$$

Note. This will be still easier, if the Multiplier be not wrote under the Multiplicand, as in the last Example.

But for 112, multiply by 12, setting the product two figures back, which likewise added to the Multiplicand, gives the product, as,

$$\begin{array}{r}
 94267 \\
 112 \\
 \hline
 1131204 \\
 \hline
 10557904
 \end{array}
 \qquad
 \begin{array}{r}
 94267 \text{ by } 112 \\
 1131204 \\
 \hline
 10557904
 \end{array}$$

78. In like manner, if the Multiplier be 21, 31, 41, &c. let the Multiplicand stand for its product by unity, and write under it, in the proper place, the product by the other figure. For example, the operation of multiplying 365 by 31 may stand as below.

$$\begin{array}{r} 365 \\ \times 31 \\ \hline 1095 \\ 11315 \end{array}$$

79. When it is required to multiply by a number consisting of nines, *viz.* 9, 99, 999, 9999, &c. write as many cyphers as there are nines in the Multiplier to the right-hand of the Multiplicand, and from the result subtract the Multiplicand, and the remainder will be the required product.

Examp. 1. Let it be required to multiply 456 by 9 ?
One cypher annex'd to 456 makes $4560 = 456 \times 10$. Art. 73.
From which subtract the Multiplicand $\underline{456} = 456 \times \underline{1}$

The remainder is the product $\underline{4104} = 456 \times 10 - 1$

Examp. 2. Required the product of 456 by 99 ?
Two cyphers annex'd to 456 make $45600 = 456 \times 100$
From which subtract $\underline{456} = 456 \times \underline{1}$

The remainder is the product $\underline{45144} = 456 \times 100 - 1$

Examp. 3. What is the product of 456 by 999 ?
Three cyphers annex'd to 456 make $456000 = 456 \times 1000$
From which take $\underline{456} = 456 \times \underline{1}$

The remainder is the answer $\underline{455544} = 456 \times 1000 - 1$

80. When it is required to multiply by a number consisting of any digit repeated *viz.* 11, 111, 1111, &c. or 22, 222, 2222, &c. or 33, 333, 3333, &c. &c. find by Art. 79. the product of the given Multiplicand by 99, 999, 9999, &c: and take the ninth part thereof, * that is divide the product by 9, then the result multiplied by the digit which repeats in the given Multiplier will be the product required.

Examp. 1. Let it be required to multiply 456 by 444 ?
The product of 456 by 999 is $\underline{455544} = 456 \times 999$ Art. 79.
The ninth part thereof is $\underline{50616} = 456 \times (\frac{1}{9}) = 111$
Multiply by the repeating digit $\underline{4}$
The product is the answer $\underline{202464} = 456 \times (111 \times 4) = 444$.

* If the Reader hath not learnt how to divide, let him postpone the practice of this Rule till he hath read the next Chapter.

Examp. 2. Multiply 4538769 by 7777777 ?
 Seven Cyphers annexed to the Multiplicand _____ } = 45387690000000

Subtract the Multiplicand	4538769
Remains	<u>45387685461231</u>
$\frac{1}{5}$ of the remainder	5043076162359
Multiply by the digit	<u>7</u>
The answer	35301533136513

81. To multiply by a Factor consisting of so many cyphers, between two digits, as there are places in the Multiplicand.

$$\begin{array}{r}
 34567 \\
 6000005 \\
 \hline
 \text{Prod.} & 207402172835
 \end{array}$$

In this and the like cases there is no more difficulty than in multiplying by a single digit ; for the number of cyphers being equal to the places in the Multiplicand, the product by the 6 falls just to the left-hand of the product by the 5 ; but if the product of the digit next the left-hand by that in the unit's place, and what is carried, be less than 10, then a cypher must be put down between the two products.

82. The following is an universal method for all cases ; whereby tho' there is not any contraction, and even some more to do, yet it makes the work so easy that there is no time lost, at least in large examples, and more certainty in the operation. Thus,

Write down the Multiplicand, then double it ; add this sum to the Multiplicand, and this again, and so on, every sum to the Multiplicand, till you have nine numbers, and it's plain that thus you have a Table of the products of the Multiplicand by all the digits, made up by a very simple and easy operation ; and then you have no more to do, but to transfer your several products out of this Table, and sum them up.

TABLE.

1	467853798
2	935707596
3	1403561394
4	1871415192
5	2339268990
6	2807122788
7	3274976586
8	3742830384
9	4210684182

B X.

E X A M P L E.

$$\begin{array}{r} 467853798 \\ 6839754 \end{array}$$

$$\begin{array}{r} 1871415192 \\ 2339268990 \\ 3274976586 \\ 4210684182 \\ 1403561394 \\ 3742830384 \\ 2807122788 \end{array}$$

$$\begin{array}{r} 3200004886285692 \end{array}$$

Altho' this method is universal, yet we need not apply it in every case, for that would not always be best; but in such Examples as this the ease and readiness with which it is done, does more than save the time spent in making the Table; with this advantage, that the work is performed with much more certainty, because it is more simple.

83. This may be contracted in many cases; for there is no necessity always to make the Table for all the nine digits. And it may happen, that by help of some of the preceding methods, we can as easily make a Table for few, or no more than we have use for in the Multiplier; nor is it any great matter in what order they stand in the Table.

For Example, to multiply 78659 by 6897. In making this Table, first take 3, then double its product for 6; and do the rest by the common way.

T A B L E.

1	78659
3	235977
6	471954
7	550613
8	629272
9	707931

Again, to multiply 783596 by 3857, first multiply by 3, then by 5 (as in Art. 74.) then add these products for 8, then subtract the first from this for 7.

TABLE.

1	783596
3	2350788
5	3917980
8	6268768
7	5485172

84. Another Method by a small moveable Table,

Make a Table of the Multiplicand only for the numbers 1, 2, 5 (using Art. 74. for 5,) and make it upon a bit of loose paper, that it may be always applied directly and immediately over the place where every particular product is to be written down, (for much of the difficulty lies in the distance and cross position of the Multiplicand to the several products) and out of this small Table find your product thus:

Suppose for a Multiplicand 685497. When the figure of the Multiplier is 2 or 5, here you have the products; then for 3 add 2 to 1 (*i. e.* the numbers against 2 and 1,) for 4, double the number against 2; for 6, add 5 and 1, or multiply 2 by 3; for 7, add 5 and 2; for 8, add 5, 2, and 1; for 9, use the method of Art. 79.

TABLE.

1	685497
2	1370994
5	3427485

85. When more numbers than two are given to be multiplied one by the other, that kind of Multiplication is called *continual*, and is thus performed, *viz.* first multiply any two of the numbers given one by the other, then multiply the product by another of the numbers given, and this product by the fourth number given (if there be so many) and in that order 'till every one of the given numbers has been made a Multiplier, so the last product is the true product required. *Example.* If 4, 18, and 22, were given to be multiplied continually, first 18 multiplied by 4 produces 72, which multiplied by 22 (the third number) produces 1584, the last product or number required. See the Work in the margin. The proof of Multiplication is by Division, as will appear by the next Chapter.

$$\begin{array}{r}
 18 \\
 18 \\
 \times 4 \\
 \hline
 72 \text{ Prod. 1.} \\
 22 \\
 \hline
 144 \\
 144 \\
 \hline
 1584 \text{ Prod. 2.}
 \end{array}$$

86. If one of the Factors is equal to the product of any two or more numbers, then multiply by those numbers continually.

That

Chap. V. of Whole Numbers:

39

That is, to multiply 42991 by 56, first multiply it by 7, and that product by 8, then the same product will arise as if it were multiplied by 56, the product of 7 by 8. *Example.*

$$\begin{array}{r}
 42991 \\
 \times 7 \\
 \hline
 300937 \\
 \times 8 \\
 \hline
 2407496
 \end{array}
 \qquad
 \begin{array}{r}
 42991 \\
 \times 56 \\
 \hline
 257946 \\
 \times 214955 \\
 \hline
 2407496
 \end{array}$$

Note. If Numbers are given that cannot be produced by the Multiplication of any two figures, then find out two figures whose product comes nearest, either under or above it, to which add what is wanting, or from which subtract what is over the given number; as,

$$\begin{array}{r}
 479 \text{ by } 47 \\
 \times 6 \\
 \hline
 2874
 \end{array}
 \qquad
 \begin{array}{r}
 479 \\
 \times 47 \\
 \hline
 3353
 \end{array}
 \qquad
 \begin{array}{r}
 927 \text{ by } 29 \\
 \times 4 \\
 \hline
 3708
 \end{array}
 \qquad
 \begin{array}{r}
 927 \\
 \times 29 \\
 \hline
 8343
 \end{array}$$

$$\begin{array}{r}
 22992 \\
 \underline{-} 479 \text{ subtracted} \\
 \hline
 22513
 \end{array}
 \qquad
 \begin{array}{r}
 22513 \\
 \underline{+} 927 \text{ added} \\
 \hline
 26883
 \end{array}$$

87. To multiply Numbers of several denominations. For example; to multiply 3*l.* 13*s.* 7*½ d.* by 8, the process may stand as below,

$$3 \cdot 13 \cdot 7 \frac{1}{2} \\
 \times 8$$

$$\underline{\text{Ans}w. \ 29 \cdot 9 \cdot 0}$$

The Work is performed as follows, saying 8 half-pence make 4 pence, which reserve in mind; again, 8 times 7 pence make 4*s.* 8*d.* (to wit, 8 six-pences make 4*s.* and there are 8 pence besides) to which adding 4 pence in mind, there will arise 5*s.* which reserve in mind, and subscribe a cypher under the place of pence; again, say 8 times 13*s.* make 5*l.* 4*s.* (to wit 8 angels make 4*l.* and 8 times 3*s.* make 1*l.* 4*s.*) to which adding 5*s.* in mind, the sum will be 5*l.* 9*s.* wherefore subscribe 9*s.* (the excess above the pounds) under the shillings, and keep 5*l.* in mind; lastly, say 8 times 3 pounds make 24 pounds, which with

D 4

5 pounds

5 pounds in mind make 29 pounds; so that the total product or answer of the question is found to be 29*l.* 9*s.*

More Examples of this kind are these:

$$\begin{array}{r}
 l. \quad s. \quad d. \\
 17 \cdot 15 \cdot 5\frac{1}{2} \\
 \hline
 7 \\
 \hline
 \text{Ans}w. 124 \cdot 8 \cdot 0\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 l. \quad s. \quad d. \\
 18 \cdot 12 \cdot 6\frac{1}{2} \\
 \hline
 8 \\
 \hline
 \text{Ans}w. 149 \cdot 00 \cdot 6
 \end{array}$$

C H A P. VI.

D I V I S I O N of Whole Numbers.

88. D I V I S I O N is that by which we discover how often one number is contained in another; or (which is the same) it shews how to divide a number propos'd, into as many equal parts as you please.

89. In Division there are always three remarkable numbers, which are commonly called by these names, the *Dividend*, the *Divisor*, and the *Quotient*.

90. The *Dividend* is the number given to be divided into equal parts.

91. The *Divisor* is the number by which the Dividend is to be divided; that is, it is the number which declares into how many equal parts the Dividend is to be divided.

92. The *Quotient* is the number arising from the Division, and shews one of the equal parts required; so if 15 were given to be divided by 5, or into 5 equal parts, the number arising, or one of the equal parts will be 3, for 5 is found three times in 15: And here 15 is the Dividend, 5 the Divisor, and 3 the Quotient.

93. As Multiplication is a manifold addition, so Division is a manifold Subduction; that is, the taking of one number out of another as often as possible. Thus, 15 divided by 5, quotes 3.

For

For if 5 be three times taken from 15, nothing will remain.

Note 1. If two numbers consist of an equal number of places the lesser cannot be contained in the greater above 9 times.

For 10 times any number consists of the said number with a cypher annexed, by *Art. 73.* that is, it has one place more than that number.

Note 2. If two numbers be given, the greater of which has one place of figures more than the lesser, but if the unit's place of the greater number be taken away, the remaining figures of that number be less than the lesser number, then the lesser number is not contained in the greater more than 9 times.

For if the lesser number be contained in the greater 10 or more times, then the figures which remain in the greater number, after the unit's place is taken away, will be equal to or greater than the lesser number.

Note 3. If an even number be divided by an odd number, the Quotient will be even.

Note 4. If an odd number be divided by an odd number, the Quotient will be odd.

Note 5. If an even number be divided by an even number, the Quotient may be either even or odd.

94. Division being the hardest lesson in Arithmetic, must be heedfully attended to by the learner, for whose ease the utmost endeavour is used to make the way smooth by Rules and Examples, beginning with the easiest first, which will be in that case when the Divisor consists of one figure only; for example, let it be required to divide 192 by 8, or 192 pounds into 8 equal parts or shares; here 192 is the Dividend, 8 is the Divisor, and the Quotient or one of the equal parts is sought.

95. Place a crooked line at each end of the Dividend, that on the left-hand serving for the place of the Divisor, and that on the right for the Quotient; then if the Divisor be a single figure, subscribe a point under the first figure of the Dividend towards the left-hand, if such first figure be either equal to, or greater than the Divisor; but if such first figure be less than the Divisor put a point under the next place of the Dividend; which number so distinguished by the point may be called the Dividual; so in the *Example* above given, 192 being the Dividend, and 8 the Divisor, set a point under 9, not under 1, because it is less than the Divisor. This done, the Dividual, or number whereof the question must be asked, is 19.

96. Having thus prepared the numbers, ask how often the Divisor is contained in the Dividual, and write the number which

which answers the question in the Quotient; then multiply the Divisor by the number placed in the Quotient, and subscribe the product under the Dividual. Lastly, having drawn a line under the product, subtract it from the Dividual, and subscribe the remainder orderly under the line. So demanding how many times the Divisor 8, is found in the Dividual 19, the answer is two times, therefore write 2 in the Quotient; then multiplying the Divisor 8, by 2 (the number placed in the Quotient) the product is 16, which subscribe orderly under the Dividual 19; and after a line is drawn under the product 16, subtract it from the Dividual 19, and place the remainder 3 under the line.

97. Put another point under the next place of the Dividend towards the right-hand, and bring down the figure or cypher standing in that place to the remainder; that is, set it next after it, so the whole will be a new Dividual: Thus a point being placed under 2, which stands in the next place of the Dividend, write 2 next after (to wit on the right-hand of) the remainder 3, so 32 is a new Dividual, or number whereof the second question must be asked, and the Work will stand as you see in the Example.

98. A new Dividual being set apart, renew the question, and proceed according to Art. 96. Thus demanding how often the Divisor 8, is found in the Dividual 32, the answer is four times; therefore write 4 in the Quotient: Then multiplying the Divisor 8, by 4 (the figure last placed in the Quotient) the product is 32, which subscribe under the Dividual 32, and after a line is drawn underneath, subtract the product 32 from the Dividual 32, and there being no remainder, subscribe 0 under the line; so the whole Work being finished the Quotient is found to be 24, and the operation stands as you see in the Example; therefore if 192 pounds be equally divided among 8 persons, the share of each person will be 24 pounds.

A second example: Let it be required to divide 936 pounds into 9 equal parts: having distinguished the first Dividual by a point, (according to Art. 95.) demand how often the Divisor 9 is found in the Dividual 9, and finding it once contained in it, write one in the Quotient; then multiplying the Divisor 9

$$8) \begin{array}{r} 192 \\ \cdot \end{array} (2$$

$$\begin{array}{r} 16 \\ - \end{array}$$

$$\begin{array}{r} 3 \\ - \end{array}$$

$$8) \begin{array}{r} 192 \\ \cdot \end{array} (2$$

$$\begin{array}{r} 16 \\ - \end{array}$$

$$\begin{array}{r} 32 \\ - \end{array}$$

$$8) \begin{array}{r} 192 \\ \cdot \end{array} (24$$

$$\begin{array}{r} 16 \\ - \end{array}$$

$$\begin{array}{r} 32 \\ - \end{array}$$

$$\begin{array}{r} 32 \\ - \end{array}$$

$$\begin{array}{r} 0 \\ - \end{array}$$

$$9) \begin{array}{r} 936 \\ \cdot \end{array} (1$$

$$\begin{array}{r} 9 \\ - \end{array}$$

$$\begin{array}{r} 0 \\ - \end{array}$$

by

by 1, the product is 9, which subscribe under the Dividual 9; after this, a line being drawn under the product 9, subtract it from the Dividual 9, and there being no remainder, place a 0 under the line, as in the *Example*.

Again, placing a point under 3 which stands in the next place of the Dividend, transcribe the said 3 next after the remainder 0 for a new Dividual; then asking how often the Divisor 9 is contained in the Dividual 3, and not finding it once contained therein, write 0 in the Quotient: And now because the product which ought to arise from the Multiplication of the Divisor by 0 (the cypher last placed in the Quotient) amounts to 0, the Dividual 3, out of which that product should have been subtracted, remains the same without Alteration; therefore after a point is subscribed under 6, the next place of the Dividend, annex 6 to the Dividual 3, so there will be a new Dividual, to wit 36: Then demanding how often the Divisor 9 is found in the Dividual 36, the answer will be 4 times; therefore place 4 in the Quotient, and multiplying the Divisor 9 by 4, the product is 36, which subscribe underneath, and subtract from the Dividual 36; so the remainder is 0; thus the whole Work being finished, the Quotient is found to be 104, as in the *Example*: Wherefore conclude, if 936 l. be equally divided among 9 persons, the share of each will be 104 l. In like manner, if 296163 be divided by 7, the Quotient will be 42309.

99. The whole Work of Division is briefly contained in this following Verse.

Dic quot, multiplica, subduc, transferque secundam.

Or thus,

*First you must ask how oft, in Quotient answer make;
Then multiply, subtract, a new Dividual take.*

And the reason of the operation will appear from the following explanation thereof.

Divisor 8) 936 (100 the first Quotient.

800 = 100 X 8 the product of the Divisor and the Quotient.

136 = first remainder.

Divisor

Divisor 8) 136 (10 the second Quotient.

$80 = 10 \times 8$ the product of the Divisor and the Quotient.

$\underline{56} =$ second remainder.

Divisor 8) 56 (7 the third Quotient.

$56 = 7 \times 8$ the product of the Divisor and the Quotient.

$\underline{\underline{00}}$

Now the sum of the several Quotients, viz. 100 + 10 + 7 = 117 the Quotient, as found by the operation above directed, viz.

$$8) \underline{936} (117$$

$\underline{13}$
 $\underline{8}$

$\underline{56}$
 $\underline{56}$

$\underline{00}$

100. When in the division the Divisor consists of a single figure only, the Quotient may be express'd, and all the operation performed in mind, without writing down any part thereof; so 82506 being given to be halved or divided into two equal parts, the Work will be thus: The Divisor 2 is found in 8 four times; in 2 once; in 5 twice, 2)82506(41253 and there will remain 1, which 1 being supposed to stand before (to wit, on the left-hand of) the cypher makes 10; then say 2 is found in 10, 5 times; and last of all in 6, 3 times; so that the true Quotient or one half of the given number 82506 is found to be 41253.

In like manner, if 82506 be given to be divided by 3, or into 3 equal parts, the Work will be thus: The Divisor 3 is found in 8 twice, and there 3)82506(27502 will remain 2, which 2 being supposed to stand before (to wit, on the left-hand of) the following 2, makes 22; then say, 3 is found in 22, 7 times; in 15, 5 times; in 0, not at all; and lastly, in 6 twice; so that the true Quotient, or one of the three equal parts required is 27502. After the same manner may Division be work'd by any single figure, without much charge to the memory.

When

When the Division is thus performed, it is usual to write the Quotient under the Dividend : Thus

3) 82506

27502

101. Here the *Learner* may ask, what shall be done with the last remainder, if any happen, when the Division is finished? A full answer to this will be found in *Chap. 17.* where the Doctrine of Fractions is explained ; yet I shall here produce an *Example* where the said case happens, *viz.* Let it be required to divide 351 by 8, or 351 pounds equally among 8 persons ; now, if the operation be prosecuted according to the former rules, the Quotient will be found to be 43, and after the Division is finished, there will remain 7, 8) 351 that is, each person must have 43 pounds, and there will be an overplus of 7 pounds, which must be also divided equally among the 8 persons ; but that cannot be done till the 7 pounds be reduced into shillings, and then those shillings must be divided by 8, to give every person his due share of the shillings contained in the said 7 pounds : Again, if there yet remain any surplusage of shillings, they must be reduced to pence, which also are to be divided by 8, to give every person his due share of pence : So that when this question is fully answered, each person's share will appear to be 43*l.* 17*s.* 6*d.* But how the before-mentioned *Reduction* is performed, will be made manifest in the next chapter.

102. When the Divisor consists of two, three, or more places, the operation is more difficult than the former ; but depends upon the same grounds ; and therefore the Learner being well versed in the preceding method, of dividing by a single figure, will the more readily understand these that follow, which are two, of which the first is the easier, but the latter more expeditious : For an *Example* of the former, let it be required to divide 4112772 by 708 ; or, which is the same, to divide 4112772 into 708 equal parts.

First a *Table* is to be made to shew at first sight any *Multiple* or product of the *Divisor*, it being taken twice, thrice, or any number of times under ten ; so having first written down the *Divisor* itself 708, and drawn a line on the right-hand of it, place 1 on the right-hand of the line directly against the *Divisor* ; then under the

<i>The Divisor</i>	708	1
<i>Multiples of the Divisor</i>	1416	2
	2124	3
	2832	4
	3540	5
	4248	6
	4956	7
	5664	8
	6372	9

Divisor

Divisor 708, subscribe the double thereof, which is 1416, and place the figure 2 directly against the said double, to wit, on the other side of the line. Again, adding 1416, (to wit, the double of the *Divisor*) to the *Divisor* itself 708, the sum is 2124 for the triple of the *Divisor*; subscribe this under the double, and place 3 on the other side of the line right against it. Again, adding 2124 (the triple of the *Divisor*) to the *Divisor* 708 produces 2832 for the Quadruple of the *Divisor*, which Quadruple subscribe under the Triple; and proceeding in like manner, at last the Table is finished, which readily shews the *Divisor*, with the *Duple*, *Triple*, *Quadruple*, *Quintuple*, *Sextuple*, *Septuple*, *Octuple*, and *Nonuple* of the *Divisor*.

Now for a proof of the said Table, adding the last number thereof, to wit, 6372 (which was found to be nine times the *Divisor*) to the *Divisor* 708, the sum is 7080, which is evidently ten times the *Divisor*; therefore the Table is true, in regard that the last Number of it is derived from all the superior numbers.

The Table of Multiples or Products of the *Divisor* being thus prepared, set down the *Dividend* on the right hand of the *Divisor*; then distinguish by a point so many of the foremost places of the *Dividend* towards the left-hand, as are either equal in value to the *Divisor*, or which being greater, yet come nearest to the value thereof; thus subscribe a point under 2, thereby setting apart 4112, being the fewest of the foremost places which contain the *Divisor* 708, so is 4112 the *Dividual* or number whereof the first question must be asked; then demanding how often the *Divisor* 708 is contained in the *Dividual* 4112, the answer will be found by the Table to be five times; for looking in the Table it appears that 6 times the *Divisor* is the next greater than the *Dividual* 4112, and 5 times is the next lesser, therefore write 5 in the Quotient, and the number in the Table which stands against 5, to wit, 3540, subscribe under the *Dividual* 4112. Then having drawn a line underneath, subtract 3540 (which is five times the *Divisor*) from the *Dividual* 4112, and subscribe the remainder 572 under the line; that done, put a point under the next place of the *Dividend* towards the right-hand,

708	1)	4112772	(5809
1416	2	3540	
2124	3	5727	
2832	4	5664	
3540	5	6372	
4248	6	6372	
4956	7	0	
5664	8		
6372	9		

hand, and because the figure 7 stands in that place, transcribe 7 next after the remainder 572, so there is 5727 for a new *Dividual*.

Then demanding how often the *Divisor* 708 is contained in the *Dividual* 5727, the answer will be found by the Table to be 8 times; for looking by the Table, 9 times the *Divisor* is the next greater, and 8 times is the next lesser than the *Dividual*; therefore write 8 in the Quotient, and the number in the Table, which stands against 8, to wit, 5664 subscribe under, and subtract from the *Dividual* 5727, placing the remainder 63 under the line.

Again, put a point under the next place of the *Dividend*, to wit, the figure 7, and transcribing 7 next after the remainder 63, the new *Dividual* will be 637; then demanding how often the *Divisor* 708 is contained in the *Dividual* 637, and not finding it once contained therein, write 0 in the Quotient, and since in this case (that is, when a cypher answers the question) the *Dividual* remains the same without alteration, the figure or cypher standing in the next place of the *Dividend*, is to be transcribed after the *Dividual* for a new *Dividual*, so writing 2 next after 637, the new *Dividual* is 6372: Then demanding how often the *Divisor* 708 is contained in 6372, the Table shews that it is contained in it 9 times; therefore writing 9 in the Quotient, and placing the number which stands against 9 in the Table, to wit, 6372 under the *Dividual* 6372, and subtracting it from the *Dividual*, there *Divisor* 188 | 1) 20304 (108 will remain 0. Whence, if 4112772 be divided by 708, or into 708 equal parts, the true Quotient or one of the equal parts required is 5809. In like manner, if 20304 be divided by 188, that is into 188 equal parts, the Quotient arising, or one of those equal parts will be 108, and the operation will stand as you see.

<i>Multiples of the Divisor</i>	188	1	20304	(108)
	376	2		
	564	3	188	
	752	4	1504	
	940	5	1504	
	1128	6		0
	1316	7		
	1504	8		
	1692	9		

The preceding Method of *Division* by the help of a Table of the Multiples or products of the *Divisor*, as it is most easy, so in some cases, namely, where the *Divisor* is great, and a Quotient of many places is required, (as in calculating Tables of Interest, Astronomical Tables, and such like) it excels all other

other ways of *Division*; both in respect of certainty and expedition; but for common practice, it is too tedious.

103. *The last and principal method of Division, when the Divisor consists of many places, which to such as have the Table of Multiplication by heart, will not be difficult:* For Example, let 56304 be a number given to be divided by 184, that is, into 184 equal parts, and the Quotient, or one of the equal parts is required.

First, Distinguish by a point (as before) so many of the foremost places of the Dividend towards the left-hand, as are either equal in value (when consider'd apart) to the Divisor, or else which being greater, yet come nearest to it: Thus subscribe a point under the figure 3, thereby setting apart 563, being the fewest of the foremost places, which contain the Divisor; so is 563 the Dividual, or number 184) 56304 (whereof the first question must be asked. Hav-

ing thus prepared the numbers, demand how often the Divisor 184 is contained in the Dividual 563; and since to answer this question and such like, there is a necessity of trial, it will be requisite to shew how this trial may fitly be made: First, therefore, compare the number of places in the Dividual, with the number of places in the Divisor, and when the number of places is the same in both, let it be asked how often the first or extreme figure of the Divisor, towards the left-hand, is contained in the first figure of the Dividual towards the same hand; so here demanding how often 1 is contained in 5, the answer is 5 times; therefore the Divisor 184 is not contained oftner than 5 times in the Dividual 563 (for 6 times 184 is manifestly greater than 563,) but whether it be contained 5 times in it or not, examination must be made, either by multiplying (in some bye-place) the Divisor 184 by the said 5, and comparing the product with the Dividual 563; or else thus, saying 5 times 1 (to wit, the 1 in the Divisor) is contained in 5, to wit, the first figure of the Dividual 563, 5 times; but then the 8, the following figure of the Divisor, cannot be found 5 times in 6, the following figure of the Dividual and consequently the Divisor 184 is not contained 5 times in the Dividual 563; wherefore make another trial to see whether it may be contained 4 times in it or not; saying, 4 times 1 is 4, which is found in 5, and there will remain 1; but then 4 times 8, which is 32, cannot be had in 16 (for the 1 before remaining being supposed to stand on the left-hand of 6 makes 16); hence again, the Divisor 184 is not contained 4 times in the Dividual 563; wherefore make another trial to see whether it may be contained 3 times in it or not; saying 3 times 1 is 3, which is found in 5,

and there will remain 2; again three times 8 is 24, which is found in 26 (for the 2 before remaining being supposed to stand before the 6 in the Dividual, makes 26) and there will remain 2. Lastly, 3 times 4 is 12, which is likewise found in 23, (for 2 remaining in the 26 being supposed to stand before the 3 in the Dividual makes 23); therefore the Divisor 184 is contained 3 times in the Dividual 563; write 3 in the Quotient, and proceeding according to Art. 96. multiply the Divisor 184 by 3 (the figure placed in the Quotient) 184) 56304 (3 Quotient so the product is 552, which subscribe orderly under the Dividual 563; then having drawn a line under the said product, subtract it from the Dividual, and subscribe the remainder, which is 11 under the line.

Again, according to Art. 97. bring down 0, which stands in the next place of the Dividend, to the remainder 11, so there is 110 for a new Dividual; then demanding how often the Divisor 184 is found in the Dividual 110, and not finding it once contained in it, write 0 in the Quotient (which is to be done as often as the question is answered by nothing); now because the product arising from the Multiplication of the Divisor by 0, (the cypher last placed in the Quotient) amounts to 0; the Dividual 110, out of 184) 56304 (306 which that product should be subtracted, remains the same without alteration; therefore, after a point is subscribed under 4, the following place of the Dividend, annex 4 to the last Dividual 110, so there will be a new Dividual, to wit, 1104; and here the question at large is, to know how often 184 is found in 1104; but to lessen the trial, because the Dividual consists of one place more than is in the Divisor, it must be asked how often the first figure of the Divisor, on the left-hand, is contained in the two foremost places of the Dividual, towards the left-hand, viz. demand how often 1 is contained in 11, and altho' it may be had 11 times, yet never begin the trial above 9 times, see Note 2. Art. 93. therefore make trial with 9, saying, 9 times 1 is 9, which is found in 11, and there will remain 2; but then 9 times 8, which is 72, cannot be found in 20 (20, because the 2 remaining being supposed to stand before 0 in the Dividual makes 20,) therefore make trial with 8, saying, 8 times 1 is 8, which is found in 11, and there will remain 3; but then 8 times 8 cannot be had in 30 (30, because the 3 remaining being supposed to stand before the 0 or cypher makes 30); therefore make trial with 7, saying, 7 times 1 is 7, which

is found in 11, and there will remain 4; but then 7 times 8 cannot be had in 40; therefore make trial with 6, saying, 6 times 1 is 6, which is found in 11, and there will remain 5; then 6 times 8 is 48, which is found in 50, and there will remain 2; therefore the Divisor 184 is contained 6 times in the Dividual 1104: Wherefore write 6 in the Quotient, and proceeding according to Art. 96. multiply the Divisor 184 by 6 (the figure last placed in the Quotient) so the product is 1104, which being subscribed under, and subtracted from the Dividual 1104, the remainder is 0; therefore the Quotient sought is 306.

104. If the figure assumed for the Quotient holds good upon trial, as aforesaid, by two or three of the foremost places of the Dividual, it will for the most part hold throughout the Dividual; but this must be a perpetual rule, that whenever the product of the Multiplication of the Divisor, by the figure placed in the Quotient, happens to be greater than the Dividual, from which it ought to be subtracted, such product must be struck out of the Work, and a lesser figure is to be placed in the Quotient.

For a second Example, Let it be required to divide 15114220 by 2987, or into 2987 equal parts.

First, the Divisor 2987 being greater than 1511, (to wit, the four foremost places of the Dividend) set a point under 4, thereby setting apart 15114 for a Dividual; then, because the Dividual consists of one place more than the Divisor, ask how often 2 (the first figure of the Divisor) is contained in 15, (the two foremost places of the Dividual) and finding the answer to be 7 times, infer thence that the Divisor 2987 cannot be contained more than 7 times in the Dividual 15114; but whether it will be contained 7 times in it or not, examination must be made, either by multiplying 2987 by 7 (in some bye-place) and comparing the product with the Dividual 15114; or else by the manner of trial before delivered in the last Example: So at length it will be discovered, that the Divisor 2987 will not be found above 5 times in the Dividual 15114; wherefore (according to Art. 96.) writing 5 in the Quotient, and multiplying 2987 by 5, subscribe the product of that Multiplication, which is 14935, under the Dividual 15114; then drawing a line under the said product, and subtracting it from the Dividual 15114, subscribe the remainder 179 under the line.

Chap. VI. *of Whole Numbers.*

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Again, (according to Art. 97.) bring down 2, the next place of the Dividend, to the said remainder 179, so the new Dividual will be 2987; that done, asking how often the Divisor 2987 is contained in the Dividual 1792, and not finding it once contained in it, write 0 in the Quotient; and here, because the Question is answered by 0, the next place of the Dividend, to wit 2, is to be brought down to the Dividual 1792, so the new Dividual is 17922. Then renewing the question, and proceeding as before, at length the Division being finished, the Quotient will be found 5060 exactly, without any remainder; but if any remainder had happened, after the Subtraction of the last product, it must have been prosecuted according to Art. 101.

In like manner, if 1208939550 be divided by 19999, or into 19999 equal parts, the Quotient, or one of those equal parts, will be found 60450, and the Work will stand as you see here.

This latter method of Division is to be preferred before any of the ways of dividing, by dash-ing out figures, where the steps of the Division are so confound-ed (besides the burden upon the memory, by a promiscuous Mul-tiplication and Subtraction) that if any error happen, it can hardly be corrected without beginning the Work a-new: But in the way before explained, the parti-cular Multiplications, Sub-trac-tions, and Remainders, which belong to every figure of the Quotient, are so distinctly and clearly expres'd, that if an error happen, the Work may easily be reformed.

105. So often as the question is repeated in Division, so many places there must be in the Quotient, (which may be dis-covered by the number of points placed under the Dividend) and so many times is one and the same kind of operation re-peated, the substance whereof is contained in the Verse before-mentioned in Art. 99.

106. When the Divisor consists of 1 or an unit in the extreme place towards the left-hand, and nothing but cyphers towards the right, the Division is performed by cutting off, with a line, so many places of the Dividend towards the right-hand, as the Divisor has cyphers; so the figures which stand on the left-hand of the line, give the Quotient, and those cut off to the right (if they be significant figures) are to be proceeded with as a surplusage or overplus remaining, according to Art. 101. So if 4720*l.* were given to be divided equally among 10 persons, the share of each 10) 472|0 (472 would be 47*l.* also if the said 4720*l.* 100) 47|20 (47 were to be divided equally among 100 1000) 4|720 (4 persons, the share of each would be 4*l.* and there would be a surplusage or remainder of 20*l.* to be also subdivided among them, after the said 20*l.* are converted into shillings, according to Art. 119. Lastly, if the said 4720*l.* were to be divided among 1000 persons, the share of each would be 4*l.* and there would be a remainder of 720*l.* to be also divided as aforesaid. See the form of the Work in the margin.

107. When the Divisor consists of any significant figure or figures in the first or foremost place or places, towards the left-hand, and nothing but a cypher or cyphers towards the right, cut off, by a line, so many places of the Dividend towards the right-hand, as the Divisor has cyphers towards the right; then divide the figures of the Dividend, which stand on the left-hand of the line, by the figures in the Divisor which remain, when the said cypher or cyphers are omitted, remembering after the Division is finished, to write down next after the last remainder, the places of the Dividend which were first cut off: So if 36732 were given to be divided by 20, the Quotient will be 1836, and there will remain 12, *viz.* if you cut off one place from the Dividend towards the right-hand (because the Divisor ends with one cypher, and then divide the rest, to wit, 3673) by 2 (according to Art. 102.) 2|0)3673|2 there will arise in the Quotient 1836, and the 1836:12 last remainder, after such Division is finished, will be 1, to which if 2 (the figure first cut off from the Dividend) be annexed, the total remainder is 12.

In like manner, if 7456787 were given to be divided by 304000, the Quotient will be 24, and there will remain 160787; *viz.* if you cut off 3 places from the Dividend towards the right-hand (3 places, because the Divisor ends with 3 cyphers) and then divide 7456 by 304, there will arise in the Quotient

24, and the last remainder, after such Division is finished, will be 160, to which if 787 (the places first cut off from the Dividend) be annexed, the total remainder or surplusage is 160787, which is to be proceeded with, as in Art. 101.

304|000) 7456|787 (24

$$\begin{array}{r} 608 \\ 1376 \\ 1216 \\ \hline 160787 \end{array}$$

108. If it be required to divide by a number consisting of nines, *viz.* 9, 99, 999, 9999, &c. it may be performed by Addition, on the same principles as the multiplying those numbers was done by Subtraction, in Art. 79. as follows.

Divide the given Dividend into periods, of as many places of figures as there are nines in the Divisor, beginning from the left-hand, and annex as many cyphers, to the right-hand of the number, as may be wanted to compleat a period.

Then write the figures of the left-hand period under those of the second period, or that which is next thereto toward the right-hand; add these two together, and place their sum under the third period; observing if the sum of the two figures in the highest place exceed 9, to place the figure that would (in common Addition) be carried, under the lowest place of the second period; add the third period to those figures which stand under it, including the carried figure, and place them under the fourth period; and so proceed, till you have placed figures under the right-hand period, and under them place such a figure, as would have been to be there placed, had the Work been to have proceeded a period farther.

Add the whole together; and, beginning at the right-hand, cancel as many figures as there were cyphers annexed to the Dividend; then, from the figures that remain, cut off with a comma, from the right-hand toward the left, as many figures as the Divisor contained nines; so shall the figures, to the left-hand of the comma, be the Quotient; and those to the right-hand thereof the remainder.

Note. If the remainder be all nines, add 1 to the Quotient.

Exampl. 1. Let it be required to divide 571665762 by 999?

The Dividend will make just 3 periods 571.665.762
The first period wrote under the second 571
The sum of 665 and 571 will be 1.236
I

Under the last place is set 1, because

572237,999

if 762 and 1236 were to have been added together, the 1 would have been so placed; then, by adding the whole, the sum will be 572237999; and three figures being cut off, from the right-hand, will give 572237,999, that is, 572237 for the Quotient, and 999 for the remainder; but since the remainder is equal to

the Divisor, the Quotient, increased by 1, viz. 572238 will be the answer.

To prove this Work, let 572238 be multiplied by 999, according to Art. 79. See the Work.

572238000

571665762 Product.

Where it is evident that the figures subtracted are the same with the sum of those added, except in the unit's place.

Note. The above process will take up less room, if the carried figures be wrote in a line by themselves, as follows:

571.665.762
571.236
I I

Examp. 2. Divide 57166576279 by 999?
 The number with a cypher annex'd = 571.665.762.799

After the operation there arises the number
 From which cancel the right-hand figure, }
 because a cypher was added to the Dividend, and it will be _____ }
 And this being properly separated gives }
 57223800 for the Quotient, and 79 for }
 the remainder _____ }
 57223800,079

$$\begin{array}{r}
 \text{Proof} & 5722380000 \\
 & 57223800 \\
 \hline
 \text{To } 57223800 \times .999 = & 57166576200 \\
 \text{Add the remainder} & 79
 \end{array}$$

The sum makes the Dividend 57166576279

Examp. 3. Divide 5710668153 by 999?
 The number with two cyphers annex'd 571.066.815.300
 571.637.452

From the operation arises 5716384
 And cancelling 2 figures 5716384
 Which properly separated will be $5716384 \cdot 537$
 viz. 5716384 for the Quotient, and 537 for the remainder.
 Proof 5716384000

$$\begin{array}{r}
 & 5716384 \\
 \times & 999 \\
 \hline
 & 5710667616 \\
 + & 537 \\
 \hline
 & 5710668153
 \end{array}$$

109. If it be required to divide by a number consisting of any Digit repeated, viz. 11, 111, 1111, &c. or 22, 222, 2222, &c. or 33, 333, 3333, &c. divide the given Dividend by the Digit which repeats in the Divisor, and multiply the Quotient by nine; then divide the product by 99, 999, 9999, &c. as in Art. 108. and the result will be the Quotient required.

Example. Let it be required to divide 202464 by 444?

4) 202464

50616 = the Quotient of 202464 divided by the repeating Digit.

9

455544 = the Product of 50616 by 9.

455

455,999 or 456, the Quotient required, by Art. 108.

110. Division and Multiplication interchangeably prove one another; for in Division if you multiply the Divisor by the Quotient, the product will be equal to the Dividend: So in the Example of Art. 103. if 184 the Divisor be multiplied by 306 the Quotient, the product is 56304, which is the same with the Dividend; but when, after the whole Division is finished, any figures remain of the last Subtraction, add them likewise to the product: So in the last Example of Art. 107. the Divisor 304000, being multiplied by the Quotient 24, produces 7296000, unto which if you add the number remaining, to wit, 160787, the sum is 7456787, which is the same with the Dividend. Again, in Multiplication, if the product be divided by the Multiplier, the Quotient will give you the Multiplicand, or if the product be divided by the Multiplicand, the Quotient will give you the Multiplier: So in the first Example of Art. 70. if the product 111024 be divided by the Multiplicand 3084, the Quotient gives the Multiplier 36.

111. There is also a common Proof of Multiplication argued from the Multiplicand, the Multiplier and the Product, by casting away nines; but by that way of proof (though rightly used) a false product will be affirmed to be true: *Example,* If 3462 be multiplied by 786, the true product is 2721132; but if I say, 4953132 or 3153132 is the product (or many others which may be given) the proof by nines will confirm them to be true products, though they are false, as will be evident to such as know the Rule, which is mentioned here, only to set a brand upon it, that it may be avoided by all lovers of truth.

112. Another proof of Division is, by adding together those lines which, in the following *Example*, are marked with Asterisks

isms (being the particular products of the Divisor, multiplied severally by each figure in the Quotient, together with the remainder of the Division) the total of which (if right) will be the Dividend.

$$736) \ 863256 \ (1172$$

736*

1272

736*

5365

5152*

.2136

1472*

.664*

863256 Proof.

113. When a whole number is given to be divided by a Divisor, which is equal to the product of the Multiplication of two single figures ; instead of dividing by that Divisor, you may first divide by one of those single figures, and then divide the Quotient by the other ; so will the last Quotient be the same, as if the Division had been finished by the Divisor first given : Thus, if 3456 be given to be divided by $48 = 6 \times 8$, divide first by 8, the Quotient will be 432 ; and then divide that Quotient (432) by 6, and the result 72 is the same as will arise by dividing 3456 by 48.

114. To divide Numbers of divers denominations : For Example, to divide 12*l.* 19*s.* 6*d.* into 5 parts, the Work may stand as below :

$$5) \ 12 \ . \ 12 \ . \ 6$$

$$\underline{2 \ . \ 10 \ . \ 6}$$

And the operation is performed in the following manner : Say the fives in 12, twice, and there remains 2, which are two pounds (for the remainder is always of the same name with the Dividend) or 40 shillings, and 12*s.* in the shillings place, make 52 shillings ; now the fives in 52, ten times, and there remains 2, which are two shillings or 24 pence, to which adding the 6*d.* in the place of pence, make 30 pence ; then the fives in 30 six times.

A Question whose Solution requires the use of the four preceding Rules.

A father left among five sons an estate, consisting of 500*l.* in Cash, with 5 bills each of 48*l.* 10*s.* 6*d.* he ordered 20*l.* to be bestowed upon his burial, and his debts to be paid, amounting

ing to 164*l.* then his free estate to be divided in this manner, *viz.* the eldest son to have the third part, and the other four sons to have equal shares: What is the share of each son?

Answ. 186*l.* 4*s.* 2*d.* to the eldest, and 93*l.* 2*s.* 1*d.* to each of the rest. See the operation.

48 . 10 . 6 the value of each bill.
5 bills.

242 . 12 . 6 total of the bills.

500 . 0 . 0 cash.

742 . 12 . 6 total.

184 . 0 . 0 deducted, *viz.* 20 + 164.

3) 558 . 12 . 6 free estate.

186 . 4 . 2 eldest son's share.

4) 372 . 8 . 4 remains.

93 . 2 . 1 the share of each of the other 4 sons.

C H A P. VII.

R E D U C T I O N.

115. **F**orasmuch as in Money there are diversities of kinds, *viz.* in *England*, pounds, shillings, pence, and farthings; also diverse kinds of Weights, Measures, &c. as has been fully declared in the second Chapter; and because it is often required to find how many pieces of one kind of Money are equal in value to a given number of another, (and so likewise of Weights, Measures, &c.) it is requisite in this place to shew how that is performed, since thereby the Rules of Multiplication and Division before delivered will be exercised. This kind of operation is called *Reduction*.

116. *Reduction* is either descending or ascending.

117. *Reduction* descending, is, when some number of integers of a greater denomination being given, it is required to find how many integers of a lesser denomination are equal in value to that given number of the greater: As when it is demanded to find how many shillings are contained in 30*l.* Likewise how many pence in 320*s.* or how many hours in 365 days, &c.

118. *Reduction* ascending is, when some number of integers of a lesser denomination being given, it is required to find how

how many integers of a greater denomination are equal in value to that given number of the lesser : As when it is proposed to find how many pence are contained in 500 farthings ; likewise how many shillings in 348 pence ; or how many days in 864 hours, &c.

119. *Reduction* descending is performed by Multiplication ; for if the given number of integers of a greater denomination, be multiplied by a number which expresses how many integers of the lesser are equal to one of the integers given, the product is the number of integers of the lesser denomination required.

So 230*l.* of *English* money will be reduced into 4600*s.* for if 230 be multiplied by 20, (the number of shillings which are equal to 1 pound) the product is 4600 ; in like manner 4600*s.* will be reduced into 55200*d.* for if 4600 be multiplied by 12 (the number of pence contained in 1 shilling) the product is 55200. Also 55200 pence being multiplied by 4 (because 4 farthings make a penny) are reduced into 220800 farthings, as by the operation in the margin is evident.

The like method is to be observed in Weights, Measures, &c. So 345 ounces *Troy* are reduced into 6900 penny-weights, and 6900 penny-weights, to 165600 grains, as by the operation in the margin you may see.

230 Pounds.
20
4600 Shillings.
12
55200 Pence.
4
220800 Farth.
345 Ounces.
20
6900 Penny W.
24
276
138
165600 Grains.

Note. By this Rule the learner is furnished with skill to resolve that case in Division, when the Dividend is less than the Divisor. *Example.* Let it be required to divide 7 pounds of *English* money, equally among 8 persons ; here it is evident that the Dividend 7 is less than the Divisor 8 ; that is, the number of pounds is less than the number of persons, and consequently each share must be less than a pound : So that in effect it is required to find how many shillings and pence belong to every person for his share : First, therefore reduce the 7 pounds into shillings, which will be 140, these divided by 8 give 17 shillings to each person, and there will yet be a remainder of 4 shillings to be also equally divided into 8 parts ; but these 4 shillings must be first reduced into pence, which will be 48, then dividing 48 by 8, the Quotient will give 6 pence more to every person : So at last it appears that if 7 pounds of *English* money be equally divided into 8 parts, the entire Quotient (or one of the equal shares) will be 17 shillings and 6 pence. In

In like manner, if 354 pounds of English money be given to be divided equally amongst 125 persons, the share of each will be found to be 2 pounds 16 shillings 7 pence 2 farthings and somewhat more, but the parts of a farthing being of no moment (and not properly to be handled in this place) are neglected.

Compare these two Examples with the Example, Art. 101.

120. In Reduction descending, the learner may receive help by the subsequent Tables.

1. ENGLISH MONEY.

Pounds	Multipled by	20	Produce	Shillings.
Shillings	Multipled by	12	Produce	Pence.
Pence	Multipled by	4	Produce	Farthings.

2. TROY-WEIGHT.

Pounds	Multipled by	12	Produce	Ounces.
Ounces	Multipled by	20	Produce	Penny-weights.
Penny-weights	Multipled by	24	Produce	Grains.

Also in APOTHECARIES-WEIGHTS.

Ounces-Troy	Multipled by	8	Produce	Drams.
Drams	Multipled by	3	Produce	Scruples
Scruples.	Multipled by	20	Produce	Grains.

3. Of AVOIRDUPOISE-WEIGHTS.

Hundred Weights	Multip. by	4	Produce	Quarters.
Quarters	Multip. by	28	Produce	Pounds.
Pounds	Multip. by	16	Produce	Ounces.
Ounces	Multip. by	16	Produce	Drams.

4. Of LIQUID MEASURES.

Hogheads	Multip. by	63	Produce	Gallons.
Gallons	Multip. by	2	Produce	Pottles.
Pottles	Multip. by	2	Produce	Quarts.
Quarts	Multip. by	2	Produce	Pints.

5. Of DRY MEASURES.

<i>Quarters</i>	<i>Multipled by</i>	8	<i>Product</i>	<i>Bushels.</i>
<i>Bushels</i>		4		<i>Pecks.</i>
<i>Pecks</i>		2		<i>Gallons.</i>
<i>Gallons</i>		2		<i>Pottles.</i>
<i>Pottles</i>		2		<i>Quarts.</i>
<i>Quarts</i>		2		<i>Pints.</i>

6. Of LONG MEASURES.

<i>English Miles</i>	<i>Multipled by</i>	8	<i>Product</i>	<i>Furlongs.</i>
<i>Furlongs</i>		220		<i>Yards.</i>
<i>Yards</i>		3		<i>Feet.</i>
<i>Feet</i>		12		<i>Inches.</i>

Inches 3

<i>Yards, or Ells</i>	<i>Mul.</i>	4	<i>Product</i>	<i>Quarters.</i>
<i>Quarters</i>		4		<i>Nails.</i>

7. Of SUPERFICIAL MEASURES of LAND.

<i>Acres</i>	<i>Mul. by</i>	4	<i>Product</i>	<i>Roods.</i>
<i>Roods</i>		40		<i>Perches, or Poles.</i>

8. Of TIME.

<i>Weeks</i>	<i>Mul.</i>	7	<i>Product</i>	<i>Days.</i>
<i>Days</i>		24		<i>Hours.</i>
<i>Hours</i>		60		<i>Minutes.</i>

121. Integers of several denominations may be reduced into the least of those denominations, according to Art. 120. by descending orderly to the next inferior denomination; and adding to each product such integers, (if there be any) as are of the same name.

So 12 pounds 13 shillings and 10 pence may be reduced into 3046 pence, in this manner, *viz.* 12 pounds multiplied by 20 (because 20 shillings make 1 pound) produce 240 shillings, to which adding 13 shillings, the sum is 253 shillings. Again, 253 shillings multiplied by 12 (because 1 shilling is equal to 12 pence,) produce 3036 pence, to which, if 10 pence be added, the sum is 3046 pence, as by the operation in the Margin is manifest.

<i>l.</i>	<i>s.</i>	<i>d.</i>
12	13	10
20		
	240	
add	13	
	253	Shillings.
	12	
	506	
	253	
	3036	
add	10	
	3046	Pence.

But after that general method is well understood, the Work of the last *Example*, and such like, may be contracted thus, *viz.* To convert 12 pounds, 13 shillings and 10 pence, all into pence: First, 12 multiplied by 0 (which stands in the unit's place of 20) produces 0; but instead of 0 write down 3 under the line (to wit, the 3 that stands in the unit's place of the 13 shillings in the sum proposed:) Then proceed to multiply 12 by 2, saying twice 2 is 4, to which adding 1 (for the ten in the said 13 shillings) it makes 5, which set on the left-hand of 3 before-written: Lastly, twice 1 is 2, which set on the left-hand of 5; and so 12 pounds 13 shillings, are converted into 253 shillings.

<i>l.</i>	<i>s.</i>	<i>d.</i>
12	13	10
20		
	253	Shillings.
	12	
	516	
	253	
	3046	Pence.

It remains to multiply the said 253 by 12 (because 12 pence make 1 shilling) and to add 10 to the product, which may be done thus: First, twice 3 is 6, to which adding 10 (to wit, 10 pence in the sum first propounded,) it makes 16; wherefore (according to the Rule of Multiplication) set 6 under the line, and keep 1 in mind: Again, twice 5 with 1 in mind making 11, write down 1, and keep 1 in mind; likewise twice 2 and 1 in mind making 5, write down 5: Then 253 multiplied by 1 making 253, which set orderly under 516: Lastly, those two products added together make 3046, which is the number of pence contained in 12*l.*, 13*s.*, 10*d.* as before was found out by the general method.

So 35 ounces 16 penny-weights and 12 grains *Troy* will be reduced into 17196 grains.

122. *Reduction* ascending is performed by Division; for if the number of integers given, be divided by such a number of the same

same integers, as are equal to one of the integers required, the Quotient is the number of integers sought for.

So 220800 farthings being divided by 4 (the number of farthings in a penny,) give 55200 pence, in the Quotient: In like manner, if 55200 pence be divided by 12 (the number of pence in a shilling) the Quotient is 4600 shillings. Lastly, 4600 shillings being divided by 20 (because 20 s. make a pound *Sterling*) the Quotient is 230 pounds *Sterling*, which are equal to 220800 farthings first given. The operation is as follows:

4) 220800

12) 55200

20) 4600

230 £.

In like manner, 34268 grains *Troy* will be reduced to 5 pounds 11 ounces 7 penny-weights and 20 grains.

123. This kind of *Reduction* may be made the easier to the learner by the following Tables.

1. Of ENGLISH MONEY.

<i>Farthings</i>	$\frac{1}{4}$	$\frac{4}{12}$	$\frac{1}{20}$	<i>Pence.</i>
<i>Pence</i>	$\frac{1}{2}$	$\frac{12}{60}$	$\frac{1}{5}$	<i>Shillings.</i>
<i>Shillings</i>	$\frac{1}{20}$	$\frac{60}{120}$	$\frac{1}{10}$	<i>Pounds.</i>

2. Of TROY-WEIGHT.

<i>Grains</i>	$\frac{1}{24}$	$\frac{24}{20}$	$\frac{1}{12}$	<i>Penny-weights.</i>
<i>Penny-weights</i>	$\frac{1}{5}$	$\frac{20}{60}$	$\frac{1}{6}$	<i>Ounces.</i>
<i>Ounces</i>	$\frac{1}{12}$	$\frac{60}{120}$	$\frac{1}{10}$	<i>Pounds-Troy.</i>

Also in APOTHECARIES-WEIGHTS.

<i>Grains</i>	$\frac{1}{20}$	$\frac{20}{3}$	$\frac{1}{8}$	<i>Scruples.</i>
<i>Scruples</i>	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{6}$	<i>Drams.</i>
<i>Drams</i>	$\frac{1}{8}$	$\frac{8}{60}$	$\frac{1}{15}$	<i>Ounces Troy.</i>

3. Of AVOIRDUPOISE-WEIGHTS.

<i>Drams</i>	$\frac{1}{16}$	$\frac{16}{16}$	$\frac{1}{16}$	<i>Ounces.</i>
<i>Ounces</i>	$\frac{1}{16}$	$\frac{16}{28}$	$\frac{1}{28}$	<i>Pounds.</i>
<i>Pounds</i>	$\frac{1}{28}$	$\frac{28}{4}$	$\frac{1}{4}$	<i>Quarters of Hund.</i>
<i>Quarters</i>	$\frac{1}{4}$	$\frac{4}{100}$	$\frac{1}{100}$	<i>Hundred Weights.</i>

4. Of

4. Of LIQUID MEASURES.

Pints	$\left\{ \begin{array}{l} \text{Divid. by } 2 \\ \text{Divid. by } 2 \\ \text{Divid. by } 2 \\ \text{Divid. by } 2 \end{array} \right\}$	$\left\{ \begin{array}{l} 2 \\ 2 \\ 2 \\ 63 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Quarts.} \\ \text{Pottles.} \\ \text{Gallons.} \\ \text{Hogheads.} \end{array} \right\}$
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5. Of DRY MEASURES.

Pints	$\left\{ \begin{array}{l} \text{Divid. by } 2 \\ \text{Divid. by } 4 \end{array} \right\}$	$\left\{ \begin{array}{l} 2 \\ 2 \\ 2 \\ 2 \\ 4 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Quarts.} \\ \text{Pottles.} \\ \text{Gallons.} \\ \text{Pecks.} \\ \text{Bushels.} \\ \text{Quarters.} \end{array} \right\}$
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6. Of LONG MEASURES.

Barley-corns	$\left\{ \begin{array}{l} \text{Divid. by } 3 \\ \text{Divid. by } 12 \\ \text{Divid. by } 3 \\ \text{Divid. by } 220 \\ \text{Divid. by } 8 \end{array} \right\}$	$\left\{ \begin{array}{l} 3 \\ 12 \\ 3 \\ 220 \\ 8 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Inches.} \\ \text{Feet.} \\ \text{Yards.} \\ \text{Furlongs.} \\ \text{English Miles.} \end{array} \right\}$
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ALSO,

Nails	$\left\{ \begin{array}{l} \text{Div. by } 4 \\ \text{Div. by } 4 \\ \text{Div. by } 5 \end{array} \right\}$	$\left\{ \begin{array}{l} 4 \\ 4 \\ 5 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Quarters of Yards.} \\ \text{Yards.} \\ \text{Ells English.} \end{array} \right\}$
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7. Of SUPERFICIAL MEASURES of LAND.

Perches, or Poles	$\left\{ \begin{array}{l} \text{Div. by } 40 \\ \text{Div. by } 4 \end{array} \right\}$	$\left\{ \begin{array}{l} 40 \\ 4 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Roods, or Quarters} \\ \text{of Acres.} \end{array} \right\}$
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Reeds	$\left\{ \begin{array}{l} \text{Div. by } 4 \end{array} \right\}$	$\left\{ \begin{array}{l} 4 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Acres.} \end{array} \right\}$
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8. Of TIME.

Minutes	$\left\{ \begin{array}{l} \text{Div. by } 60 \end{array} \right\}$	$\left\{ \begin{array}{l} 60 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Hours.} \end{array} \right\}$
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Hours	$\left\{ \begin{array}{l} \text{Div. by } 24 \end{array} \right\}$	$\left\{ \begin{array}{l} 24 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Days.} \end{array} \right\}$
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Days	$\left\{ \begin{array}{l} \text{Div. by } 7 \end{array} \right\}$	$\left\{ \begin{array}{l} 7 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Weeks.} \end{array} \right\}$
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Note, That if after Division is finished, in Reduction ascending, there be any remainder, it is of the same denomination with the Dividend.

Note also, That Reduction descending and ascending mutually prove one another, by inverting the question; for as in 56 pounds Sterling, there will be found 53760 farthings by Reduction descending: So for proof thereof 53760 farthings will be reduced to 56 pounds, by Reduction ascending.

124. Questions to exercise Reduction.

1. In 257*l.* how many shillings? *Ans.* 5140.
2. In 3076*l.* how many shillings? *Ans.* 61520.
3. In 902*s.* how many pence? *Ans.* 10824.
4. In 2179*s.* how many farthings? *Ans.* 104592.
5. In 49*l.* 13*s.* 7*d.* how many pence? *Ans.* 11923.
6. In 2053*l.* 14*s.* 9*d.* 2*f.* how many farthings? *Ans.*

1971590.

7. In 354*lb.* of *Troy-weight*, how many grains of Goldsmith's weight? *Ans.* 2039040.
8. In 300 *English miles*, how many yards? *Ans.* 528000.
9. In 1 *English mile*, how many barley-corns length? *Ans.* 190080.

10. In 560 acres, how many perches? *Ans.* 89600.
11. In 225 acres, 3 roods, and 30 perches, how many perches? *Ans.* 36150.
12. In 11923 pence, how many pounds? *Ans.* 49*l.* 13*s.* 7*d.*

13. In 5764684 farthings, how many pounds? *Ans.* 6004*l.* 17*s.* 7*d.*

14. In 234678 perches, how many acres? *Ans.* 1466 acres, 2 roods, 38 perches.

15. In 525960 minutes of an hour, how many days? *Ans.* 365 days and 6 hours, (or one year very near.)

16. In 10080 pints, how many hogsheads? *Ans.* 20.

17. In 34678 grains of Apothecaries weight, how many ounces *Troy*? *Ans.* 72 ounces, 1 dram, 2 scruples, and 18 grains.

18. In 106735 pints of wheat, how many quarters? *Ans.* 208 quarters, 3 bushels, 2 pecks, 1 gallon, 1 pottle, 1 quart, 1 pint.

19. In 3969301 barley-corns length, how many miles? *Ans.* 20 miles, 7 furlongs, 12 yards, 2 feet, 4 inches, and 1 barley-corn's length.

20. In 1900800 barley-corns length, how many miles? *Ans.* 10.

21. In 24 C. weight 3 quarters, 26*lb.* how many pounds Avordupoise? *Ans.* 2798.

Note. Hundred weights, Quarters and Pounds, may be speedily reduced into pounds, thus: Set down the Hundred weights four several times in the form following, and take in the odd weight.

C. qrs.

24 . 3 . 26

24

24

24

110 odd weight.

*Ans*w. 2798 pounds, as above.

22. In 12 crowns, how many shillings and pence? *Ans*w. 60*s.* and 720*d.*
23. In 50 half-crowns, how many pence and farthings? *Ans*w. 1500*d.* and 6000*f.*
24. In 306 crowns, how many half-crowns and pence? *Ans*w. 612 half-crowns, and 18360 pence.
25. How many shillings and pence are in 17 guineas? *Ans*w. 357 shillings, and 4284 pence.
26. How many crowns and six-pences are in 28*l.*? *Ans*w. 112 crowns, and 1120 six-pences.
27. In 6000 farthings, how many pence and half-crowns? *Ans*w. 1500 pence, and 50 half-crowns.
28. In 18360 pence, how many half-crowns and crowns? *Ans*w. 612 half-crowns, and 306 crowns.
29. How many shillings and guineas are in 4284 pence? *Ans*w. 357 shillings and 17 guineas.
30. How many crowns and pounds are in 1120 six-pences? *Ans*w. 112 crowns; and 28 pounds.
31. In 30 chalders of coals, each 36 bushels, how many pecks? *Ans*w. 4320.
32. If a piece of ground contains 24 acres, and an inclosure (of 17 acres 3 roods) be taken out of it, how many perches are there in the remainder? *Ans*w. 1000.
33. I have a bank note of 20*l.* a note of hand for 6*l.* 10*s.* and, in several coins, as follows: In copper, 13 farthings, and 45 half-pence; in silver, 25 two-pences, 36 three-pences, 56 groats, 96 six-pences, 67 shillings, 97 half-crowns, 126 crowns; in gold, 25 quarter-guineas, 65 half-guineas, 77 guineas, and 34 moidores: I would know what I am worth? *Ans*w. 245*l.* 9*s.* 2*d.*

C H A P. VIII.

Of the Rule of THREE DIRECT.

§25. **T**HE *Rule of Three* is so called, because by three numbers known or given, it teaches to find a fourth unknown; it is also called the *Golden Rule*, for the excellency thereof: Lastly, it is called the *Rule of Proportion*, for the reason hereafter declared.

§26. The *Rule of Three* is either Single or Compound.

§27. The Single Rule is, when three terms or numbers are proposed, and a fourth proportional to them is demanded.

§28. Four numbers are said to be proportionals, when the first contains the second, or is contained by the second, in the same manner as the third contains the fourth, or is contained by the fourth. So these four numbers are said to be proportionals, 8, 4, 12, 6; for as 8 contains 4 twice, so does 12 contain 6 twice; and therefore 8 is said to have such proportion to 4, as 12 has to 6; likewise these are proportionals, 4, 8, 6, 12. For as 4 is the half of 8, so is 6 the half of 12; and therefore 4 is said to have such proportion to 8, as 6 has to 12.

§29. The terms or numbers of the *Rule of Three* (to wit the three numbers given, and the fourth sought) consist of two different denominations, *viz.* two of the three given terms have one name, and the other given term with the term required have another: So this question being demanded, if four students spend 19 pounds in certain months, how much money will serve 8 students for the same time, and at the same rate of expence? Here students and pounds are the two denominational of the terms in the question, *viz.* 4 and 8 (being two of the terms proposed) have the denominations of students, and 19 the other term given, together with the term required, have the denomination of pounds.

§30. In the *Rule of Three*, two of the three given terms imply a supposition, and the third moves a question: So in the afore-mentioned question a supposition is made, that 4 students spend 19 pounds, and a question is moved with the number 8, to wit, how many pounds will 8 students spend?

§31. In the *Rule of Three*, the numbers given must be so ranked, that the known number or term upon which the question is made, may possess the third place; also of the other two, that which has the same denomination with the third, must be

in the first place : Lastly, the other known term, which is of the same denomination with the fourth term sought, (or answer of the question) must possess the second place : So in the question before-mentioned, the terms 4, 19, and 8, are to be thus placed, *viz.* 8 is the term upon which the question is moved, and therefore to possess the third place in the rule ; 4 is of the same denomination with 8, *viz.* of students, and therefore to be in the first place : Lastly, 19 being of the same denomination with the term sought for, *viz.* of money, is to be in the second place : And so they will be placed thus :

$$\text{Students. Pounds: Students.}$$

$$\text{If } 4 : 19 :: 8$$

That is to say, if 4 students spend 19 pounds, what will 8 students spend ? And here, for the better discerning of the term or number upon which the question is asked, you may observe, that for the most part it is the known number in the question, which immediately follows these or such like words, *viz.* *How many? how much? what will? how long? how far, &c.*

132. *The Rule of Three* is either *Direct* or *Inverse*.

133. *The Rule of Three Direct* is, when the sense or tenor of the question requires that the fourth number sought should have such proportion to the second, as the third number has to the first ; so in the aforementioned question, if 4 students spend 19 pounds, how many pounds will 8 students spend at the same rate of expence ? It is evident, that the thing requir'd is to find a number which may have such proportion to 19, as 8 has to 4 ; that is, as 8 is the double of 4, so ought the fourth number to be the double of 19 ; for if 19 pounds be required to maintain 4 students a certain time, as much more must needs be required for the maintenance of 8 students the same time ; and therefore in this case, we may say in a direct proportion, as 4 is to 8, so is 19 to the number required.

134. In the *Direct Rule of Three*, if you multiply the second term by the third, (or which is all one) the third term by the second, and then divide the product by the first; the Quotient will give the fourth term or fourth proportional required. So in the question before proposed, if you multiply 19 by 8 the product is 152, which, if you divide by 4, the Quotient will give you 38, the fourth term demanded, and the Work will stand thus :

<i>Stud. 1.</i>	<i>Stud.</i>
If 4 : 19 :: 8	
	8
	4) 152 (38 Pounds.
	12
	32
	32
	0

A second Example may be this, if 8 yards cost 9 pounds, how much will 3 yards cost? *Answ.* 3*l.* 7*s.* 6*d.*

This question being stated according to *Art. 131.* will stand as here you see; then multiplying (as before) the second term 9 by the third term 3, the product is 27, which being divided by the first term 8, the Quotient is 3 pounds, and there is a remainder of three pounds, which must be reduced into sixty shillings, and after those shillings are divided by 8, and the rest of the Work prosecuted according to *Art. 114.* the entire Quotient or answer of the question is 3*l.* 7*s.* 6*d.*

$$\begin{array}{r} y. \quad l. \quad y. \\ \text{If } 8 : 9 :: 3 \\ 8) \underline{27} \\ 3 \\ 3 \cdot 7 \cdot 6 \end{array}$$

A third Example, if 51 ounces of silver plate be sold for 13 pounds Sterling, what is the price of 1 ounce of that plate? *Answ.* 5*s.* 1*d.* and somewhat more. The operation is thus: After the three known terms of this question are rightly ordered, they will stand as here you see in the Example; then multiplying the second time 13 by the third term 1, the product will be also 13, (for Multiplication by 1 makes no alteration;) which 13 being divided by 51, after the manner of operation delivered in the Note upon *Art. 119.* the entire Quotient or answer of the question will at length be found to be 5*s.* 1*d.* and somewhat more; but the surplusage being less than a farthing, is omitted as useless.

$$\begin{array}{r} oz. \quad l. \quad oz. \\ \text{If } 51 : 13 :: 1 \\ 51) \underline{260(5\text{Shillings.}} \\ 255 \\ \hline 5 \\ 12 \\ \hline 5 \\ 51 \\ \hline 9 \end{array}$$

Example 4. What must be paid to a Labourer for his wages for 27 weeks at the rate of 4*s.* for 1 week? *Answ.* 5*l.* 8*s.*

After the three given terms are rightly placed, they will stand as you see in the Example; then multiplying the third term 27 by the second term 4, the product is 108, which divide by the first term 1; but since Division by 1 makes no alteration, the Quotient is also 108, so that the fourth term sought is 108 shillings, which being reduced to pounds, according to *Art. 122.* gives 5*l.* 8*s.* for the answer of the question.

$$\begin{array}{r} \text{Weeks. Shill. Weeks.} \\ 1 : 4 :: 27 \\ 4 \\ \hline 108 \end{array}$$

135. In the *Rule of Three*, if after the question is stated according to Art. 131. any of the three given terms be a compound term consisting of diverse denominations; as pounds, shillings, and pence; or weeks, days, hours, &c. such compound term must first be reduced into the lowest of those denominations (by Art. 121.) to the end that the three given terms may be three single numbers; also of these three single numbers, the first and third must always be of one and the same denomination: For if it happen that they express things of different names, that of the two which has the greater name (or denomination) is to be reduced into the same name with the lesser (by Art. 119.) These preparations being observed, the rest of the Work is to be prosecuted according to Art. 134. *Examp.* What will 48 ounces 17 penny-weight and 20 grains of silver-plate amount to at the rate of 5s. 6d. the ounce?

Answ. 13L 8s. 10d. 3f. very near.

This question being stated according to Art. 131. will stand as you see in the *Example*, to wit, if 1 ounce cost 5s. 6d. what will 48 oz. 17 p.wt. 20gr. cost? Here, because the third term is compounded of diverse denominations, it must be reduced into the lowest of those denominations, to wit, grains; so, by Art. 121. there

will be found 23468	oz.	s.	d.	oz.	p.w.	gr.
grains for the third term:	1	:	5 . 6	48	17	20
Likewise, because the	20	12	.			20
second term 5s. 6d. is	—	—	—	—	—	—
a compound term, whose	20	66	.	977		
lowest name is pence, it	24	.		24		
must be reduced into	—	—	—	—	—	—
pence (by the aforesaid	480			3928		
Rule:) So there will be				1954		
found 66 pence for the						
second term: Again be-						
cause the first term has						
the name ounce, and the third term the name grain, the						
first term 1 ounce must be converted into 480 grains (which						
are equal to 1 ounce) then will the three terms or single num-						
bers stand as here you see, viz. If 480						
grains cost 66 pence, how many pence gr. pence. gr.						
will 23468 grains cost? Now proceed 480 : 66 :: 23468						
ing according to Art. 134. there will						
arise in the Quotient 3226 pence, besides a remainder of 408						
pence, which being reduced to 1632 farthings, and those divided						
by the first term 480, the Quotient will be 3 farthings: So						
that the entire Quotient is 3226 pence, 3 farthings, and some-						
F 3						what

what more; (but the parts of a farthing being of no moment, may be neglected. Lastly, the said 3226 pence being reduced according to Art. 122. give 13l. 8s. 10d. 3f. so that 13l. 8s. 10d. 3f. and somewhat more, will be the answer of the question.

136. For the proof of the *Direct Rule of Three*, multiply the fourth term by the first; which done, if that product be equal to the product of the second term multiplied by the third, the Work is right, otherwise it is erroneous: So in the first Example, 38 the fourth term being multiplied by the first term 4, the product is 152, which is also the product of 19 multiplied by 8. But if it happen that after the fourth term, or answer of the question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the product of the first term, multiplied by such fourth term; and then the sum must be equal to the product of the second and third terms, (the second term consisting of the same denomination with the fourth:) So in the last Example the fourth term is 3226, and there happens to be a remainder of 408, which being added to the product of the Multiplication of the said 3226 by the first term 480, gives 1548888, which is the same with the product of the third term 23468 multiplied by the second term 66, as will appear when work'd.

137. Questions to exercise the *Rule of Three Direct*.

1. If 17 yards of cloth cost 19l. 2s. 6d. what will 35 yards cost at that rate? *Answ.* 39l. 7s. 6d.
2. If 35 yards cost 39l. 7s. 6d. how many yards may be bought at that rate for 19l. 2s. 6d.? *Answ.* 17 yards.
3. If 35 yards cost 39l. 7s. 6d. what are 17 yards worth at that rate? *Answ.* 19l. 2s. 6d.
4. If 17 yards be sold for 19l. 2s. 6d. how many yards will 39l. 7s. 6d. buy at that rate? *Answ.* 35 yards.
5. What must I pay for the carriage of 17 hundred weight, 3 quarters, and 11 pounds *Avoirdupoise*, at the rate of 7 shillings the hundred weight? *Answ.* 6l. 4s. 11d. 1f.
6. If 6l. 4s. 11d. 1f. be paid for the carriage of 17 hundred weight, 3 quarters and 11 pounds, what was paid for the carriage of 1 pound weight? *Answ.* 3 farthings.
7. What must I pay for 39 ounces, 7 penny-weight, and 18 grains of white plate, at the rate of 5s. and 5d. the ounce? *Answ.* 10l. 13s. 4d. and 3 quarters of a farthing.
8. What must 1l. (or 20s.) pay towards a tax, when 326l. 6s. 8d. is affested at 4l. 16s. 2d. 3f.? *Answ.* 2s. 6d.

3 f.

9. What

9. What will the interest of 876*l.* 17*s.* 6*d.* amount to for 1 year, at the rate of 6*l.* for 100*l.* for the same time? Answ. 52*l.* 12*s.* 3*d.*

10. If 3 yards in length of English Measure be equal to 4 ells Flemish, how many Flemish ells are contained in 120 yards English? Answ. 160 Flemish ells.

11. If 4 Flemish ells in length be equal to 3 English yards, how many English yards in 300 Flemish ells? Answ. 225 English yards.

12. If 3 ells in length of English measure be equal to 5 Flemish ells, how many Flemish ells in 120 English ells? Answ. 200 Flemish ells.

13. If 5 Flemish ells in length be equal to 3 English ells, how many English ells in 145 Flemish ells? Answ. 87 English ells.

14. If 3 ounces of silk-weight be equal to 4 ounces of Venice weight, how many ounces Venice are equal to 60 ounces of silk-weight? Answ. 80 ounces Venice.

15. A merchant delivered at London 120*l.* Sterling to receive 207*l.* Flemish at Amsterdam, what was 1*l.* Sterling valued at in Flemish money? Answ. 1*l.* 14*s.* 6*d.*

16. If a bill of exchange be accepted at London, for payment of 400*l.* Sterling for the value deliver'd at Amsterdam, at 1*l.* 13*s.* 6*d.* Flemish for 1*l.* Sterling, how much Flemish money was deliver'd at Amsterdam? Answ. 670*l.* 8*s.* 4*d.* Flemish.

17. When the exchange from Antwerp to London is at 1*l.* 4*s.* 7*d.* Flemish for 1*l.* Sterling, how much Sterling must I pay at London to receive 236*l.* Flemish at Antwerp? Answ. 192*l.* Sterling.

18. A merchant delivered at London 370*l.* Sterling by exchange for Roan, at 74*d.* Sterling for 50*s.* Tournois; how much Tournois ought he to receive at Roan? Answ. 60000*s.* Tournois.

19. In 370 ducats, at 4*s.* 2*d.* the ducat, how many French crowns at 6*s.* 2*d.*? Answ. 250 crowns. For if 74*d.* give 1 crown, 1850*d.* (or 370 ducats) will give 250 crowns.

20. In 516 dollars, at 4*s.* 5*d.* the dollar, how many guineas at 1*l.* 1*s.* 6*d.* the piece? Answ. 106 guineas. For if 258*d.* give 1 guinea, 27348*d.* (or 516 dollars,) will give 106 guineas.

21. At a noble per week, how many months board may I have for 50*l.*? Answ. 37 months, 2 weeks.

22. Bought a firkin of butter containing 56*lb.* for 18*s.* 8*d.* what is that per pound? Answ. 4*d.*

23. If a yard of Cambrick cost 12*s.* what cost 4 pieces each 20 yards? *Answ.* 48*l.*
24. If a gallon of beer cost 4*d.* what costs a barrel? *Answ.* 12*s.*
25. How much must I pay for the carriage of 10*lb.* C. wt. at the rate of 1*d.* per lb? *Answ.* 4*l.* 18*s.*
26. The cloathing of a regiment of 740 men comes to 3000*l.* how much is that for each man? *Answ.* 4*l.* 1*s.* 0*d.* 3*f.*
27. If a bushel of coals cost 10*d.* how many chalders for 100*l.*? *Answ.* 66 chalders, and 24 bushels.
28. If a man's yearly income be 300*l.* what is it per day? *Answ.* 16*s.* 5*d.* 1*s* 3*f.*
29. What cost 49392 case-knives at 4*s.* 4*d.* per dozen? *Answ.* 89*l.* 16*s.*
30. At 3*½ d.* per lb. what comes 30 C. weight 3 quarters 25 lb. of cheese to? *Answ.* 50*l.* 11*s.* 9*½ d.*
31. What is cheese per C. weight at 3*½ d.* per lb.? *Answ.* 1*l.* 12*s.* 8*d.*
32. A man bought a piece of cloth for 16*l.* 10*s.* at 15*s.* per yard, how many yards did it contain? *Answ.* 22 yards.
33. A draper bought 4 bales of cloth, each bale containing 6 pieces, and each piece 27 yards, at 16*l.* 4*s.* per piece, what was the price of the whole, and what the rate per yard? The whole cost 388*l.* 16*s.* and one yard cost 12*s.*
34. If 1 C. weight of cheese cost 37*s.* 4*d.* what is that per lb.? *Answ.* 4*d.*
35. If a yard of cloth is worth 14*s.* what is the worth of 5 pieces, each 19 yards? *Answ.* 66*l.* 10*s.*
36. Bought 12 pieces of cloth, each 12 yards, at 10*s.* 6*d.* per yard, what come they to? *Answ.* 75*l.* 12*s.*
37. A debtor owing several persons in all 1490*l.* 5*s.* 10*d.* compounds with, and pays them as far as his effects will go, which amount to no more than 931*l.* 8*s.* 7*½ d.* how much do the creditors by this composition receive per pound? *Answ.* 12*s.* 6*d.*
38. If 2 oz. of silk cost 2*s.* 6*d.* what costs 7 lb.? *Answ.* 7*l.*
39. If 1 oz. of silver cost 5*s.* 6*d.* what is the price of a tankard that weighs 1 lb. 10 oz. 10 penny-weights, 4 grains? *Answ.* 6*l.* 3*s.* 9*d.* 2*³⁶/₃₅₃ f.*
40. If an ingot of silver weighs 36 oz. 10 p.w. what is it worth at 5*s.* per ounce? *Answ.* 9*l.* 2*s.* 6*d.*
41. What will an estate of 4000*l.* per annum allow a gentleman to spend per day? *Answ.* 10*l.* 19*s.* 2*¹⁹/₃₄ d.*

42. If

42. If a gentleman has an estate of 245*l.* 10*s.* a year, how much may he spend one day with another, to lay up 60 guineas at the year's end? *Answ.* 10*s.*

43. A goldsmith sold a tankard for 10*l.* 12*s.* at the rate of 5*s.* 4*d.* per oz. what was the weight of it? *Answ.* 39 oz. 15 p. wts.

44. If 1 lb. of tobacco cost 15*d.* what cost 3 hogheads weighing (together) 15*C.* wt. 1 quarter, 19 lb.? *Answ.* 107*l.* 18*s.* 9*d.*

45. If a pint of wine cost 10*d.* what costs 3 hogheads? *Answ.* 63*l.*

46. If 17*C.* 3 quarters, 17 lb. of tobacco cost 133*l.* 13*s.* 4*d.* what costs 1 oz.? *Answ.* 1*d.*

47. If 1*C.* wt. of lead cost 15*s.* 1*d.* what costs 5 fotheres? *Answ.* 77*l.* 11*s.* 10*½ d.*

48. If 19 dozen pair of shoes cost 25*l.* 13*s.* what costs one pair? *Answ.* 2*s.* 3*d.*

49. If the expences in house-keeping six weeks amount to 9*l.* 3*s.* 6*d.* how long will 100*l.* last at that rate? *Answ.* 65 $\frac{145}{387}$ weeks.

50. If a dozen ells of Holland are valued at 3*l.* 6*s.* how much will 8 pieces, (each piece containing 54 ells) amount to at the same rate? *Answ.* 118*l.* 16*s.*

C H A P. IX. *Of the INVERSE RULE of THREE.*

I38. THE *Rule of Three Inverse*, is, when the fourth term required ought to proceed from the second term, according to the same rule or proportion that the first proceeds from the third: So this question being propounded, if 8 horses will be maintained 12 days with a certain quantity of provender, how many days will the same quantity maintain 16 horses? Here, as 8 is half 16, so ought the fourth term required to be half 12; for if certain bushels of provender serve 8 horses 12 days, 16 horses will eat as much provender in half that time: And therefore you cannot say here in a direct proportion (as before in the *Rule of Three Direct*) as 8 to 16, so is 12 to another number *Horses. Days. Horses.* which ought to be in that case as great again as 12; but contrarwaise by an

inverted

inverted proportion, beginning with the last term first; as 16 is to 8, so is 12 to another number, which ought to be in this case half 12. And by the due observation of this definition together with that of the *Rule of Three Direct* (laid down in Art. 133.) when any question belonging to the single *Rule of Three* is proposed, you may readily discern by which of those Rules it ought to be resolved; for if the three terms given look for a fourth in a direct proportion as they stand ranked in the Rule, you must resolve the question by the direct Rule; contrarywise when the proportion is inverted or turned backwards, it ought to be resolved by the *Inverse Rule of Three*, which here follows.

139. In the *Inverse Rule of Three*, after the three given terms are rightly placed and reduced (if there be need) according to Art. 135. multiply the first term by the second, (or which is the same) the second term by the first, and then divide the product by the third term, so the Quotient will give you the fourth term required, or answer of the question; thus in the question premised in the last Article, if you multiply 12 by 8, the product is 96, which if you divide by 16, the Quotient gives you 6, the fourth term required, as by the subsequent operation is manifest.

$$\begin{array}{r}
 \text{Horses. Days. Horses,} \\
 8 : 12 :: 16 \\
 \hline
 16) 96 (6 \\
 96 \\
 \hline
 0
 \end{array}$$

140. For the more ready discovering, whether a question propounded belongs to the *Rule of Three Direct*, or to the *Rule Inverse*, observe the following directions; viz. First, by the sense and tenor of the question consider, whether more be required or less; that is, whether the number sought for, must be greater or less than the second term: Secondly, esteeming the first and third terms as extremes in respect of the second, this will be a general Rule; namely, when more is required, the lesser extreme is the Divisor; but when less is required, the greater extreme is the Divisor. Lastly, the Divisor being found out, it will be apparent whether the Rule be Direct or Inverse; for when the Divisor is the first term, it is a *Rule Direct*; but when the Divisor is the third term, the Rule is *Inverse*.

Inverse. Another Example of the Rule *Inverse* may be this: If 12 Mowers do mow certain acres in 4 days, in what time will 23 Mowers perform the same work? *Answ.* 2 days, 2 hours and somewhat more. Here, the three known terms being rightly placed, will stand, as you see in the Example; and since it is evident that 23 men will require less time than 12 men to finish the same work, therefore (by the Rule afore-going) the greater of the two extreme numbers 23 and 12 must be the Divisor; and because the Divisor 23 stands in the third place, this question is to be work'd by the *Rule Inverse*; wherefore multiplying the first term 12 by the second term 4, the product is 48, which being divided by the third term 23, the Quotient gives 2 days, and there is a remainder of 2 days, which being reduced to hours, and those divided by 23, the Quotient will be 2 hours; and there is yet a remainder of 2 hours to be subdivided into 23 parts if you please; so that the fourth term sought, or answer of the question, is, 2 days, 2 hours, and somewhat more.

Again, take this for a third Example: If I lend my friend 356 pounds for 1 year and 35 days, (the year being supposed to consist of 365 days) how long time ought he to lend me 500 pounds to requite my courtesy? *Answ.* 284 days, and somewhat more, there being a remainder, to wit, 400, after the Division is finished, as by the subsequent operation is manifest.

$$\begin{array}{r} l. \\ \text{If } 356 : \\ \hline \end{array} \quad \begin{array}{r} y. \\ \text{. } 35 :: \\ \hline \end{array} \quad \begin{array}{r} d. \\ 500 \\ \hline \end{array}$$

$$\begin{array}{r} 365 \\ \text{add } 35 \\ \hline \end{array}$$

multiply { 400
 { 356

$$5|00)1424|00(284 \text{ Days.}$$

141. The proof of the *Inverse Rule of Three* is this, multiply the third term by the fourth, then if this product be equal to the product of the first term multiplied by the second, the Work is true, otherwise erroneous; so in the *Example* of Art.

Art. 139. the product of 16 and 6 is equal to the product of 8 and 12. But if it happen that, after the fourth term, or answer of the question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the product of the third term, multiplied by the fourth, and then the sum must be equal to the product of the first and second terms (such second term being of the same particular denomination with the fourth:) So in the last *Example*, the fourth term is 284 days, and there remains 400 after the division is finish'd, this 400 being added to the product of the Multiplication of the third term 500 by the fourth term 284, gives 142400, which is equal to the product of the first term 356, multiplied by the second term 400 days.

142. In the *Rule of Three*, as well *Direct* as *Inverse*; when the Divisor with either of the other two given numbers may be severally divided by some common measure, without leaving any remainder, the quotients may be taken for new terms, and proceeding in like manner as often as possible, the operation will be much contracted: So if it be demanded, what 52 yards of cloth will cost at the rate of 21*l.* for 14 yards; the answer will be found 78 pounds in manner following.

y.	l.	y.
If 14 .	21 .	52
2 .	3 .	52
1 .	3 .	26 . (78)

In the first rank you may observe that the Divisor 14 and the second term 21, being severally divided by their common measure 7, the three new terms (in the second rank) will be 2, 3, 52. Again, in the second rank, the Divisor 2 and the third term 52 being severally divided by their common measure 2, the three new terms (in the third rank) will be 1, 3, 26. Lastly, working with these according to the *Rule of Three Direct*; the answer to the question, (or fourth term) will appear to be 78.

Another Example. If 21 men can finish a work in 16 days, what time must be allowed to 12 men for the finishing of such a work? *Answ.* 28 days.

Men.	Days.	Men.
21 .	16 .	12
7 .	16 .	4
7 .	4 .	1 (28 Days.)

In the first rank you may observe, that the Divisor 12 (for the *Rule is Inverse*,) and the first term 21 being severally divided

vided by their common measure 3, the three new terms in the second rank will be 7, 16, 4. Again, in the second rank, the Divisor 4, and the second term 16 being severally divided by their common measure 4, the three new terms in the third rank will be 7, 4, 1. Lastly, working with these as the *Rule of Three Inverse* requires, the answer to the question (or fourth term) will be found 28.

143. Questions to exercise the *Rule of Three Inverse.*

1. There was a certain Building raised in 8 months, by 120 workmen; but the same being demolished, it is required to be rebuilt in 2 months: How many men must be employed about it? *Answ.* 480 men.

2. If 28 s. will pay for the carriage of an hundred weight 150 miles, how far may 6C. wt. be carried for the same money? *Answ.* 25 miles.

3. If for 5l. 5s. I have 14 C. wt. carried 136 miles, how many miles may I have 24 C. wt. carried for the same money? *Answ.* $79\frac{1}{2}$ miles.

4. If a footman performs a journey in 3 days, when the days are 16 hours long, how many days will he require of 12 hours long to go the same journey in? *Answ.* 4 days.

5. How many yards of plush is sufficient to make a cloak of equal magnitude with one which hath in it 4 yards of 7 quarters wide, when the plush is but 3 quarters wide? *Answ.* $9\frac{1}{3}$ yards of plush.

6. How many yards of canvas, that is ell-wide, will be sufficient to line 20 yards of Say, that is three quarters wide? *Answ.* 12 yards.

7. If a man performs a journey in 6 days, when the day is 8 hours long; in what time will he do it, when the day is 12 hours long? *Answ.* 4 days.

8. If I lent my friend 100l. for 6 months (allowing the month to be 30 days) how long ought he to lend me 1000l. to requite my kindness? *Answ.* 18 days.

9. If 6 mowers can mow a field in 12 days, in what time will 24 mowers do it? *Answ.* 3 days.

10. Suppose 800 soldiers were placed in a garrison, and their provisions were computed sufficient for 2 months, how many soldiers must depart, that the provisions may serve them 5 months? *Answ.* 480 men.

11. Admit that I lent to a friend on his occasion 100l. for 6 months, and he promised me the like kindness when I desired it; but

ut when I came to request it, he could lend me only 75*l.* the question is, how long I may keep his money, to recompence my courtesy to him? *Ans.* 8 months.

C H A P. X.

The Double Golden Rule Direct, performed by Two single Rules.

144. **T**HE Compound Golden Rule is, when more than 3 terms are proposed; therefore, under the compound Golden Rule is comprehended the Double Golden Rule, and divers Rules of plural proportion.

145. The Double Golden Rule is, when five terms being given, a sixth proportional to them is demanded: As in this question, if 4 students spend 19 pounds in 3 months, how much will serve 8 students 9 months? Or this, if 9 bushels of provender serve 8 horses 12 days, how many days will 24 bushels last 16 horses?

146. The five terms given in this Rule consist of two parts; viz. a supposition expressed in the three first terms; and a demand made in the two last: So in the first Example, this clause (if 4 students spend 19 pounds in 3 months) is the supposition, and this (how much will serve 8 students 9 months;) is the Demand? Likewise, in the other Examples, this clause (if 9 bushels of provender serve 8 horses 12 days) is the supposition, and this (how long, or how many days will 24 bushels last 16 horses,) is the proposed demand?

147. Here for ranking the terms given in their due order, first observe amongst the terms of supposition, which of them has the same denomination with the term required: Then reserving that term for the second place, write the other two terms of supposition one above another in the first place; and lastly, the terms of demand likewise one above another in the third place of the Rule, in such sort that the uppermost may have the same denomination with the uppermost of those in the first place. *Example.* If 4 students spend 19 pounds in 3 months, how much will serve 8 students 9 months? Here the three terms of supposition are 4, 19 and 3, and of these, term 19 has the same denomination with the term required, viz. of pounds;

pounds; for you are to enquire how much money is requisite for the maintenance of 8 students 9 months: Wherefore referring 19 for the second place, write 4 and 3 one above another, thus; then on the right-hand of 4 : 19
4, write 19 in the second place; this done, 3
the work will stand as in the Margin: Last of all, the terms of demand being 8 and 9, and 8 having the denomination of students, place it in the same line with 4 and 19, and write 9 under it; all this performed, the terms in this question rank themselves as follows, viz. Thus,

$$\begin{array}{r} 4 \cdot 19 : 8 \\ 3 \quad \quad \quad 9 \end{array}$$

Or thus,

$$\begin{array}{r} 3 \cdot 19 : 9 \\ 4 \quad \quad \quad 8 \end{array}$$

In like manner, if the second question, in Art. 145. were propounded; the terms thereof ought to be disposed thus,

$$\begin{array}{r} 8 \cdot 12 : 16 \\ 9 \quad \quad \quad 24 \end{array}$$

Or thus,

$$\begin{array}{r} 9 \cdot 12 : 24 \\ 8 \quad \quad \quad 16 \end{array}$$

148. Questions belonging to the *Double Golden Rule* may be resolved by two single Rules of Three.

149. When Questions of this nature are resolved by two single Rules, the Proportions are as follow:

1. As the uppermost term of the first place, is to the middle term; so is the uppermost term of the last place to a fourth number.

2. As the lower term of the first place, is to that fourth number; so is the lower term of the last place to the term required.

So in this Example before recited, using tacitly the lower term of the first place as a common number in the first proportion: 4 : 19 : 8 3 : 9
Say thus,

1. If 4 students spend 19 pounds (in 3 months) what will serve 8 students the same time?

Or thus, If 4 students spend 19 pounds, what will 8 spend?

Which Rule of Three will be discovered to be direct (by Art. 140.) therefore the fourth proportional proceeding from the

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the said three given numbers 4, 19, and 8 is 38, (by Art. 134.) Again, to find the term required, using tacitly the uppermost term of the third place as a common number in this last proportion; say as follows:

2. If in 3 months 38 pounds are spent (by 8 students) how much will serve them for 9 months?

Or thus, if 3 give 38, what will 9 yield?

Which *Rule of Three* will likewise be discovered to be direct (by Art. 140.) therefore the fourth proportional proceeding from the said 3 numbers, 3, 38, and 9, you will likewise find (by Art. 134.) to be 114; for 38 being multiplied by 9 the product is 342, which divided by 3, yields in the quotient 114: So that if 4 students spend 19 pounds in 3 months, 114 pounds will serve 8 students 9 months.

150. In like manner, if two single Rules of Three be formed (according to the preceding Art. 149.) out of the five numbers given in the last-mentioned question, the same being ranked according to the latter manner of ordering the said numbers in Art. 147. each of the said two Rules of Three will be a Rule Direct, and the same answer of the question, to wit, 114 pounds will be discovered, as may be seen by the subsequent operation.

$$\begin{array}{r} 3 : 19 : : 9 \\ 4 : \quad \quad \quad 8 \end{array}$$

$\begin{array}{r} 3 : 19 :: 9 \\ 9 \end{array}$	$\begin{array}{r} 4 : 57 :: 8 \\ 8 \end{array}$
$\begin{array}{r} 3) 171. (57 \\ 15 \end{array}$	$\begin{array}{r} 4) 456 (114 \\ 4 \end{array}$
$\begin{array}{r} 21 \\ 21 \\ \hline 0 \end{array}$	$\begin{array}{r} 05 \\ 4 \\ \hline 16 \\ 16 \\ \hline 0 \end{array}$

151. The *Double Golden Rule* is either *Direct* or *Inverse*.

152. The *Double Golden Rule Direct*, is, when both the single Rules do each of them look for a fourth term in a direct proportion; as in the Example, Art. 149. where each of the two single Rules of Three is Direct.

For

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For another *Example* take this. If the carriage of 8*C.* weight 128 miles cost 48 shillings, for how much may I have 4*C.* weight carried 32 miles after the same rate? The terms of this question, according to *Art.* 147. rank themselves in this order.

$$\begin{array}{r} 128 \quad . \quad 48 \quad . \quad 32 \\ \quad 8 \quad . \quad \quad \quad 4 \end{array}$$

Now taking tacitly the lower term of the first place as a common number, form the first Rule of Three, according to *Art.* 149. saying,

1. If the carriage of a certain weight (to wit, 8*C.*) 128 miles, costs 48 shillings, what will the carriage of the same weight 32 miles cost?

Here it is easy to discern, that the fewer miles any weight is carried, the less money will pay for the carriage of that weight; therefore the fourth number sought by the said Rule of Three must be less than the second number 48? And in regard that by *Art.* 140. when less is required, the greater extreme (whether it be the first or third number) must be the Divisor; therefore the first number 128 is the Divisor, and consequently the Rule of Three above proposed is a Rule Direct; then finding out the fourth number, by *Art.* 134. to be 12 shillings, proceed to the second proportion, and say,

2. If the carriage of 8*C.* (32 miles) cost 12 shillings, how much must I give to have 4*C.* carried the same distance?

And here likewise finding a fourth number to be looked for in a direct proportion, discover that fourth by *Art.* 134. to be 6*s.* which is the term demanded, and the answer to the question propounded: So if the carriage of 8*C.* 128 miles cost 48*s.* the carriage of 4*C.* 32 miles will cost 6*s.* according to the same rate.

C H A P. XI.

The Double Golden Rule Inverse, performed by two single Rules.

153. **T**H E *Double Golden Rule Inverse*, is, when one of the single Rules looks for a fourth term in an inverted proportion: As in the last *Example* proposed in *Art.* 143. For if you rank the terms of that question, according to *Art.* 147. Thus,

$$\begin{array}{r} 8 \\ \times \\ 9 \end{array} \quad \begin{array}{r} 12 \\ + \\ 24 \end{array} \quad \begin{array}{r} 16 \\ 24 \end{array}$$

And then work by two single Rules of Three, formed according to Art. 149. you'll find by Art. 140. that the first of the said two Rules of Three will be Inverse, and the latter Direct. For saying first, if 8 horses be maintained 12 days (by 9 bushels of provender,) how many days will 16 horses be kept by so much provender? Here the answer 6 days will be found out by the Rule of Three Inverse: Secondly, saying, if 9 bushels of provender be eaten up (by 16 horses) in 6 days, in how many days will 24 bushels be spent? Here the answer 16 days will be found out by the Rule of Three Direct.

But if you order the given terms of the same question Thus,

$$\begin{array}{r} 9 \\ \times \\ 8 \end{array} \quad \begin{array}{r} 12 \\ + \\ 16 \end{array} \quad \begin{array}{r} 24 \\ 16 \end{array}$$

And then work by two single Rules of Three, formed according to Art. 149. you'll find by Art. 140. that the first of the said two Rules of Three will be Direct, and the latter Inverse. For saying, first, if 9 bushels of provender last 12 days (to maintain 8 horses) how many days will 24 bushels serve the same number of horses? The answer 32 days will be found out by the Rule of Three Direct. Secondly, saying, if 8 horses are maintained 32 days, (by 24 bushels of provender,) how long will 16 horses be kept by the same quantity of provender? Here the answer 16 days will be found out by the Rule of Three Inverse.

Therefore, whensoever a question belonging to the Double Rule of Three is separated into two single Rules of Three, (according to the preceding Rules) if one happens to be a Rule Inverse, that Double Rule is called the Double Rule Inverse.

C H A P. XII. *The RULE of FELLOWSHIP.*

154. **T**H E Rules of plural proportion are those, by which we resolve questions that are discoverable by more Golden Rules than one, and yet cannot be performed by the Double Golden Rule mentioned before in the two last chapters. Of these Rules there are divers kinds and varieties according

cording to the nature of the question proposed; for here the terms given are sometimes four, sometimes five, sometimes more, and the terms required sometimes more than one, &c.

155. Two particular Rules of plural proportion are these, the Rule of Fellowship, and the Rule of Alligation.

156. The Rule of Fellowship is that, by which in accounts among divers men (their several stocks together with the whole gain or losf being given) the gain or losf of each particular man may be discovered: As in this Example, *A* and *B* were sharers in a parcel of merchandize, in the purchase of which *A* laid out 7*l.* and *B* 11*l.* and they having sold this commodity, find that their clear gains amount to 54*s.* Now here the question to be resolved by this Rule, is, what part of that 54*s.* belongs to *A*, and what to *B*, according to the rate of the several sums and stocks which they adventured? Again, *A*, *B*, and *C*, freight a ship from the *Canaries* for *England* with 108 tuns of wine, of which *A* had 48, *B* 36, and *C* 24, the mariners meeting with a storm at sea, were constrained for the safety of their lives, to cast 45 tuns thereof over-board: Here the question to be resolved, is, how many of the 45 tuns every particular merchant has lost, according to the rate of his adventure.

157. The Rule of Fellowship is either single, or double.

158. The single Rule is, when the stocks proposed continue in the adventure (or common bank) equal times, to wit, one stock as long time as another.

159. In the single Rule of Fellowship, take the total of all the stocks for the first term, the whole gain or losf, for the second, and the particular stocks for the third term; that done, repeating the Rule of Three so often, as there are particular stocks in the question, the fourth terms produced upon those several operations, are the respective gains or losses of those particular stocks propounded: So in the first Example above-mentioned 7*l.* and 11*l.* are the stocks proposed, whose total is 18*l.* which take for the first term: Again, 54*s.* the common gain, is the second term, and 7*l.* the first particular stock, is the third term of the first proportion; whereupon say, as 18*l.* to 54*s.* so 7*l.* to another number, which by the Direct Rule of Three is 21*s.* viz. the part of the gain due to *A*, who expended the 7*l.* stock. Then for the second proportion, say, as 18*l.* to 54*s.* so 11*l.* to another number, which by the Rule of Three Direct is 33*s.* viz. the part of the gain due to *B*, for his 11*l.* stock.

$$\text{As } 18\text{:}54 :: \left\{ \begin{array}{l} 7\text{:}21 \\ 11\text{:}33 \end{array} \right.$$

Again, in the other premised Example, the particular Loss that happens to *A* is 20 tuns, to *B* 15, and to *C* 10 tuns.

$$\text{For as } 108 : 45 :: \begin{cases} 48 : 20 \\ 36 : 15 \\ 24 : 10 \end{cases}$$

160. The Double Rule of Fellowship is, when the stocks proposed are double numbers, *viz.* when each stock has relation to a particular time: Example, *A*, *B*, and *C*, hold a pasture in common, for which they pay 45*l.* per Annum. In this pasture, *A* had 24 oxen during 32 days; *B* had 12 there 48 days; and *C* fed 16 oxen there 24 days: Now the question to be resolved by this Rule, is, what part each of these tenants ought to pay of the 45*l.* rent? And here you may observe, that the stocks propounded are double numbers, *viz.* each stock of oxen has reference to a particular time; for the respective stock of *A* is 24 oxen, and its particular time is 32 days; again, the stock of *B* is 12 oxen, and the respective time is 48 days: And lastly, the stock of *C* is 16 oxen, and its peculiar time is 24 days, which, as you see, are double numbers.

161. In the Double Rule of Fellowship, multiply each particular stock by its respective time, and take the total of their products for the first term, the whole gain or loss for the second, and the said particular products of the double numbers for the third term: This done, repeating, as before, the Rule of Three, so often as there are products of the double numbers; the fourth terms produced upon those several operations, are the numbers you look for: So in the above Example, the product of 24 and 32, is 768, the product of 12 and 48 is 576, and the product of 16 and 24 is 384, the sum of these products is 1728, which is the first term in the question; then 45*l.* the rent, is the second term, and 768 the first product, is the third term of the first proportion: Wherfore I say, as 1728 to 45*l.* so 768 to another number, which I find by the Direct Rule of Three to be 20*l.* *viz.* the part of the rent that *A* ought to pay: Then for the second proportion say, as 1728 to 45*l.* so 576 to 15*l.* which is the part that *B* ought to pay: And lastly, as 1728 to 45*l.* so is 384 to 10*l.* *viz.* the part that *C* must pay.

$$\text{As } 1728 : 45 :: \begin{cases} 768 : 20 \\ 576 : 15 \\ 384 : 10 \end{cases}$$

A second Example: Three merchants *A*, *B*, and *C*, enter partnership, and agree to continue in a joint adventure 16 months; *A* puts into the common stock at the beginning of the

the said term 100 pounds, at 8 months end he takes out 40 pounds, and 4 months after such taking out he puts in 140 pounds. *B* puts in at first 200 pounds, at 6 months end he puts in 50 pounds more, and 4 months after the putting in of the 50 pounds, he takes out 100 pounds. *C* puts in at first 150 pounds, at 4 months end he takes out 50 pounds, and 8 months after such taking out puts in 100 pounds. Now, at the end of the said 16 months, they had gained 357 pounds, the question is, how much of the said gains belongs to every merchant for his share?

In questions of this nature, two things are principally to be observed : 1. The whole time of partnership. 2. The respective time belonging to each man's particular stock ; so here, it is evident that the whole time is 16 months, and the particular stocks and times belonging to every merchant will be as follows, *viz.*

<i>A</i> . had 100 <i>l.</i> in the common stock for 8 months,	{	800
therefore 100 <i>l.</i> multiplied by 8 produces _____		
Also 60 <i>l.</i> for 4 months, therefore 60 multiplied by	{	240
4 produces _____		
Also 200 <i>l.</i> for 4 months, therefore 200 multiplied by	{	800
4 produces _____		
The total of the products of money and time for	{	1840
<i>A</i> is _____		

<i>B</i> , had 200 <i>l.</i> in the common stock for 6 months,	{	1200
therefore 200 multiplied by 6 produces _____		
Also 250 <i>l.</i> for 4 months, therefore 250 multiplied	{	1000
by 4 produces _____		
Also 150 <i>l.</i> for 6 months, therefore 150 multiplied	{	900
by 6 produces _____		
The total of the products of money and time for	{	3100
<i>B</i> , is _____		

<i>C</i> , had 150 <i>l.</i> in the common stock for 4 months,	{	600
therefore 150 multiplied by 4 produces _____		
Also 100 <i>l.</i> for 8 months, therefore 100 multiplied	{	800
8 produces _____		
Also 200 <i>l.</i> for 4 months, therefore 200 multiplied	{	800
4 produces _____		
The total of the products of money and time for	{	2200
<i>C</i> , is _____		

Then adding the said three Totals together, to wit, 1840, 3100, and 2200, the sum is 7140; wherefore proceeding as in the last Example, say, by the Rule of Three Direct, as 7140 is to the total gain 357 pounds; so is 1840 to 92 pounds the gain of A: Again, as 7140 is to 357; so is 3100 to 155 the gain of B: Lastly, as 7140 is to 357; so is 2200 to 110 the gain of C.

162. The Rule of Fellowship is proved by addition of the terms required, whose sum ought to be equal to the second term in the question, otherwise the whole Work is erroneous: So in the first Example in Art. 159. 21*s.* and 33*s.* being added together, are equal to 54*s.* the second term in that question. Likewise in the last Example of the same Article, as also in the first Example of the last Article, the sum of 20, 15, and 10, the terms required, are equal to 45, the second term propounded,

C H A P. XIII.

Of the RULE of ALLIGATION,

163. **T**H E Rule of Alligation is that, by which we resolve questions, that concern the mixing of divers simples together.

164. Alligation is either Medial or Alternate.

165. Alligation Medial, is, when having the several quantities and rates of diverse simples proposed, we discover the rate of a mixture compounded of those simples. So 10 bushels of wheat at 4*s.* or (which is all one) 48*d.* the bushel; 40 bushels of rye at 3*s.* or 36*d.* the bushel; and 50 bushels of barley at 2*s.* or 24*d.* the bushel; being mixed with 20 bushels of oats at 12*d.* the bushel, the Rule of Alligation Medial shews you the price of that mixture.

166. In Alligation Medial, first sum up the given quantities, then find the total value of all the simples: That done, the proportion will be as follows:

As the sum of the quantities is to the total value of the simples:

So is any part of the mixture proposed, to the required rate or price of that part.

Repeating again the above Example, I demand how much one bushel of that mixture is worth? Now the sum of 10, 40,

50, and 20, (the given quantities) is 120 bushels; and the value of the 10 bushels of wheat at 4*s.* the bushel amounts to 48*d.* Again, the value of the 40 bushels of rye at 3*s.* the bushel, is 1440*d.* The value of the 50 bushels of barley at 2*s.* the bushel, is 1200*d.* And the value of 20 bushels of oats at 1*s.* the bushel, is 240*d.* All these values being added together, their total is 3360*d.* Then say, by the Rule of Three Direct, if 120 bushels give 3360*d.* what will 1 bushel yield? The Rule answers 28*d.* Therefore a bushel of that mixture may be afforded for 28*d.* that is, 2*s.* 4*d.* which is the resolution of the Question proposed.

In like manner, if it be demanded, what 8 bushels or a quarter of that mixture is worth, the answer will be 224*d.* which being divided by 12, and by that means reduced into shillings is 18*s.* 8*d.*

167. In Alligation Medial, the trial of the work is by comparing the total value of the several simples with the value of the whole mixture: For when those sums accord, the operation is perfect: So in the above Example:

l. s. d.
10 bushels of wheat at 4 <i>s.</i> the bushel, is _____ 2 : 0 : 0
40 bushels of rye at 3 <i>s.</i> the bushel, is _____ 6 : 0 : 0
50 bushels of barley at 2 <i>s.</i> the bushel, is _____ 5 : 0 : 0
And 20 bushels of oats at 1 <i>s.</i> the bushel, is _____ 1 : 0 : 0
All which amount to _____ 14 : 0 : 0

Which is likewise the value of 120 bushels, at 28*d.* or 2*s.* 4*d.* the bushel, for that also amounts to 14*l.*

168. Alligation Alternate, is, when having the several rates of diverse simples given, we discover such quantities of them, as are necessary to make a mixture, which may bear a certain rate propounded.

Example. A man being determined to mix wheat at 4*s.* or 48*d.* the bushel, with rye of 3*s.* or 36*d.* the bushel, with barley of 2*s.* or 24*d.* the bushel, and with oats of 1*s.* or 12*d.* the bushel; the Rule of Alligation Alternate will discover to you how much rye, how much barley, and how much oats he ought to add to the wheat, in such sort, that the mixture of them all together may bear a certain rate or price proposed.

169. In questions of Alligation Alternate, you must rank the terms after such a manner, that the given rate of the mixture may represent the root, and the several rates of the simples may stand as branches issuing from that root: So the above Example being laid down, demand how much rye, barley, and oats, ought to be added to the wheat, that the mixture of all together may bear the rate or price of 28*d.* or 2*s.* 4*d.* the bushel:

bushel: And therefore, drawing a line of connexion, place 28 d. the given rate of the mixture, upon the left-hand thereof, by itself, representing the root, and likewise write the other rates proposed, viz. 48 d. 36 d. 24 d. and 12 d. one above another upon the right-hand of that line of connexion, which rates are conceived to issue from 28 d. as branches from the root, the fabrick whereof appears plainly in the margin.

$$\begin{cases} 48 \\ 36 \\ 24 \\ 12 \end{cases}$$

170. Having ranked the terms in their due order, link the branches together by certain arches, in such sort, that one that is greater than the root or rate of the mixture, may always be coupled with another that is less than the same: So in the pre-mised Example, 48 may be linked with 12, and 36 with 24, or otherwise 48 may be coupled with 24, and 36 with 12, and then the work will stand thus:

$$28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$$

$$\text{Or thus, } 28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$$

171. Having alligated the branches and found the differences betwixt them and the root, write the differences of each branch just against its respective yoke-fellow. So the branches of the Example aforesaid being linked after the first manner, and the difference between 28 and 48 being 20, place 20 just against 12, the respective yoke-fellow of 48. Again, 16 being the difference between 28 and 12, write it just against 48. In like manner 8 being the difference between 28 and 36, place it right against 24. And lastly, 4, the difference between 28 and 24, write just against 36: In the end the whole fabrick of the Work (as the branches are thus linked) will stand as in this Example.

$$\begin{array}{c|c} & \begin{array}{c} 48 \\ 36 \\ 24 \\ 12 \end{array} \\ \hline 28 & \left\{ \begin{array}{l} 20 \\ 16 \\ 8 \\ 4 \end{array} \right. \end{array}$$

172. And the differences thus placed will answer the question; for if to 16 bushels of wheat at 4 s. per bushel, be added 4 bushels of rye at 3 s. 8 bushels of barley at 2 s. and 20 bushels of oats at 1 s. the price of a bushel of the mixture will be 2 s. 4 d.

For

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For 16 bushels of wheat at 4*s.* are worth 64*s.*

4	rye	3 <i>s.</i>	12
8	barley	2 <i>s.</i>	16
20	oats	1 <i>s.</i>	20

Therefore 48 bushels of the mixture are worth 112*s.*

And 1 bushel thereof is worth $\frac{112}{48} s. = \frac{7}{3} s. = 2 s. 4 d.$

173. But the branches being linked after the other manner, the Work will be thus disposed:

48	4
36	16
24	20
12	8

For in this Case 48 has 24 for its yoke-fellow, and the respective comrade of 36 is 12; and here the interchangeable placing of the differences (as in the premised Examples) is that which is more particularly termed Alternation.

174. The numbers thus found will also answer the question.

For 4 bushels of wheat at 4*s.* are worth 16*s.*

16	rye at 3 <i>s.</i>	48 <i>s.</i>
20	barley at 2 <i>s.</i>	40 <i>s.</i>
8	oats at 1 <i>s.</i>	8 <i>s.</i>

Therefore 48 bushels of the mixture are worth 112*s.* as before.

175. When one branch is linked to diverse other branches, and not to one alone, the differences ought to be as often transcribed, as it is so diversely linked. So in the premised Example, you may (if you please) conceive 12 to be coupled both with 48 and 36; likewise 24 may be conceived to be linked with the same 48 and 36; wherefore the difference between 28 and 12 being 16, write it against both 48 and 36. In like manner the difference between 28 and 24 being 4, write it likewise over-against the same numbers 48 and 36. Again, 20 being the difference betwixt 28 and 48, place it just against 24 and 12; and 8 being the difference between 28 and 36, write it likewise over-against the same numbers 24 and 12: All this performed, the whole frame of the work will stand as in the Margin:

48	16 4.
36	16 4.
24	20 8.
12	20 8.

176. When, in one and the same line, there are found more differences than one, add them together, and write the sum just against the same differences beyond a straight line drawn towards the right-hand of the Work.

So in the last Example, the sum of 16 and 4 (the differences placed just against the first branch) being 20, write it over-against the same differences, beyond the new line drawn upon the right-hand of the Work, and so consequently the rest in their due order, as appears by the Work hereunto annexed.

	16 4. 16 4. 20. 8. 20. 8.	20. 20. 28. 28.
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Which numbers are a third answer to the question.

For	20 bushels of wheat	at 4 <i>s.</i>	are worth	80 <i>s.</i>
	20	rye	at 3 <i>s.</i>	60 <i>s.</i>
	28	barley	at 2 <i>s.</i>	56 <i>s.</i>
	28	oats	at 1 <i>s.</i>	28 <i>s.</i>

Therefore 96 bushels of the mixture are worth . 224 s.

And 1 bushel is worth $\frac{324}{864} = \frac{112}{486}$ as before.

177. The reason of these combinations and the alternate placing of their differences will appear from this plain consideration, *viz.* that whatever is lost by selling any quantity whose given price exceeds the mean, is gained again on the quantity alligated thereto, whose given price is less than the mean.

Thus, in the Example work'd in Art. 171, and proved in Art. 172.

**By selling 16 bush. at
which are worth 28 d. } you lose $16 \times 20 = 320$ d.
48 d. }**

And by selling 20 bush. at which are worth but 28 d. you gain $20 \times 16 = 320$ d.

Again, by selling 4 bush. at 28 d. } which are worth 26 d. { you lose $4 \times 8 = 32$ d.

But by selling 8 bush. at which are worth but
 $28 d.$ } you gain $8 \times 4 = 32 d.$

178. Take this for another Example: It is required to mix wheat at 48 d. the bushel, with rye of 36 d. the bushel, with barley of 24 d. the bushel, and with oats of 12 d. the bushel; and the question now is, how much rye, barley, and oats ought

to be added to the wheat, that the entire mixture may be afforded at 16 d. the bushel? Here the branches of this question (according to Art. 170.) ought to be linked: Thus,



And as for the Alteration of the differences, it is evident (by Art. 176.) that the difference between 16 and 12 being 4, ought to be thrice transcribed, viz. first, just against 48, then against 36, and last of all against 24. Again, 32 the difference between 16 and 48, as also 20 the difference between 16 and 36; and lastly, 8 the difference betwixt 16 and 24, ought all to be placed just against 12.

$\left\{ \begin{matrix} 48 \\ 36 \\ 24 \\ 12 \end{matrix} \right.$	$\left \begin{matrix} 4 \\ 4 \\ 4 \\ 32. \end{matrix} \right.$	$\left \begin{matrix} 4 \\ 4 \\ 4 \\ 60 \end{matrix} \right.$

179. Again, determining to mix wheat at 48 d. the bushel, with rye of 36 d. the bushel, with barley of 24 d. the bushel, and with oats of 12 d. the bushel, I desire to know how much of each I ought to take, that I may afford the whole mixture at 40 d. the bushel: Here the whole Work being ordered according to the Rules foregoing, will stand as follows.

$\left\{ \begin{matrix} 48 \\ 36 \\ 24 \\ 12 \end{matrix} \right.$	$\left \begin{matrix} 4. \\ 8 \\ 8 \\ 8 \end{matrix} \right.$	$\left \begin{matrix} 16. \\ 28. \end{matrix} \right \begin{matrix} 48 \\ 8 \end{matrix}$

180. A man intending to mix wheat at 48 d. the bushel, with rye of 36 d. the bushel, with barley of 24 d. the bushel, with pease of 16 d. the bushel, and with oats of 12 d. the bushel, desires to know how much rye, barley pease, and oats he ought to add to the wheat, that the whole mass of corn so mixed may be afforded at 20 d. the bushel. This question being thus proposed, the terms of it (by the Rules foregoing) may be alligated, and the differences of the terms alternated, as follows:

$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right.$	$\begin{array}{c} 4 \\ 4 \\ 4. 8. \\ 28. 16. 4. \\ 4 \end{array}$	$\begin{array}{c} 4 \\ 4 \\ 12 \\ 48 \\ 4 \end{array}$
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181. When two kinds of things only are given to be mixed, this Rule of Alligation will give but one answer; for instance, Suppose it were required to mix brandy at 8*s.* per gallon, with cyder at 1*s.* per gallon, so as to make the mixture worth 5*s.* per gallon? The operation would stand as below.

$$5 \left\{ \begin{array}{l} 8 \\ 1 \end{array} \right) \quad \begin{array}{c} 4 \\ 3 \end{array}$$

And the answer will be 4 gallons of brandy, and 3 of cyder.

182. Nevertheless let it be observed that any other two numbers that are in the same proportion to each other, as 4 and 3, will also answer the question. Thus 8 and 6, 12 and 9, 16 and 12, &c. are answers to the question: For in these the quantity of the mixture is the double, triple, quadruple, &c. of the quantity in the former; and so are the parts of which it is composed.

183. If three kinds of things are given to be mixed, the Rule of Alligation will give but one answer; but then, as before, all numbers that are in the same proportion between themselves, as the numbers which compose that answer, will also satisfy the question.

184. But this is not all, for by the help of an artifice, now to be explained, innumerable other answers may be obtained, the numbers composing which are not in the same proportion as the above.

Let it be required to mix brandy at 8*s.* per gallon, with wine at 7*s.* per gallon, and cyder at 1*s.* per gallon, so that the mixture may be worth 5 shillings per gallon? The Work by the Rule of Alligation will stand as follows.

$5 \left\{ \begin{array}{l} 8 \\ 7 \\ 1 \end{array} \right)$	$\begin{array}{c} 4 \\ 4 \\ 3, 2 \end{array}$	$\begin{array}{c} 4 \\ 4 \\ 5 \end{array}$
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Which shews that 4 gallons of brandy, 4 of wine, and 5 of cyder will answer the question.

For

For	4 gallons of brandy at 8 s. per gal. are worth	<u>32 s.</u>
4 ditto	wine at 7 s. ditto	<u>28 s.</u>
5 ditto	cyder at 1 s. ditto	<u>5 s.</u>

Therefore 13 ditto of the mixture are worth 65 s.
And 1 ditto is worth $\frac{6}{13} s. = 5 s.$

Now let us suppose, that it should be determined to use 5 gallons of cyder in the mixture constantly, but to use any quantities of brandy and wine that will answer the question.

Then may the quantity of brandy be increased or diminished by 2, the difference between the prices of the wine and mixture; if at the same time the quantity of the wine be diminished or increased by 3, the difference of the prices of the brandy and mixture.

That is to say, to (4) the quantity of brandy given by the above answer, add 2; and from (4) the quantity of wine, take 3; so shall the sum 6, and difference 1, be respectively quantities of brandy and wine, which mixed with 5 gallons of cyder will answer the question.

For	6 gallons of brandy at 8 s. are worth	<u>48 s.</u>
1 ditto	wine at 7 s. is worth	<u>7 s.</u>
5 ditto	cyder at 1 s. are worth	<u>5 s.</u>

Therefore 12 ditto of the mixture are worth 60 s.
And 1 ditto is worth $\frac{6}{13} s. = 5 s.$

Again, if from (4) the quantity of brandy given in the first answer, 2 be subtracted; and if to (4) the quantity of wine, 3 be added; then will 2 and 7, the remainder, and sum, be respectively quantities of brandy and wine, which will answer the question.

For	2 gallons of brandy at 8 s. are worth	<u>16 s.</u>
7 ditto	wine at 7 s. are worth	<u>49 s.</u>
5 ditto	cyder at 1 s. are worth	<u>5 s.</u>

Therefore 14 ditto of the mixture are worth 70 s.
And 1 ditto is worth $\frac{7}{13} s. = 5 s.$

185. Now if instead of the numbers of the first answer 4, 4, and 5, larger numbers in the same proportion, viz. 12, 12, and 15, were taken, the following 8 answers would be found by increasing and diminishing the quantities of brandy and wine as above directed, the quantity of cyder remaining constantly 15 s. viz.

Brandy 18 . 16 . 14 . 12 . 10 . 8 . 6 . 4 . 2 .

Wine 3 . 6 . 9 . 12 . 15 . 18 . 21 . 24 . 27 .

Cyder 15 . 15 . 15 . 15 . 15 . 15 . 15 . 15 . 15 .

And

And if, instead of these, still larger numbers in that proportion, or in proportion as any of the last found answers, be assumed; a greater number of other answers may be found.

186. If instead of supposing the quantity of cyder invariable, the quantity of brandy be taken for such; then an infinite number of answers may be found, by continually increasing the quantity of wine by (4) the difference between the prices of the cyder and mixture; and the quantity of cyder by (2) the difference between the prices of the wine and mixture.

Thus, assuming the second answer 6, 1, and 5, as the basis of our work, and esteeming the 6 gallons of brandy as invariable, the following system of answers will arise.

Brandy 6 . 6 . 6 . 6 . 6 . 6 . 6 . 6 &c.

Wine 1 . 5 . 9 . 13 . 17 . 21 . 25 . 29 . 33 &c.

Cyder 5 . 7 . 9 . 11 . 13 . 15 . 17 . 19 . 21 &c.

187. Lastly, esteeming the quantity of wine as invariable, the quantity of brandy must be increased by (4) the difference, of the prices of the cyder and mixture; and the quantity of cyder must be increased by (3) the difference of the prices of the brandy and mixture.

Thus taking the third answer 2, 7, and 5, as the basis, and making 7 invariable, these answers arise.

Brandy 2 . 6 . 10 . 14 . 18 . 22 . 26 . 30 &c.

Wine 7 . 7 . 7 . 7 . 7 . 7 . 7 . 7 &c.

Cyder 5 . 8 . 11 . 14 . 17 . 20 . 23 . 26 &c.

188. When there are 4 kinds of things to be mixed, and two of them are of greater value, and the other two of lesser value than the mixture; the Rule of Alligation will give three answers, as in Art. 172, 174, 176. with either of which, or with any numbers that are in the same proportion among themselves as those, as a basis, innumerable other answers, consisting of numbers which are in different proportions among themselves, may be found, by making any two of them invariable, and changing the rest in the manner as above.

Note 1. That the number by which the quantity of any simple is to be varied, is always the difference between the price of the mixture, and the price of the other simple, which in any operation is considered as variable.

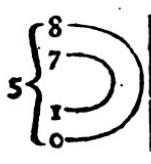
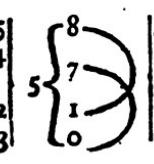
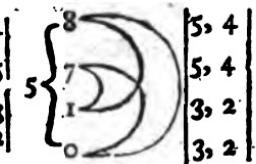
Note 2. That if the simples, which in any operation, are considered as variable, be both of greater, or both of less value than the mixture; then while the one is increased, the other must be diminished; but if the one be of greater value than the mixture, and the other of lesser, then they must be both increased, or both diminished.

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189. Let it be required to mix brandy at 8*s.* wine at 7*s.* cyder at 1*s.* and water at nothing *per gallon* together; so that the mixture may be worth 5*s.* *per gallon*?

The three answers, by the Rule of Alligation, are as follow.

		
5, 4, 9 5, 4, 9 3, 2, 5 3, 2, 5		

Now taking the last answer for the basis of our operation, and making the quantities of wine and cyder invariable, we shall have

Brandy	9	.	14	.	19	.	24	.	29	.	34	.	39	.	8 <i>s.</i>
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Cyder	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>
Water	5	.	8	.	11	.	14	.	17	.	20	.	23	.	8 <i>s.</i>

Making the Brandy and Cyder invariable.

Brandy	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Wine	9	.	14	.	19	.	24	.	29	.	34	.	39	.	8 <i>s.</i>
Cyder	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>
Water	5	.	7	.	9	.	11	.	13	.	15	.	17	.	8 <i>s.</i>

Making the Wine and Water invariable.

Brandy	9	.	13	.	17	.	21	.	25	.	29	.	33	.	8 <i>s.</i>
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Cyder	5	.	8	.	11	.	14	.	17	.	20	.	23	.	8 <i>s.</i>
Water	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>

Making the Brandy and Water invariable.

Brandy	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Wine	9	.	13	.	17	.	21	.	25	.	29	.	33	.	8 <i>s.</i>
Cyder	5	.	7	.	9	.	11	.	13	.	15	.	17	.	8 <i>s.</i>
Water	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>

Making the Brandy and Wine invariable.

Brandy	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	8 <i>s.</i>
Cyder	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>
Water	5	.	5	.	5	.	5	.	5	.	5	.	5	.	8 <i>s.</i>

Or taking 4 other numbers in the same proportion as 9, 9, 5 and 5, viz. 36, 36, 20 and 20.

Brandy

Brandy	36	36	36	36	36	36	36	36
Wine	36	36	36	36	36	36	36	36
Cyder	40	35	30	25	20	15	10	5
Water	4	8	12	16	20	24	28	32

Lastly, making the Cyder and Water invariable.

Brandy £c.	44	42	40	38	36	34	32	30	28	£c.
Wine £c.	24	27	30	33	36	39	42	45	48	£c.
Cyder £c.	20	20	20	20	20	20	20	20	20	£c.
Water £c.	20	20	20	20	20	20	20	20	20	£c.

190. Not only the sets of numbers thus found, but their sums and differences will also be answers:

Brandy, Wine, Cyder, Water.

Thus from or to	42	27	20	20						
Take or add	9	9	10	1						
The remainder	33	18	10	19						
And sum	51	36	30	21						

} will be answers to the question.

191. Alligation Alternate, is either Partial, or Total.

192. Alligation Partial; is, when having the several rates of diverse simples, and the quantity of one of them given, we discover the several quantities of the rest, in such sort, that a mixture of those simples being made according to the quantity given, and the quantities so found, that mixture may bear a certain rate proposed.

Exampl. Suppose it were required to mix brandy at 8s. per gallon, and wine at 7s. with 10 gallons of cyder at 1s. so that the mixture may be worth 5s. per gallon.

Proceed as before in Art. 170, &c. to alligate the prices, and alternate their differences,

5	8	4	4	
	7	4	4	
	1	3	2	
				5

Then having by this means obtained such quantities of the several simples as will make a mixture of the price proposed, say,

As (5) the quantity of cyder found by alternation,
Is to (10) the quantity of ditto given in the question;
So is (4) the quantity of wine and brandy per alternation,
To (8) the quantity of ditto required.

Note. This proportion may be applied to any of the answers found by the other directions.

193. Now having found one answer by the above proportion, others may be found by Art. 184. Thus,

Brandy 12 : 10 . 8 . 6 . 4 : 2

Wine 2 . 5 : 8 . 11 . 14 . 17

Cyder 10 . 10 : 10 . 10 . 10 . 10

By which means, other five answers are obtained as above. See the proof of the two extreme answers below.

12 gallons of brandy at 8s. are worth 96s.

2 ditto wine at 7s. ditto 14s.

10 ditto cyder at 1s. ditto 10s.

Therefore 24 ditto of the mixture are worth 120s.

And 1 ditto is worth $\frac{120}{24}$ s. = 5s.

Again 2 gallons of brandy at 8s. are worth 16s.

17 ditto wine at 7s. ditto 119s.

10 ditto cyder at 1s. ditto 10s.

Therefore 29 ditto of the mixture are worth 145s.

And 1 ditto is worth $\frac{145}{29}$ s. = 5s.

194. Alligation Total, is, when instead of the quantity of any particular simple, the quantity of the mixture is given, together with the prices of the mixture, and the several simples of which it is to be composed.

Examp. Let it be required to mix brandy at 8s. wine at 7s. and cyder at 1s. per gallon together, so that the mixture may contain 26 gallons, and be worth 5s. per gallon?

The prices being alligated, and their differences being alternated as before, viz.

$\left\{ \begin{array}{l} 8 \\ 7 \\ 1 \end{array} \right.$	$\begin{array}{r} 4 \\ 4 \\ 3.2 \end{array}$	$\begin{array}{r} 4 \\ 4 \\ 5 \\ \hline 13 \end{array}$
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Add the numbers which compose the answer together, and say,

As (13) the total of the mixture per alternation,

To (26) the total of the mixture given in the question;

So are (4, 4, and 5) the several quantities of the simples per alternation,

To (8, 8, and 10) the quantities of the said simples required.

195. One answer being thus obtained, others may be found as follows:

1°. Let the quantity of that simple, whose value alone is greater, or less than the value of the mixture, be increased or diminished,

diminished, by the difference of the differences between the prices of the other two simples and the price of the mixture.

2°. Of the remaining two simples, let the quantity of that simple whose value is farthest from the value of the mixture, be increased or decreased (according as the former is) by the sum of the differences, between the prices of the other two simples, and that of the mixture.

3°. Let the quantity of the remaining simple be decreased or increased, also, by the sum of the differences, between the prices of the other two simples, and that of the mixture; but observe, that the quantity of this simple is to be decreased, when those of the two former are increased; and the contrary.

In the Example before us, the value of cyder is alone less than the value of the mixture: Also the differences between the prices of the other two simples, brandy and wine and that of the mixture are severally 3 and 2, the difference of which differences is 1; therefore (10) the quantity of the cyder, may be increased or diminished by 1, that is, it may become 11 or 9.

Of the other 2 simples; the value of the brandy is farthest from that of the mixture; also the prices of the other two simples, wine and cyder, differ from the price of the mixture, severally, by 2 and 4, whose sum is 6; therefore (8) the quantity of the brandy, may be increased or diminished by 6, and become 14 or 2.

Lastly, the quantity of the wine (8) may be diminished or increased by (7) the sum of 3 and 4, the respective differences between the prices of the brandy and cyder, and that of the mixture; that is, it may become 1 or 15.

Thus are two more answers to the question obtained, which, with the former, may stand as below.

Brandy	14	.	8	.	2	.
Wine	1	.	8	.	15	.
Cyder	11	.	10	.	9	.

The proof of the two last answers follows:

14 gallons of brandy at 8s. are worth 112s.

1 ditto wine at 7s. ditto 7s.

11 ditto cyder at 1s. ditto 11s.

Therefore 26 ditto of the mixture are worth 130s.

And 1 ditto is worth $\frac{130}{26}$ s. = 5s.

Again 2 gallons of brandy at 8s. are worth 16s.

15 ditto wine at 7s. ditto 105s.

9 ditto cyder at 1s. ditto 9s.

Therefore 26 ditto of the mixture are worth 130s. as before.

196. Note; That if there be 4 or more simples out of which the mixture is to be compounded, then 1 or more of them must be considered as invariable; so that there may be only 3 variable; and those, so that one of them may be of a contrary value, with respect to the price of the mixture, from the other two.

Example. Tis required to mix such a quantity of brandy at 8s. wine at 7s. cyder at 1s. and water at 0s. per gallon, as will make a hogshead, or 63 gallons of the mixture, worth 5s. per gallon?

By the process in Art. 189. the two following proportions were found, viz.:

	Brandy, Wine, Cyder, Water.
Among the first found answers	9 : 9 : 5 : 5
Among the third	9 : 14 : 5 : 7
And their sums (which are answers by Art. 190.) ——————	{ 18 : 23 : 10 : 12

make just 63 the quantity given.

Then 1°. making the water invariable we shall have per Art. 195.

Brandy	36	:	30	:	24	:	18	:	12	:	6.
Wine	2	:	9	:	16	:	23	:	30	:	37.
Cyder	13	:	12	:	11	:	10	:	9	:	8.
Water	12	:	12	:	12	:	12	:	12	:	12.

2°: Making the cyder invariable, will produce

Brandy	32	:	25	:	18	:	11	:	4.
Wine	7	:	15	:	23	:	31	:	39.
Cyder	10	:	10	:	10	:	10	:	10.
Water	14	:	13	:	12	:	11	:	10.

3°: Making the Wine invariable, gives

Brandy	19	:	18	:	17.
Wine	23	:	23	:	23.
Cyder	2	:	10	:	18.
Water	19	:	12	:	5.

Lastly, making the brandy invariable, we have

Brandy	18	:	18	:	18.
Wine	24	:	23	:	22.
Cyder	3	:	10	:	17.
Water	18	:	12	:	6.

197. If you are inclined to find more answers you may, for Water, make any number invariable from 5 to 19.

Cyder, ditto 2 to 18.

Wine, ditto 2 to 39.

Brandy, ditto 4 to 36.

For since, in some of the above answers, those quantities have been produced; therefore 'tis highly probable, that all the intermediate numbers may have place in some of the answers, and perhaps some out of those limits.

Example. Suppose it were required to make a hogshead of mixture, of the same simples, and at the same prices; but so that there should be just 16 gallons of wine.

When the water was made invariable, this answer among the others arose.

Brandy	24	And now making 16 invariable, two other answers will arise, <i>viz.</i>	25	24	23
Wine	16		16	16	16
Cyder	11		3	11	19
Water	12		19	12	5

198. But if, instead of gallons, you will mix by pints; then (making the same number of gallons, *viz.* 24, 16, 11, 12, or their equivalent number of pints, *viz.* 192, 128, 88, 96, the basis of the operation) more answers may be produced, as follows.

Bran. &c. 197. 196. 195. 194. 193. 192. 191. 190. 189. 188. 187. &c.

Wine &c. 128. 128. 128. 128. 128. 128. 128. 128. 128. &c.

Cyder &c. 48. 56. 64. 72. 80. 88. 96. 104. 112. 120. 128. &c.

Wat. &c. 131. 124. 117. 110. 103. 96. 89. 82. 75. 68. 61. &c.

The two extreme answers are proved below.

197 pints of brandy at 1 s. per pint are worth 9 l. 17 s.

128 ditto wine at 0 s. 10½ d. ditto 5 l. 12 s.

48 ditto cyder at 0 s. 1½ d. ditto 0 l. 6 s.

131 ditto water at 0 s. 0 s.

Therefore 504 ditto or 63 gal. of the mixture are worth 15 l. 15 s.

And 1 gallon thereof is worth $\frac{3}{8} l. = 5 s.$

Again 187 pints of brandy at 1 s. per pint are worth 9 l. 7 s.

128 ditto wine at 0 s. 10½ d. ditto 5 l. 12 s.

128 ditto cyder at 0 s. 1½ d. ditto 0 l. 16 s.

61 ditto water at 0 s. 0 s.

Therefore 504 ditto of the mixture are worth 15 l. 15 s.
as before.

C H A P. XIV.

The Application of Alligation to the mixing of Metals, according to their different Fineness; and to the Composition of Medicines.

199. **A LLOY** is a sort of coarse silver or copper, or some base metal, with which goldsmiths mix gold or silver, to abate the fineness thereof.

An ounce of gold is divided into 24 parts, called caracts ; and an ounce of silver into 20 parts, called penny-weights ; therefore to distinguish fineness of metals, such gold as will abide the fire without loss, is accounted 24 caracts fine ; if it lose 2 caracts in trial, it will then be 22 caracts fine.

Silver is valued in ounces, and a pound of silver which loseth nothing in trial, is called 12 ounces fine ; but if it lose 2 penny-weights, it is then said to be 11 ounces 18 penny-weights fine.

Examp. A goldsmith hath 20 ounces of gold at 20 caracts fine, and would mix it with some at 22 caracts fine, and some of 24 caracts fine ; how much of 22 and 24 caracts fine, and how much alloy, must he mix with the 20 ounces of 20 caracts fine, so as the whole mass may be 18 caracts fine.

First, set down the values in order as usual, with the mean value, and in the place of the alloy, because it is not accounted of any value, place a cypher ; then alligate the values, and alternate their differences as usual, *per Art. 171, &c.*

18	20	18	18
	22	18	18
	24	18	18
	0	2, 4, 6	12

Then by *Art. 192.*

$$\text{As } 18 : 20 :: \left\{ \begin{array}{l} 18 \\ 18 \\ 12 \end{array} : \begin{array}{l} 20 \\ 20 \\ 13 \frac{1}{2} \end{array} \right\} \text{ The quantity of } \left\{ \begin{array}{l} 22 \text{ Car. fine.} \\ 24 \text{ Car. fine.} \\ \text{Alloy.} \end{array} \right\}$$

The Application of Alligation to the Composition of Medicines.

200. Medicines and Simples, in respect of their qualities, are considered in some of these five ways, *viz.* either as they are hot or cold, moist or dry, or as they are temperate; so that such simples or medicines as work heat in our bodies, are said to be hot; such cold, as are the cause of coldness, &c.

The mean or middle between the extreme qualities of heat and coldness, also between dryness and moisture, is called temperate, or the temperature; from which every one of the said qualities, hot, cold, moist, and dry, differs in four degrees; so that a medicine or simple is said to be either temperate, or else hot, cold, moist, or dry, in the first, second, third or fourth degree.

201. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed as you see from A to B, the differences between 5, (the middle number) and the superior numbers 6, 7, 8, 9, will be 1, 2, 3, 4, which may represent the 4 degrees of the qualities hot and dry; likewise the differences between 5 and the inferior numbers 4, 3, 2, 1, will be 1, 2, 3, 4, which may represent the 4 degrees of the qualities cold and moist, the temperature represented by 0, being the mean or middle from whence the said degrees do swerve.

Ind.	Deg.	Ind.	Deg.
B	9	4	Qualities hot and dry.
	8	3	
	7	2	
	6	1	
	5	0	Temperature.
	4	1	
	3	2	Qualities cold and moist.
	2	3	
A	1	4	

202. Since the Rule of Alligation Alternate requires, that of two things miscible, the one must exceed the mean proposed, and the other be less; therefore the questions of Alligation in this kind are to be worked with the numbers in the aforesaid column AB: For by them the degrees and qualities are discovered, being placed as you see in the column adjacent to AB, and

and for distinction sake, those numbers in the said column AB, may be called the Indices or Exponents of the degrees; which Indices are to be used in the same manner, as the prices of merchandizes in the questions of Alligation Alternate, and therefore those Examples may be compared with these.

203. Prop I. Having divers simples whose qualities are known, to make a composition or mixture of them, in such manner that the quality of the medicine may be some mean among the qualities of the simples, and the quantity of it any quantity assigned.

Example 1. An Apothecary has four sorts of simples, A, B, C, D, whose qualities are as follow, viz. A is hot in the fourth degree, B is hot in the second, C is temperate, and D is cold in the third degree; the question is to know what quantities of each of them ought to be taken, to make a medicine, whose quantity may be 12 ounces, and the quality in the first degree of heat? Seek in the aforesaid column AB, for the Indices or Exponents of the qualities of the simples given, viz. for A which is hot in the fourth degree, take 9; for B which is hot in the second, take 7; for C which is temperate, take 5; and for D which is cold in the third degree, take 2: That done, rank those numbers in the same manner as the prices of merchandizes in the questions of the 13th Chap. viz. descend from the highest degree of heat to the temperature, and so proceed downwards to the degrees of cold, setting 6 the Index or Exponent of the mean quality propounded, which is a degree of heat, as common to them all: Then, by crooked lines or otherwise, connect two such Indices, whereof one may be greater than the mean, and the other less, and proceeding according to Art. 171, &c. you'll find that to make a medicine of 9 ounces, and the quality resulting to be in the first degree of heat, you must take 1 ounce of A (being that simple which was hot in 4°.) 4 ounces of B, 3 ounces of C, and 1 ounce of D, as will be manifest by the proof.

Degree	Index	The Proof.
9	A	$9 \times 1 = 9$
7	B	$7 \times 4 = 28$
6	C	$5 \times 3 = 15$
2	D	$2 \times 1 = 2$
	9	9) 54 (6

204. By the *Rule of Proportion* you may increase the medicine to the quantity of 12 ounces, and yet the quality to continue in the first degree of heat, according to the following operation.

<i>Qun.</i>	<i>Qun.</i>	<i>Qun.</i>
9 : 1	:	12 : 1 $\frac{1}{3}$
9 : 4	:	12 : 5 $\frac{1}{3}$
9 : 3	:	12 : 4,
9 : 1	:	12 : 1 $\frac{1}{3}$

The Quantity assigned 12 Ounces.

By the other connexions of the qualities, other quantities of every simple would arise; but that, and the method of finding answers in other proportions, have been sufficiently manifested in the last Chapter.

Example 2. Suppose there are five simples, A, B, C, D, E, whose qualities are as follow, viz. A is hot in 3° . B is hot in 2° . C is hot in 1° . D is cold in 1° . E is cold in 3° . and it is required to mix 4 ounces of B, with such quantities of the rest, that the quality of the medicine may be temperate?

Proceed as before, so you'll find that to make a medicine of 13 ounces, and the quality of the form resulting to be temperate, you must take 1 ounce of A, 1 ounce of B, 4 ounces of C, 6 ounces of D, and 1 ounce of E: Then since the quantity of B, in the composition propos'd, is limited, *viz.* 4 ounces, find numbers which may be in such proportion to 4 (the quantity of B assigned) as the numbers 1, 4, 6, 1, (the quantities of A, C, D, E, in the aforesaid composition of 13 ounces) are to 1 (the quantity of B in the said composition) in manner following.

Our,

Ounces	Ounces	Ounces	Ounces
1 : 1	:	4	: 4 of A
1	:	4	: 16 of C
1	:	6	: 24 of D
1	:	1	: 4 of E

$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\}$ to be mixed with 4 ounces of B.

205. Prop. II. A Medicine being compounded of several simples, whose qualities and quantities are known, to find the degree of the form resulting, viz. the exact temperament of the medicine.

Example 1. Suppose a medicine to be compounded of two simples, viz., 6 ounces of B hot in 4° . and 3 ounces of C hot in 3° . and it is required to find the temperament of the medicine, viz. the degree and quality resulting from such mixture? Seek in the aforesaid column AB for the Indices, of the respective degrees and qualities of the simples given, and dispose them orderly in ranks right against their respective quantities; then multiply each Index by its respective quantity, and divide the sum of the products by the sum of the quantities: So will the quotient be the Index of the degree, and quality of the medicine.

$$\begin{array}{r}
 \text{Indices} \\
 \text{B} \quad \text{C} \\
 \hline
 9 + 6 = 15 \\
 8 + 3 = 11 \\
 \hline
 9) 18 (8\frac{2}{3}
 \end{array}$$

So in the said Example, the quotient will be found $8\frac{2}{3}$, which is the Index of $3\frac{2}{3}$ degrees of heat; and therefore the said medicine is hot in $3\frac{2}{3}$ degrees.

Example 2. Suppose a medicine to be compounded of 4 simples, whose qualities and quantities are known, viz. 2 ounces of A hot in 3° . 3 ounces of B hot in 2° . 4 ounces of C temperate, and 5 ounces of D cold in 4° . and let it be required to find the quality resulting from such mixture. According to the aforesaid Rule, multiply each Index by its respective quantity, and divide the sum of the products by the sum of the quantities, so the quotient is $4\frac{2}{7}$, which is the Index of $4\frac{2}{7}$ degrees of cold (for the difference between 5 the Index of the temperature, and $4\frac{2}{7}$ the Index found, is $\frac{5}{7}$ degrees of cold) which is the quality of the said medicine.

Ind.

<i>Ind.</i>	<i>Ounc.</i>	<i>P. of</i>
$3 \times 2 =$	16	
$7 \times 3 =$	21	
$5 \times 4 =$	20	
$1 \times 5 =$	5	
		<hr/>
14)	62 (4 $\frac{1}{2}$)	

Example 3. Suppose a medicine to be compounded of several simples, whose qualities and quantities are as follow, viz. 4 ounces of a simple which is cold in 2°. and moist in 1°; 5 ounces hot in 3°. and (in respect of dryness and moisture) temperate; 3 ounces hot in 2°. and dry in 2°; 6 ounces hot in 1°. and moist in 4°; 4 ounces cold in 3°. and moist in 2°; the question is to know the temper resulting?

In the resolution of this question there must be two distinct operations, each of them like that in the last Example, viz.

1. Find in the same manner as before, the degree and quality resulting from the commixture of the qualities hot and cold; so you'll discover $5\frac{1}{2}$ which is the Index of $\frac{1}{2}$ degrees of heat. (for the difference between 5, the Index of the temperature, and $5\frac{1}{2}$ the Index found, is $\frac{1}{2}$ degrees of heat.)

<i>Ind.</i>	<i>Ounc.</i>	<i>P. of</i>
$3 \times 4 =$	12	
$8 \times 5 =$	40	
$7 \times 3 =$	21	
$6 \times 6 =$	36	
$2 \times 4 =$	8	
		<hr/>
22)	117 ($5\frac{1}{2}$)	

<i>Ind.</i>	<i>Ounc.</i>	<i>P. of</i>
$4 \times 4 =$	16	
$5 \times 5 =$	25	
$7 \times 3 =$	21	
$1 \times 6 =$	6	
$3 \times 4 =$	12	
		<hr/>
22)	80 (3 $\frac{1}{2}$)	

2. Seek in the same manner, the temper resulting from the mixture of the qualities dry and moist; and you'll find $3\frac{1}{2}$, which is the Index of $\frac{1}{2}$ degree of moisture; so that the quality of the said medicine is $\frac{1}{2}$ degree of heat, and $\frac{1}{2}$ degree of moisture, as by the operation is manifest.

206. *Prop. III. To augment or diminish a Medicine in quality, according to any degree assigned.*

Suppose a medicine to be compounded as follows, viz. 1 dram of a simple hot in 4°. 2 drams hot in 3°. 2 drams hot in 2°. 1 dram hot

hot in 1° . 1 dram cold in 1° . and 1 dram cold in 2° . Then will the quality of the said medicine be in $1\frac{1}{2}$ degree of heat (as is manifest by the second Proposition). Now let it be required to augment the said medicine in quality, viz. to add such a quantity of some one of the ingredients (or some other simple) which may raise the quality of the medicine $\frac{1}{2}$ degree; so that the temperament of the medicine after it is increased in quantity, may be in 2° of heat. Make choice of such a simple, the Index of whose quality may exceed the Index of the quality assigned, viz. take that simple which is hot in 3° . whose Index is 8, then proceed according to the Example of the first Proposition; so will you find that if 1 dram of the aforesaid medicine be mixed with $\frac{1}{2}$ dram of that simple which is hot in 3° . the temper resulting from such mixture will be in 2° of heat.

Lastly, By the Rule of Three, say if 1 dram require $\frac{1}{2}$ dram, what shall 8 drams (the quantity of the medicine first given) require?

Ans. 4 drams. So that if 4 drams of a simple which is hot in 3° . be mixed with 8 drams of a medicine hot in $1\frac{1}{2}$ degree, the temper resulting will be in 2° . of heat, as by the operation manifest.

Ind.	Drams.	
7 {	$6\frac{1}{2}$	
8	8	
1 : $\frac{1}{2}$:: 8 : 4	

The Proof.

Ind.	Drams.	Prod.
$6\frac{1}{2}$	\times 8	$= 52$
8	\times 4	$= 32$

12) 84 (7)

If it be required to diminish a medicine in quality, you are to make choice of such a simple, the Index of whose quality may be less than the Index of the quality assigned, and then to proceed as before.

Here observe, that if in questions of this nature, the quantities of the simples be expressed by weights of diverse denominations, they must be reduced to that weight which is of the lowest denomination in the question.

The

The augmenting or diminishing of a medicine, in respect of quantity : Also the finding of the value of any quantity of a medicine, the prices of the ingredients being known, will be familiar to such as understand the Rule of Proportion, and therefore I shall not insist upon them.

C H A P. XV.

The Rule of False.

207. **T**HE *Rule of False* is always performed by false and supposititious numbers taken at pleasure after the proposition is made, and the question stated : For things are said to be found out by the *Rule of False*, when by false terms supposed, we discover the true terms required.

208. The *Rule of False*, is either of single, or double position.

The Rule of single position is, when at once, *viz.* by one false position we have means to discover the true resolution of the proposed question.

For Example : *A*, *B*, and *C*, determining to buy together a certain quantity of timber, that should cost them 36*l.* agree among themselves that *B* shall pay of that sum a third part more than *A*, and that *C* shall pay a fourth more than *B*. Now the question is, what particular sum each of these parties ought to pay of the 36*l.* To resolve this question ; first, put the case that *A* ought to pay 6*l.* of the 36*l.* and then *B* must pay 8*l.* because he pays one third part more than *A*. And lastly, *C* ought to pay 10*l.* because he is to lay out one fourth part more than *B*. This done, although by Addition of these three sums, *viz.* 6, 8, and 10, I find that I have made a wrong position (their total amounting only to 24*l.* which should have been 36*l.*) nevertheless by those supposititious numbers, I have the means to discover the true sums which the several parties ought to pay : For by the *Rule of Three Direct.*.

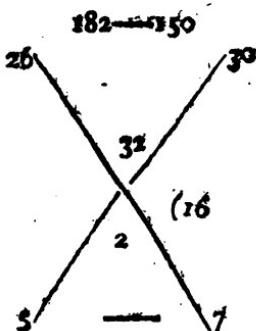
1. As 24 to 36, so is 6 to 9*l.* the part that *A* must pay.
2. As 24 to 36, so is 8 to 12*l.* the part that *B* ought to pay.
3. As 24 to 36, so is 10 to 15*l.* the part of the 36*l.* that *C* must pay.

209. Here for trial of this Rule, the total of the sums, found ought to agree with the sum given: So in the above Example, 9, 12, and 15, being added together, amount to 36, the sum proposed.

210. The Rule of Double Position, is, when two false positions are supposed for the resolution of the question propounded. As in this, a workman having thresh'd out 40 quarters of grain (part of it wheat and the rest barley) received for his labour 28*s.* being paid after the rate of 12*d.* for every quarter of wheat, and 6*d.* for each quarter of barley: Now here the question is, how many of those 40 quarters were wheat, and how many barley? First suppose at random, that there were 26 quarters of wheat and 14 of barley, and find how much money is due to the workman at the rate of 12*d.* the quarter of wheat, and 6*d.* the quarter of barley; which is 33*s.* (*viz.* 26*s.* for the 26 quarters of wheat, and 7*s.* for the 14 quarters of barley,) which he ought to have received, if the supposition were right; but because it differs from 28*s.* the true sum that he received, keep in mind 5 the difference, which call the first error, or the error of the first position: Again, propound for the second position, that there were 30 quarters of wheat, and 10 quarters of barley; and then the second error will be 7; for there is then due to the workman for the 30 quarters of wheat 30*s.* and for the 10 quarters of barley 5*s.* in all 35*s.* which differs from 28*s.* the true sum that he received, by 7*s.* and by these two false positions, together with their errors may be discovered how many quarters of wheat, and how many of barley the workman thresh'd, as shall be further explained as follows.

211. In the Rule of Double Position, having drawn two lines across, and placed the terms of the false position, (*viz.* those that have the same denomination) at the uppermost end of that cross, as also each error under its respective position at the lower end of the same cross, multiply each error by the contrary position; that is to say, the second error by the first position, and the first error by the second position; this done, when both the errors are of one and the same kind (*viz.* both excesses or both defects) subtract the less product out of the greater, and then the remainder is your dividend: But if the errors be of different kinds, (*viz.* one of them an excess, and the other a defect) add those products together, and the sum will be your dividend, which if you divide by the difference of the errors, (when they are of one and the same kind) or by their sum (when they are of different kinds) the quotient will give you the number you look for, having the same denomination with the false positions placed at the upper end of the cross.

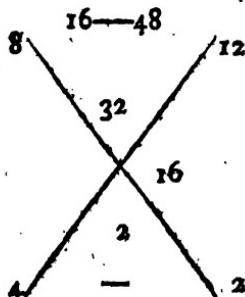
Example 1. The above question being again proposed, place these terms, viz. 26 (having the denomination of the quarters of wheat in the first position) and 30 (having the same denomination in the second position) at the upper end of the cross: As also 5 and 7 the two errors respectively under them at the lower end of the same cross, as you may see it exemplified by the following pattern.



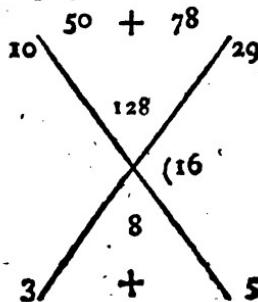
This done, having multiplied 26 by 7, the product is 182, and likewise 30 by 5, the product is 150, which being deducted out of 182 (because the errors here are both of the same kind, that is, are each of them an excess above 28s. the sum that the workman receiv'd,) the remainder is 32, which being divided by 2 (the difference between 5 and 7 the two errors,) leaves in the quotient 16, for the quarters of wheat that the workman thresh'd, whose complement to 40, viz. 24, are the quarters of barley, that he likewise thresh'd; so the workman receiving 28s. for his wages in threshing out 40 quarters of grain (being part wheat, part barley) at 12d. the quarter of wheat, and 6d. the quarter of barley; thresh'd in all 16 quarters of wheat, and 24 quarters of barley.

Example 2. The same question being again propounded, suppose for the first position that there are 8 quarters of wheat, and 32 quarters of barley, and then the first error will be 4s. for 8s. being accounted for the 8 quarters of wheat, and 16s. for the 32 quarters of barley, make in all 24s. which want 4s. of 28s. the sum received: Again, supposing that there are 12 quarters of wheat, and 28 quarters of barley, the second error will be 2s. for 12s. being allowed for the 12 quarters of wheat, and 14s. for the 28 quarters of barley, the sum is 26s. which comes 2s. short of 28s. the right sum: Now then, 8 being multiplied by 2, the product is 16; likewise 12 by 4 produces

48, out of which if you deduct 16 (because the errors in this case happen to be both defect under 28s. the sum received,) the remainder is 32, which being divided by 2 (the difference of the errors) gives you in the quotient 16, viz. the quarters of wheat, as before.

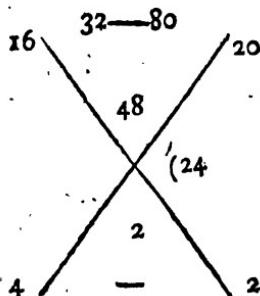


Example 3. The same demand being the third time produced, take for the first position 10 quarters of wheat, and 30 quarters of barley, and then proceeding as before, the first error will prove 3s. which upon that position is wanted of 28s. the right sum: Again, for the second position take 26 quarters of wheat, and 14 quarters of barley, and then the second error will be 5s. by which that position exceeds 28s. the true sum: Now multiplying 10 by 5, the product is 50, and 26 by 3, the product is 78. And here (because the errors are of different kinds, one of them being a defect, and the other an excess) add 50 and 78 the two products together, whose sum is 128, which being divided by 8, the sum of 3 and 5 the two errors, gives in the quotient 16, for the quarters of wheat, as before in the former resolutions: So that what positions soever you take in this question, you will always find, that the workman thresh'd 16 quarters of wheat, and 24 quarters of barley, which is the resolution of the question propounded.



212. Here the proof is the same with the process used in finding out the errors : So in the Example premised, 16 and 24 being the numbers found, and 16*s.* being allowed for the 16 quarters of wheat, likewise 12*s.* for the 24 quarters of barley, their sum is 28*s.* which was the sum received by the workman.

Example 4. A certain man being demanded what was the age of each of his four sons ? Answered, that his eldest son was 4 years older than the second ; his second son was 4 years older than the third ; his third son was 4 years older than the fourth, or youngest ; and his fourth or youngest was half the age of the eldest : The question is, what was the age of every son ? Here guess the age of the eldest son to be be 16, then it may be infer'd from the question, that the age of the second son was 12, the age of the third 8, and the age of the fourth or youngest 4 ; this 4 should be half 16 (for the question says, that the age of the youngest was half the age of the eldest) but it wants 4 thereof ; wherefore I make a second position, and take 20 for the age of the eldest, then the age of the second must necessarily be 16, the age of the third 12, and the age of the fourth 8, which should be half 20, but it wants 2 : Now (according to the Rule) multiplying 16 (the first position) by 2 (the second error) the product is 32 : Also multiplying 20 (the second position) by 4 (the first error) the product is 80, and because the errors are both of one kind, to wit, both defective ; I subtract the lesser product from the greater, so the remainder is 48 for a dividend ; also subtracting the lesser error from the greater, the remainder is 2 for a divisor : Lastly, dividing 48 by 2, the quotient is 24, and such was the age of the eldest son ; therefore the age of the second was 20, the age of the third 16, and the age of the fourth 12 ; which is half the age of the eldest, as was declared by the question.



C H A P. XVI.

Of Prime and Composite Numbers.

213. ONE Number is said to *measure*, or be a *Measur* of another, when it is contained in it a certain number of times precisely ; so that being taken out of it as oft as possible, there shall nothing remain over. Thus, 4 measures 12; because it is contained in it precisely 3 times. Observe also, that one number is said to measure another by that number which is the quotient : So 4 measures 12 by 3; and reciprocally, 3 measures 12 by 4 : And hence any number with the quote, by which it measures another, may be called the reciprocal measures of that other number.

214. A number is called the common measure of two or more numbers, when it measures each of them ; so 3 is a common measure of 6, 9, 12. And if it is the greatest number that measures them, it is called their greatest common measure ; as unity is their least.

215. A lesser number is an aliquot part of a greater, when the lesser exactly measures the greater ; and the greater is then called a multiple of the lesser. Thus 4 is an aliquot part of 12, and 12 is a multiple of 4.

216. A lesser number is an aliquant part of a greater, when the lesser doth not exactly measure the greater. Now 5 is an aliquant part of 12, because 2 times 5 is 10, and 3 times 5 is 15 ; and consequently 5 doth not exactly measure 12.

217. A number is called a prime number, which has no measure but itself and unity ; and which consequently is the product of no other numbers. So 2, 3, 5, 7, &c. are prime numbers.

218. A composed number, is that which some number measureth, as 4, 6, 9, 10, 12, &c.

219. Numbers prime to one another, are such, as have no other common measure but an unit. Thus 15 and 32 are prime to each other ; for tho' 5 and 3 will measure 15, yet neither 5 nor 3 will measure 32.

220. Numbers composed to one another, are such, as have some number for their common measure. So 15 and 21 are composed to one another, for 3 will measure 15 by 5, and 21 by 7.

221. All even numbers, except 2, are composite. But of odd numbers some are prime, as 3, 5, 7 ; and some composite,

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as 9, 15, 21 : And since the series of even and odd numbers comprehend all numbers, it follows, that

222. All prime numbers are odd, except the prime 2.

223. If a number measures each of two or more several numbers, it will measure their sum : And if it measure them all but one, it cannot measure the sum.

224. If a number measures the sum of two numbers, and also measures one of those numbers, it will measure the other ; or, if it measures the sum of several numbers, and also each of the parts but one, it must measure that one also.

225. The number A, that measures another B, measures all the numbers that B measures, i. e. all the multiples of B ; and what is measured by A, is so by all the aliquot parts of A.

226. No even number can measure any odd number.

227. If one number A, measures each of two other numbers B and C, it will also measure the remainder after B is taken out of C as oft as possible.

228. Of whatever factors any number is composed by Multiplication, it is resolvable into the same by Division, i. e. it is measurable by each of these factors, or the product of any two or more of them.

229. If the two right-hand figures of any number are measurable by 4, the whole number is measurable by 4.

230. If the three right-hand figures of any number are measurable by 8, the whole is measurable by 8.

231. If the sum of the digits or figures, constituting any number, is a multiple of 3 or 9, then the whole number is a multiple of 3 or 9.

232. If the right-hand place of any number be a cypher, or 5, the whole is a multiple of 5.

233. If the right-hand place of any number be a cypher, the whole is a multiple of 10.

234. If a number cannot be divided by some prime number, not greater than the square root thereof, that number is a prime.

235. The unit's place of all prime numbers, except 2 and 5, must be filled by one of the following digits 1, 3, 7, or 9 ; for all even numbers, and those which end with the digit 5, are composed.

236. To find which of the odd numbers ending with 1, 3, 7, or 9, under 1000, are primes ; and to find the component parts of the rest, from such a Table as the following, the problem may be easily solved ; for if any number not divisible by 2 or 5, and not exceeding the given number 1000, be sought

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sought in the Table thus, viz. the hundreds at the top or bottom, and the tens and units in the margin; then under the hundreds, and on a line with the tens and units, will be found P, if the number be a prime; or its least component prime, if the number be composite.

Then divide the number by its least component prime, and use the quotient in like manner.

So 829 is found to be a prime; and 483 is found to be divisible by 3: Now 483 divided by 3, quotes 161, which by the Table is a multiple of 7; and consequently 161 divided by 7, quotes 23, which is a prime; whence the component primes of 483, are 3, 7, 23.

237. But to make this table useful for all numbers under 1000, let even numbers be divided by 2 continually, as oft as possible; and let those numbers which end in 5, be divided by 5 as oft as possible, and the results sought in the Table, as before.

Thus, if 616 be divided by 2 three times, it quotes 77, which by the Table gives 7 for a measure, and the quote is 11, which by the Table is a prime; consequently the component primes of 616, are 2, 2, 2, 7, 11.

Again, 435 divided by 5, quotes 87, which by the Table gives 3 for a measure, which quotes 29 a prime; whence 435 is composed of 5, 3, 29.

238. The calculating of such a Table being not difficult, I shall next shew the method of performing it.

Let the paper be first rul'd with as many columns, as the limiting number contains hundreds; then number the hundreds at top and bottom, and the tens and units in the margins.

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	0	1	2	3	4	5	6	7	8	9	
1	P	P	P	P	P	P	P	P	P	P	1
3	P	P	P	P	P	P	P	P	P	P	3
7	P	P	P	P	P	P	P	P	P	P	7
9	P	P	P	P	P	P	P	P	P	P	9
11	P	P	P	P	P	P	P	P	P	P	11
13	P	P	P	P	P	P	P	P	P	P	13
17	P	P	P	P	P	P	P	P	P	P	17
19	P	P	P	P	P	P	P	P	P	P	19
21	P	P	P	P	P	P	P	P	P	P	21
23	P	P	P	P	P	P	P	P	P	P	23
27	P	P	P	P	P	P	P	P	P	P	27
29	P	P	P	P	P	P	P	P	P	P	29
31	P	P	P	P	P	P	P	P	P	P	31
33	P	P	P	P	P	P	P	P	P	P	33
37	P	P	P	P	P	P	P	P	P	P	37
39	P	P	P	P	P	P	P	P	P	P	39
41	P	P	P	P	P	P	P	P	P	P	41
43	P	P	P	P	P	P	P	P	P	P	43
47	P	P	P	P	P	P	P	P	P	P	47
49	P	P	P	P	P	P	P	P	P	P	49
51	P	P	P	P	P	P	P	P	P	P	51
53	P	P	P	P	P	P	P	P	P	P	53
57	P	P	P	P	P	P	P	P	P	P	57
59	P	P	P	P	P	P	P	P	P	P	59
61	P	P	P	P	P	P	P	P	P	P	61
63	P	P	P	P	P	P	P	P	P	P	63
67	P	P	P	P	P	P	P	P	P	P	67
69	P	P	P	P	P	P	P	P	P	P	69
71	P	P	P	P	P	P	P	P	P	P	71
73	P	P	P	P	P	P	P	P	P	P	73
77	P	P	P	P	P	P	P	P	P	P	77
79	P	P	P	P	P	P	P	P	P	P	79
81	P	P	P	P	P	P	P	P	P	P	81
83	P	P	P	P	P	P	P	P	P	P	83
87	P	P	P	P	P	P	P	P	P	P	87
89	P	P	P	P	P	P	P	P	P	P	89
91	P	P	P	P	P	P	P	P	P	P	91
93	P	P	P	P	P	P	P	P	P	P	93
97	P	P	P	P	P	P	P	P	P	P	97
99	P	P	P	P	P	P	P	P	P	P	99
	0	1	2	3	4	5	6	7	8	9	

First, Then in the place of the first odd prime 3, put P; and then raise 3 to all its powers under 1000, the number given, and in their places put 3.

Secondly, The next odd number is 5, which is passed by, because its multiples end in 5.

Thirdly, The next odd number is 7, whose place is yet vacant, and is therefore to have P in it; then multiply the numbers belonging to every place that is filled by 7, except such as give products greater than the limiting number, and in the place of the product put the figure that was in the place of the multiplicand, or else the multiplicand itself, if it be a prime. Then in the place of the square of 7, put 7; and by that square multiply the same multiplicands, putting in the places of the products as before. Also in the places of the powers of 7, put 7, and by these powers multiply the same multiplicands, putting in the places of the products as before.

Fourthly, The next odd number is 9, whose place is filled, and therefore to be passed over.

Fifthly, The next odd number is 11, whose place is yet vacant, and is therefore to have P in it; then by 11, multiply the numbers belonging to every place that is filled, except such as give products greater than the limiting number, and in the place of the product, put the figure that was in the place of the multiplicand, or else the multiplicand itself, if it be a prime; then in the place of the square and other powers of 11, put 11; and by the square of the other powers of 11, multiply the same multiplicands, putting in the places of the products, as before, and so use 13, 17, 19, 23, &c. omitting 15, 21, 25, 27, &c.

239. To reduce any number given into its component primes.

Rule. Divide the number given by 2 continually as oft as possible, without a remainder, and the result by 3 as oft as possible, and this last result by 5, 7, 11, 13, and all the prime numbers, 'till there arise a quotient less than the divisor; and then the divisors and the last dividend are the primes, composing the number given.

Exampl. 1. Reduce 122760 to its component primes.

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$$\begin{array}{r}
 2) \underline{122760} \\
 2) \underline{61380} \\
 2) \underline{30690} \\
 3) \underline{15345} \\
 3) \underline{5115} \\
 5) \underline{1705} \\
 11) \underline{341} \\
 31
 \end{array}$$

Answer 2, 2, 2, 3, 3, 5, 11, 31,

In the operation I have omitted the division by 7, because it would not measure 341; and afterward omitted dividing by 13 and 17, because they are no aliquot parts of 31.

Examp. 2. Reduce 360 to its constituent primes.

$$\begin{array}{r}
 2) \underline{360} \\
 2) \underline{180} \\
 2) \underline{90} \\
 3) \underline{45} \\
 3) \underline{15} \\
 5
 \end{array}$$

Answer 2, 2, 2, 3, 3, 5.

240. To find all the just divisors of any number.

Rule. Make as many columns as there are different component primes in the given number: And at the head of each column place each prime, and where there are more than one prime of the same sort, place in the same column the powers of that prime, till the index of the last power be the number of those like component primes; lastly, multiply every number in the first column by every number in the second, placing each product in the second; and then every number in the first and second by every number in the third, placing each product in the third, and so with the 4th, 5th, &c. columns, till all are thus used.

And so shall all the numbers in these several columns, be all the divisors required.

Example.

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Example. Find all the just divisors of 360.

2) 360	2	3	5
2) 180	4	9	10
2) 90		6	20
3) 45		12	40
3) 15		24	15
5) 5		18	45
		36	30
		72	60
			120
			90
			180
			360

Answer 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.

241. To find all the aliquot parts of any number given : Find all the just divisors of the number given by Art. 240. excluding the number itself, which divisors with unity are all the aliquot parts,

So, all the aliquot parts of 360, are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180.

242. A number which can be measured by two or more numbers, is called their common multiple ; and if it be the least number that can be so measured, it is called their least common multiple.

243. To find the least common multiple of two, or more given numbers.

Rule. Set down the given numbers on a line, and divide them severally by 2, 3, or any other number which will measure two or more of them, and place their quotients with the numbers not divided on a line beneath ; divide them continually by the same or any other numbers, till the results be units, reserving their several divisors : which divisors multiply into each other continually, and the product shall be the number sought.

Note. If the numbers are prime to each other, then the continual product of the given number will be their least common multiple,

Exampl. 1. Find the least common multiple of 2, 3, and 4.

Divisors	2	2
	<hr/>	<hr/>
	2	4
	<hr/>	<hr/>
	3	3
	<hr/>	<hr/>
	1	<hr/>

Answer 12 is their least common multiple.

Exampl. 2. Find the least common multiple of 2, 4, 6, 10, 12, and 15.

Divisors	2	2
	<hr/>	<hr/>
	2	4
	<hr/>	<hr/>
	3	3
	<hr/>	<hr/>
	5	12
	<hr/>	<hr/>
	1,	5
	<hr/>	<hr/>
	1,	60

Answer 60 is the least common multiple.

The

The Doctrine of Vulgar Fractions.

C H A P. XVII.

NOTATION of VULGAR FRACTIONS.

244. THE Doctrine of Fractions depends upon this supposition, that unity, or at least one whole thing whatsoever it be, may in mind be conceived divisible into any number of equal parts. Some will not allow 1 or unity to be a number, when it is considered in the abstract, and separated from matter : But since that prince of Arithmeticians, *Diophantus of Alexandria*, in several of his subtil problems mentions unity as a number, and propounds it to be divided into numbers ; I shall take the like liberty to esteem 1 or unity as a number, and likewise suppose it divisible into any number of equal parts.

245. A broken number, otherwise called a fraction, is only part of an integer or whole number, as if you would express in figures the length of a piece of cloth, that contains three fourths, or (which is all one) three quarters of a yard, you may write it thus $\frac{3}{4}$; that is, an entire yard being suppos'd to be divided into four equal parts, the length of the piece proposed is three of those four parts : In like manner, (a foot being divided into 12 inches) you may write 6 inches thus $\frac{6}{12}$ that is, six twelfth parts of a foot ; or if the foot be divided into one hundred equal parts, to express twenty-five of those parts, set them down thus $\frac{25}{100}$, that is, twenty-five hundredth parts of a foot.

246. A fraction consists of two parts, the Numerator and the Denominator, which are placed one above the other, and separated by a little line.

247. The Numerator is the number set above the line, and the Denominator is the number placed underneath : So in the aforementioned fraction $\frac{3}{4}$, the number 3 placed above the line is the Numerator.
 3 Numerator.
 4 Denominator.
 under it is the Denominator. Also in this fraction $\frac{6}{12}$, the Numerator is 6, and the Denominator is 12. The Denominator is

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is so called, because it denominates or declares into how many equal parts the integer or whole number is supposed to be divided; and the Numerator is so named, because it numbers or expresses how many of these equal parts of the integer are signified by the fraction.

248. If two fractions have the same denominator and different numerators, that fraction is the greater whose numerator is the greater: But if two fractions have the same numerator and different denominators, that fraction is the greater whose denominator is the least; for the fewer the parts are, into which the whole is divided, the greater must one of those parts be.

249. Hence by multiplying the numerator, the fraction will be multiplied; and by dividing the numerator the fraction will be divided.

For instance, if the fraction $\frac{3}{12}$ be proposed, and its numerator 3 be multiplied by 2, the product $\frac{3 \times 2}{12} = \frac{6}{12}$ is plainly the double of $\frac{3}{12}$.

And if, in the same fraction, the numerator 3 be divided by 3, the quotient $\frac{1}{12}$ is evidently the third part of $\frac{3}{12}$.

250. Also by multiplying the denominator the fraction will be divided, and by dividing the denominator the fraction will be multiplied.

For example, if 12 the denominator of the fraction $\frac{3}{12}$ be multiplied by 3, the product ($\frac{3}{12} \times 3 = \frac{9}{12}$) $\frac{9}{12}$ is the third part of $\frac{3}{12}$.

And if, in the same fraction, the denominator 12 be divided by 2, the quotient ($\frac{12}{2} = \frac{6}{12}$) $\frac{6}{12}$ is the double of $\frac{3}{12}$.

251. But if both numerator and denominator, be multiplied or divided by the same number, the fraction arising will be of the same value.

252. A fraction is either proper, or improper.

253. A proper fraction is that whose Numerator is less than the Denominator; such are the fractions before-mentioned $\frac{3}{12}, \frac{2}{15}, \frac{5}{10}$, and the like.

254. A proper fraction is either single, or compound.

255. A single fraction is that which consists of one Numerator, and one Denominator; such are $\frac{3}{4}, \frac{6}{12}, \frac{25}{100}$, and the like.

A single fraction often arises in division of whole numbers, for when the division is finished, if any number remain, it is to be esteemed as the Numerator of a fraction, which has the Divisor for a Denominator, and is to be annexed to the integer or integers in the quotient; as part of the quotient; which fraction always expresses certain parts (or at least a part) of an integer or entire unit, which has the same denomination with one

of the integers in the quotient : So if 17 pounds be given to be equally divided among 5 persons, there will arise 3 entire pounds in the quotient, and there will be a remainder or surplusage of 2 pounds, which 2 is to be placed, 5) 17 (3 $\frac{2}{5}$ as the Numerator of a fraction, over the Divisor 5 as a Denominator ; so will the fraction be $\frac{2}{5}$, and the complete quotient will be $3\frac{2}{5}$, that is 3 pounds and 2 fifth parts of a pound for every person's share.

A single fraction does likewise arise, when a lesser whole number is given to be divided by a greater ; for in such case the Dividend is to be made the Numerator of a fraction, and the Divisor the Denominator ; which fraction is the true quotient, and always expresses certain parts (or at least a part) of an integer, which has the same name with the Dividend. So if 3 pounds Sterling be given to be divided equally among 4 persons, the share of each, that is, the quotient will be $\frac{3}{4}$, to wit, 3 fourth parts of a pound.

In like manner, if 5 be given to be divided by 8 the quotient is $\frac{5}{8}$; so that the Numerator of a fraction is always a Dividend, the Denominator is a Divisor, and the Fraction itself is the Quotient.

256. A compound Fraction (otherwise called a Fraction of a Fraction) is that which has more Numerators and Denominators than one, and may be discovered by the word (*of*) which is interpos'd between the parts of such compound Fraction : So $\frac{2}{3}$ of $\frac{3}{4}$, is a Fraction of a Fraction, or compound Fraction, and expresses two thirds of three fourths of an integer, *viz.* a pound Sterling being supposed the integer, and first divided into four parts, three of those four parts are equal to 15s. Again, if the said 15s. be divided into three parts, two of those three parts are equal to 10s. therefore the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of a pound Sterling does express 10s. In like manner, the compound Fraction $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a pound Sterling, that is, one fourth of three fourths of four fifths of a pound Sterling expresses 3s. as will be manifested hereafter.

257. An improper Fraction is that, whose Numerator is either greater, or at least equal to the Denominator : So this Fraction $\frac{16}{4}$, that is 16 fourths, is called an improper Fraction, and so is this $\frac{4}{2}$; and a Fraction of this kind may well be named improper, because it will not admit the definition of a true Fraction ; since it is always greater than an entire unit, or at least equal to it ; so sixteen farthings, or $\frac{16}{4}$ of a penny are equal to 4 entire pence ; and 4 farthings or $\frac{4}{2}$ of a penny are equal to 1 penny : Therefore when the Numerator

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is greater than the Denominator, such improper Fraction signifies more than 1 or an integer; but when the Numerator is equal to the Denominator, (be it what number soever) such improper fraction is always equal to unity, or 1 integer.

258. A mixt number consists of entire units (or integers) or at least of unity (or 1 integer) and a fraction annexed: Thus $5\frac{1}{2}$, $1\frac{3}{4}$, and such like, are called mixt numbers: So that if a piece of timber be five feet and eleven inches in length, you may write that length thus, $5\frac{11}{12}$; in like manner, one mile and three quarters or fourths of a mile may be expressed thus $1\frac{3}{4}$.

C H A P. XVIII.

REDUCTION of VULGAR FRACTIONS.

259. **T**HE same parts of Numeration, as have been work'd in whole Numbers in the preceding Chapters, are likewife to be performed in Fractions: But first of all Reduction of Fractions in diverse kinds must be known, which being the principal skill in the Doctrine of Fractions, ought to be diligently observed by the learner.

260. Two numbers being given, their greatest common Divisor, that is, the greatest number which will measure or divide each of the numbers given without leaving any remainder, may be found out in this manner, *viz.* Divide the greater number by the less, then divide the Divisor by the remainder (if there be any) and so continue dividing the last Divisors by the remainders, until there be no remainder (neglecting the quotients); so is the last Divisor the greatest common Divisor to the numbers given.

Thus, if the greatest common Divisor to the numbers 91 and 117 be sought, divide the greater number 117 by 91, the remainder is 26; by which dividing 91, the remainder is 13; by which dividing 26 the remainder is 0; so is 13 the greatest common Divisor to the numbers 117 and 91, as is manifest from dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 117 precisely 9 times. In like manner 29 will be found a

$$\begin{array}{r} 91 \quad 117 \quad (1 \\ \underline{-} \quad \quad \quad 26) \quad 91 \quad (3 \\ \quad \quad \quad \quad \quad 78 \\ \underline{-} \quad \quad \quad \quad \quad 13 \\ \quad \quad \quad \quad \quad (2 \\ \quad \quad \quad \quad \quad \quad 26 \\ \underline{-} \quad \quad \quad \quad \quad \quad 0 \end{array}$$

common

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common Divisor to 116 and 145; and 51 a common Divisor to 561 and 612.

261. A single Fraction may be abbreviated or reduced into the least terms, by dividing the Numerator and Denominator by their greatest common measure (or Divisor;) for the quotients will be the Numerator and Denominator of a Fraction equal to the former, and in the least terms.

So if the Fraction $\frac{91}{117}$ be given to be reduced into the least terms, search out the greatest common Divisor to 91 and 117 by the last Rule, which will be found 13, and then dividing 91 by 13, the quotient will be 7 for a new Numerator; also dividing 117 by 13, the quotient will be 9 for a new Denominator: So the fraction $\frac{91}{117}$ is reduced into the least terms, viz. into the fraction $\frac{7}{9}$. In like manner $\frac{116}{145}$ will be reduced to $\frac{4}{5}$. And $\frac{45}{52}$ to $\frac{15}{17}$: But here you are to observe, that if the greatest common Divisor to the Numerator and Denominator be 1, such fraction is in its least terms already: So the fraction $\frac{15}{31}$ cannot be reduced into lower terms, because the greatest common Divisor will be found 1; the like may happen to infinite others: And though the last be a general Rule for Reduction of Fractions into their least terms, yet there are other practical Rules, which in some cases will be more ready, (especially to beginners) viz.

262. When the Numerator and Denominator are even numbers, they may be measured or divided by 2. Therefore in such case you may take the half of the Numerator for a new Numerator, also the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the line which separates the Numerator from the Denominator, and cross the same with a down-right stroke near the Fraction, as you may see in the Margin; then take the half of 16, which is 8, for a new Numerator, and the half of 64, which is 32, for a new Denominator: Again, the half of 8 is 4, for a new Numerator, also the half of 32 is 16, for a new Denominator; and proceeding in the like manner, there will be found $\frac{1}{4}$ equivalent to $\frac{16}{64}$.

263. When the Numerator and Denominator each of them end with 5, or one of them ends with 5, and the other with a cypher; they may be both measured or divided by 5. So $\frac{225}{475}$, will be reduced into $\frac{9}{19}$, and $\frac{50}{425}$ into $\frac{10}{85}$, as by the operation in the Margin is manifest.

$$\begin{array}{r} 225 \mid 45 \quad 9 \\ \hline 475 \mid 95 \quad 19 \end{array}$$

$$\begin{array}{r} 50 \mid 10 \quad 2 \\ \hline 425 \mid 85 \quad 17 \end{array}$$

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264. Whenever you can find any other number, which will exactly divide the Numerator and Denominator (though it be not the greatest common Divisor) you may divide the Numerator and Denominator by such number as before : So $\frac{28}{84}$ may be first reduced to $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced to $\frac{1}{3}$ by 7, as by the operation is manifest.

$$\begin{array}{r} 28 \mid 7 \mid 1 \\ \hline 84 \mid 21 \mid 3 \end{array}$$

265. When the Numerator and Denominator, each of them end with a cypher or cyphers, cut off equal cyphers in both, and the Fraction will be reduced to lesser terms : So $\frac{400}{800}$ is reduced to $\frac{1}{2}$, and $\frac{700}{900}$ to $\frac{7}{9}$.

$$\begin{array}{r} 4 \mid 00 \\ 5 \mid 00 \\ \hline 7 \mid 00 \\ 90 \mid 00 \end{array}$$

266. An expression of the following form, viz. $\frac{4 \times 21 \times 15}{30 \times 18 \times 14}$, which denotes the continual product of 4, 21, and 15, divided by the continual product of 30, 18, and 14, may be abbreviated, by dividing any of the factors or parts of the Numerator and Denominator by some number that is a common measure of those parts : Thus because 2 will measure 4 (a part of the Numerator) and 14 (a part of the Denominator) the Fraction will be reduced to $\frac{2 \times 21 \times 15}{30 \times 18 \times 7}$; and because 5 will measure 15 and 30 it will become $\frac{2 \times 21 \times 3}{6 \times 18 \times 7}$; also because 3 will

measure 21 and 18 it will be $\frac{2 \times 7 \times 3}{6 \times 6 \times 7}$: Now since the number 7 is a factor in both Numerator and Denominator, it may be farther reduced to $\frac{2 \times 3}{6 \times 6}$; and since 2 a part of the Numerator will measure 6 a part of the Denominator it will become $\frac{1 \times 3}{3 \times 6}$; and because 3 will measure 6 it will be $\frac{1 \times 1}{3 \times 2}$, or $\frac{1}{6}$.

Thus,

Thus, if it were required to abbreviate $\frac{12 \times 35}{49 \times 40}$

1. Dividing by 7 reduces it to $\frac{12 \times 5}{7 \times 40}$

2. Dividing by 5 reduces it to $\frac{12 \times 1}{7 \times 8}$

3. Dividing by 4 reduces it to $\frac{3 \times 1}{7 \times 2}$, or $\frac{3}{14}$.

But the operation may be performed much shorter by cancelling the figures that are divided, and writing the quotients of those divisions either above or below them: Thus, in the last Example after dividing by 7,

the Fraction might stand thus.

$$\begin{array}{r} 5 \\ \frac{12 \times 35}{49 \times 40} \\ 7 \end{array}$$

After dividing by 5, thus

$$\begin{array}{r} 1 \\ \frac{12 \times 35}{49 \times 40} \\ 7 \quad 8 \end{array}$$

And after dividing by 4, thus

$$\begin{array}{r} 3 \\ \frac{12 \times 35}{49 \times 40} \\ 7 \quad 8 \\ \quad 2 \end{array} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}$$

Suppose it were required to abbreviate $\frac{15 \times 77 \times 36}{28 \times 45 \times 33}$, the result with all the figures used in the operation will stand as

follows

$$\begin{array}{r} 1 \quad 1 \quad 1 \\ 3 \quad 11 \quad 0 \\ \frac{15 \times 77 \times 36}{28 \times 45 \times 33} = \frac{1}{1} = 1. \\ 4 \quad 9 \quad 11 \\ 1 \quad 1 \quad 1 \end{array}$$

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267. The value of a single fraction in the known parts of the integer, may be found out in this manner, *viz.* multiply the Numerator of the Fraction propos'd, by the number of known parts, of the next inferior denomination, which are equal to the integer, and divide that product by the Denominator, so is the quotient the value of the Fraction in that inferior denomination; and if there happen to be any Fraction in the quotient, you may find the value of it, in the next inferior denomination, by the same Rule, and so proceed till you come to the least known parts.

So the value of $\frac{9}{16}$ of a pound Sterling will be found 11*s.* 3*d.* *viz.* multiply the Numerator 9 by 20 (the number of shillings which are equal to 1 pound Sterling) the product is 180, which being divided by the Denominator 16, the quotient is $11\frac{4}{16}$ shillings. In like manner, the value of $\frac{4}{12}$ of a shilling will be found 3 pence; for multiplying the Numerator 4 by 12 (the number of pence in a shilling) the product is 48, which being divided by the Denominator 16, the quotient is 3 pence. Also the value of $\frac{7}{12}$ of a pound Sterling, will be found 10*s.* $9\frac{3}{4}d.$

$$\begin{array}{r}
 & & & 9 \\
 & & & 20 \\
 16) & 180 & (11\frac{4}{16} \\
 & 16 & \hline \\
 & & 20 \\
 & & 16 \\
 & & \hline \\
 16) & 48 & (3 \\
 & 48 & \hline \\
 & & 0
 \end{array}$$

268. A mixt number may be reduced to an improper Fraction equivalent to the mixt number, in this manner, *viz.* multiply the integer or integers in the mixt number, by the Denominator of the Fraction annexed to the integer or integers, and unto the product add the Numerator of the said Fraction; so is the sum, the Numerator of an improper Fraction, whose Denominator is the same with that of the said Fraction annexed.

So $4\frac{1}{2}$ will be reduced to the improper Fraction $\frac{5}{2}$ for 4 being multiplied by 2 the product is 48, to which adding the Numerator, 1, the sum is 59 for a new Numerator, which being placed over the Denominator 12, gives the improper Fraction $\frac{59}{12}$; which is equivalent to $4\frac{1}{2}$. (as will appear by Art. 271.) In like manner $7\frac{1}{2}$ will be reduced to $\frac{15}{2}$.

269. A whole number is reduced to an improper Fraction, by placing the whole number given as a Numerator, and 1 as a Denominator.

So 14 integers will be reduced to the improper Fraction $\frac{14}{1}$, and one integer to the improper Fraction $\frac{1}{1}$.

270. A whole number is reduced to an improper Fraction, which shall have any Denominator assigned, by multiplying the whole number given by the Denominator assigned, and placing the product as a Numerator over the said Denominator.

As if $\frac{1}{3}$ be given to be reduced to an improper Fraction, whose Denominator shall be 4, multiply $\frac{1}{3}$ by 4, the product is 52, which being placed over 4, gives the improper Fraction $\frac{52}{4}$ equivalent to $\frac{1}{3}$ (as will appear by the next Art.) In like manner $\frac{1}{3}$ may be reduced to $\frac{5}{7}$.

271. An improper Fraction may be reduced to its equivalent whole number, or mixt number, in this manner, viz. divide the Numerator by the Denominator, and the quotient will give the whole number or mixt number sought: So the improper Fraction $\frac{59}{12}$ will be reduced to this mixt number $4\frac{11}{12}$; for if 59 be divided by 12 the quotient is $4\frac{11}{12}$. Also this improper Fraction $\frac{12}{5}$ will be reduced to the whole number 13.

272. Fractions having unequal Denominators, may be reduced to Fractions of the same value, which shall have equal Denominators, by this Rule and the next following, viz. when two Fractions having unequal Denominators, are proposed to be reduced into two other fractions of the same value, which shall have a common Denominator; multiply the Numerator of the first Fraction (that is either of them) by the Denominator of the second, and the product shall be a new Numerator (correspondent to the Numerator of that first Fraction;) also multiply the Numerator of the second Fraction by the Denominator of the first, the product is a new Numerator (correspondent to the Numerator of the second Fraction;) lastly, multiply the Denominators one by the other, and the product is a common Denominator to both the new Numerators.

Thus, if the Fractions $\frac{2}{3}$ and $\frac{4}{5}$ be proposed, multiply 2 by 5, the product 10 is a new Numerator 2 4 correspondent unto 2: Also multiply 4 by 3, the 5 3 product 12 is a new Numerator correspondent to 4: Lastly, multiply 3 by 5, and the product 15 will be 10 12 a common Denominator to the new Numerators: So the Fractions $\frac{10}{15}$ and $\frac{12}{15}$ are found out, which 5 3 have equal Denominators, and each of these new Fractions is equal to its correspondent Fraction first 15 given, viz. $\frac{10}{15}$ is equal to $\frac{2}{3}$, and $\frac{12}{15}$ is equal $\frac{4}{5}$; as is manifest by Art. 261.

273. When three or more Fractions having unequal Denominators, are given to be reduced to other Fractions of the same value with those given, but such as shall have one common Denominator; multiply continually the Numerator of the

first Fraction into all the Denominators, except its own, i.e. the Denominator of that Fraction ; and reserve the last product for a new Numerator, instead of that first Numerator : In like manner, multiply continually the Numerator of the second Fraction into all the Denominators, except the Denominator of the second Fraction ; and reserve the last product for a new Numerator, instead of the second Numerator : Proceed in like manner to find out new Numerators for the rest of the given Fractions. Lastly, multiply continually all the Denominators one into another, and the last product will be a common Denominator to all the new Numerators.

As for Example, if these three Fractions $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, having unequal (or different) Denominators, be given to be reduced to three other Fractions of the same value, which shall have equal Denominators, (or one common Denominator.) First, multiply continually the first Numerator 3 into the second and third Denominators 5 and 7 ; saying 3 times 5 makes 15, which multiplied by 7 produces 105, for a new Numerator instead of the first Numerator 3 : Secondly, multiply continually the second Numerator 2 into the first and third Denominators 3 and 7 ; saying, twice 3 is 6, which multiplied by 7 produces 42, for a new Numerator instead of the second Numerator 2 : Thirdly, multiply continually the third Numerator 5 into the first and second Denominators 3 and 5 ; saying, 5 times 5 makes 25, which multiplied by 3 produces 75, for a new Numerator instead of the third Numerator 5 : Fourthly and lastly, multiply continually all the Denominators 3, 5, and 7, one into another ; saying, 3 times 5 makes 15, which multiplied by 7 produces 105 for a Denominator to each of the three new Numerators 105, 42, and 75, before found out : And so these three Fractions $\frac{105}{105}$, $\frac{42}{105}$, and $\frac{75}{105}$ are discovered, which have one common Denominator 105 ; and every one of them is equal in value to its correspondent Fraction first given, viz. $\frac{1}{3}$ is equal to $\frac{1}{3}$; Also $\frac{2}{5}$ is equal to $\frac{2}{3}$; and $\frac{3}{7}$ is equal to $\frac{3}{3}$; as may easily be proved by Art. 261.

After the same manner, these four Fractions $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{4}{9}$, are reducible to these $\frac{105}{315}$, $\frac{126}{315}$, $\frac{135}{315}$, and $\frac{144}{315}$, which have 315 for a common Denominator ; and are equal in value respectively to the four Fractions given to be reduced.

Note. Although by the foregoing Art. 272. and 273. any multitude of Fractions may be reduced to a common Denominator ; yet because Fractions in their least terms are fittest for use, I shall shew how lesser Denominators than those that are discovered by the said Rules, may frequently be found out.

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274. When 2 or more Fractions having unequal Denominators, are given to be reduced to other equal Fractions having the least common Denominator possible; find by Art. 243. the least common multiple of all the given Denominators, so shall that be the common Denominator required.

Divide the said common Denominator severally by the Denominators of the given Fractions, and multiply each Numerator respectively by the quotient arising from its own Denominator, and the products will be the Numerators required.

Exampl. 1. Reduce $\frac{5}{2}$ and $\frac{7}{8}$ to equivalent Fractions, having the least common Denominator possible.

$$\begin{array}{r} \text{By Art. 243. } 6 \mid 12 : 18 \\ \hline 2 \mid 2 : 3 \\ \hline 3 \mid 1 : 3 \\ \hline \end{array} \quad \text{Therefore } 6 \times 2 \times 3 = 36 \text{ the common Denominator.}$$

$$\text{Then } \frac{5}{2} = 3. \text{ And } \frac{5}{8} = 2.$$

And $(3 \times 5 =) 15$. And $(7 \times 2 =) 14$, are the new Numerators.

Therefore $\frac{15}{36}$ and $\frac{14}{36}$ are the Fractions required.

Exampl. 2. Reduce $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{11}$ and $\frac{5}{13}$ to equivalent Fractions having the least common Denominator possible.

$$\begin{array}{r} 3 \mid 3 : 6 : 9 : 12 : 18 : 36 \\ \hline 3 \mid 1 : 2 : 3 : 4 : 6 : 12 \\ \hline 2 \mid 2 : 1 : 4 : 2 : 4 \\ \hline 2 \mid 1 : . : 2 : 1 : 2 \\ \hline \end{array}$$

Therefore $3 \times 3 \times 2 \times 2 = 36$ is the common Denominator.

Then 36 divided severally by $3, 6, 9, 12, 18$ and 36. Will quote $\frac{3}{12}, \frac{6}{12}, \frac{9}{12}, \frac{12}{12}, \frac{18}{12}$ and $\frac{36}{12}$. Which Quotients multiplied $\{ 2, 5, 4, 7, 11 \}$ respectively by $\{ 2, 5, 4, 7, 11, 13 \}$.

Produce the new Numerators $24, 30, 16, 21, 22$ and 13.

And $\frac{24}{36}, \frac{30}{36}, \frac{16}{36}, \frac{21}{36}, \frac{22}{36}$ and $\frac{13}{36}$ are the Fractions required.

Exampl. 3. Reduce $\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{5}{3}, \frac{2}{5}$ and $\frac{1}{4}$ to the least common Denominator possible.

	8 . 12 . 9 . 3 . 6 . 4
2	4 . 6 . 9 . 3 . 3 . 2
2	2 . 3 . 9 . 3 . 3 . 1
3	1 . 3 . 9 . 3 . 3
3	1 . 3 . 1 . 1

I

Therefore $2 \times 2 \times 2 \times 3 \times 3 = 72$ is the common Denominator.

Then 72, divided severally by 8 . 12 . 9 . 3 . 6 . 4

Will quote 9 . 6 . 8 . 24 . 12 . 18

Those multiplied by 5 . 7 . 4 . 2 . 5 . 1

Produce 45 . 42 . 32 . 48 . 60 . 18

Answer $\frac{4}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$ and $\frac{1}{2}$.

275. A compound Fraction (otherwise called a Fraction of a Fraction) may be reduced to a single Fraction in this manner, viz. multiply all the Numerators continually, and take the product for a new Numerator; also multiply all the Denominators continually, and the product will be a new Denominator.

Thus, if the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ be given to be reduced to a single Fraction; multiply the Numerators 2 and 3, one by the other, so is the product 6 a new Numerator. Also multiplying the Denominators 3 and 4 one by the other, the product 12 is a new Denominator, so $\frac{6}{12}$ or $\frac{2}{4}$ of $\frac{3}{4}$ is the single Fraction sought, being equivalent to $\frac{2}{3}$ $\frac{6}{12}$ or $\frac{1}{2}$ of $\frac{3}{4}$, the compound Fraction given to be reduced.

In like manner, this compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ will be reduced to $\frac{24}{60}$, or $\frac{2}{5}$; for the Numerators 2, 3, 4, being multiplied continually, produce the new Numerator 24; and the Denominators 3, 4, 5, multiplied continually, produce the new Denominator 60: Lastly, the new Fraction $\frac{24}{60}$ (by Art. 261.) will be reduced to $\frac{2}{5}$, which is equal to $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$: But to make the meaning hereof more evident, suppose the integer to be 1 pound of English money; then

$\frac{2}{3}$ of 1*l.* (viz. of 20*s.*) is 16*s.*

$\frac{3}{4}$ of those (viz. of 16*s.*) is 12*s.*

$\frac{5}{6}$ of those $\frac{3}{4}$ (viz. of 12*s.*) is 8*s.* or $\frac{2}{3}$ *l.* whereby 'tis manifest that $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ *l.* is equal to $\frac{2}{3}$ *l.*

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276. In operations of this kind it will frequently happen, that the Reduction will be performed more speedily, if the factors be only connected by (\times) the sign of Multiplication, without actually performing the operation of multiplying:

Thus, in the last Example say $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5} = \frac{2 \times 3 \times 4}{3 \times 4 \times 5}$; then

this expression will by Art. 266. be reduced to $\frac{2}{5}$, only by cancelling (3×4) the Factors common to both Numerator and Denominator.

277. By this Rule a Fraction or mix't number of a lesser name may be reduced to a Fraction of a greater name. As if $3\frac{1}{2}$ pence be proposed to be reduced to a compound Fraction of a pound Sterling, the operation will be in this manner, viz. $3\frac{1}{2}$ or $\frac{7}{2}$ of $\frac{1}{2}$ penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound Sterling; which compound Fraction will, by the aforesaid Rule, be reduced to $\frac{7}{120}$. In like manner, $42\frac{3}{5}$ minutes of an hour are equal to $\frac{42}{5}$ of an hour.

For $42\frac{3}{5} = \frac{213}{5}$, by Art. 268.

And $\frac{213}{5}$ of $\frac{1}{60} = \frac{675}{16 \times 60}$, by Art. 275. and 276.

But $\frac{675}{16 \times 60} = \frac{135}{16 \times 12} = \frac{45}{16 \times 4} = \frac{45}{64}$ by Art. 266.

Here you may also observe, that when a compound Fraction is one of the given terms in any question, it is first of all to be reduced to a single Fraction.

278. Two or more Fractions being given, there may be whole numbers found, which shall have the same reason or proportion as the Fractions given, viz. when the Fractions given have unequal Denominators, reduce them to equivalent Fractions, which have a common Denominator (by Art. 274.) then, rejecting the common Denominator, the Numerators will have the same reason or proportion as the Fractions first proposed.

So $\frac{3}{4}$ and $\frac{5}{6}$ being given, will first of all be reduced to their equivalent Fractions $\frac{18}{24}$ and $\frac{20}{24}$; then rejecting the common Denominator 24, the Numerators 24 and 25 have the same reason with $\frac{3}{4}$ and $\frac{5}{6}$, viz. as $\frac{3}{4}$ is to $\frac{5}{6}$, so is 24 to 25: Also if the Fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ were proposed, there would be found 1, 2, and 4, which are in the same proportion one to the other as those Fractions given: In like manner, if mixt numbers be given, there may be whole numbers found which shall have the same reason or proportion, as the mixt numbers; so $5\frac{2}{3}$ and $3\frac{1}{2}$ being given, will be first reduced to the improper Fra-

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tions $\frac{1}{7}$ and $\frac{2}{9}$ (by Art. 268.) Also the said $\frac{1}{7}$ and $\frac{2}{9}$ will be reduced to $\frac{13}{63}$ and $\frac{14}{63}$; then rejecting the common Denominator 24, the Numerators 136 and 87 will have the same reason as $5\frac{2}{3}$ and $3\frac{1}{3}$, viz. as 136 is to 87, so is $5\frac{2}{3}$ to $3\frac{1}{3}$. Also $16\frac{1}{2}$ and 18 being given, there will be found $3\frac{3}{4}$ and $3\frac{1}{2}$, which being divided by their common measure 3, will give $1\frac{1}{4}$ and $1\frac{1}{2}$, which have the same reason as $16\frac{1}{2}$ and 18.

C H A P. XIX.

ADDITION of VULGAR FRACTIONS, and MIXT NUMBERS.

279. WHEN the numbers given to be added are single Fractions, and have equal Denominators, add all the Numerators together; so is the sum the Numerator of a Fraction, whose Denominator is the same with the common Denominator, which new Fraction is the sum of the Fractions given to be added.

So $\frac{3}{9}$ and $\frac{2}{9}$ being proposed to be added, their sum will be found $\frac{5}{9}$, viz. the sum of the Numerators, 3 and 2 is 5, which being placed over the common Denominator 9, gives $\frac{5}{9}$; In like manner, the sum of these Fractions $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, and $\frac{1}{3}$, will be found $\frac{4}{3}$, which (by Art. 271.) will be found equivalent to $1\frac{1}{3}$; so that $1\frac{1}{3}$ is the sum of the Fractions given to be added.

280. When the Fractions, propos'd to be added, have unequal Denominators, they are first to be reduced to Fractions of the same value, which shall have a common Denominator (by Art. 274.) and then they may be added as above.

So if $\frac{2}{3}$ and $\frac{3}{5}$ were given to be added, their sum will be found $1\frac{1}{5}$; for $\frac{2}{3}$ and $\frac{3}{5}$ will be reduced to their equivalent Fractions $\frac{10}{15}$ and $\frac{9}{15}$, which having equal Denominators may be added according to Art 279. and so the sum will be found $1\frac{1}{5}$: In like manner, the sum of these Fractions $\frac{1}{2}$ and $\frac{1}{4}$ will be found $\frac{3}{4}$. Also the sum of these six Fractions $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, after they are reduced to a common Denominator (according to Example 2d. in Art. 274.) will be found $\frac{12}{3}$, that is, $3\frac{1}{2}$.

281. When any of the Fractions given to be added is a compound Fraction, such compound Fraction is first of all to be reduced to a single Fraction (by Art. 275.) and then you may proceed as before.

So $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{1}{4}$ being given to be added, their sum will be found $\frac{2}{3} \frac{1}{6}$; for the compound Fraction $\frac{2}{3}$ of $\frac{1}{4}$ will be reduced to $\frac{1}{6}$; which added to the single Fraction $\frac{2}{3}$ gives $\frac{2}{3} \frac{1}{6}$. Here you may observe, that the Fractions given to be added in all the former cases, are supposed to be Fractions of integers, which have one and the same particular Denomination, *viz.* if one of the fractions given to be added, be a Fraction of a pound Sterling, all the rest ought to be Fractions of a pound Sterling, and the like is to be understood of other Denominations.

282. When Fractions of Integers of different Denominations are given to be added, they are first of all to be reduced to Fractions of Integers that have one and the same particular Denomination, (by Art. 277.) and then they may be added by the first or second Rule of this Chapter.

So if $\frac{2}{3}$ of a pound Sterling, $\frac{1}{2}$ of a shilling, and $\frac{1}{4}$ of a penny were given to be added, reduce the two latter into Fractions of a pound Sterling, *viz.* $\frac{1}{2}$ of a shilling is $\frac{1}{2}$ of $\frac{1}{12}$ of a pound Sterling, which compound Fraction being reduced to a single Fraction gives $\frac{1}{24}$ l. Likewise $\frac{1}{4}$ of a penny is $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of a pound Sterling, which compound Fraction being reduced, gives $\frac{1}{48}$ l. Lastly $\frac{2}{3}$, $\frac{1}{24}$ l., and $\frac{1}{48}$ l., being added according to Art. 280. their sum will be found $\frac{2}{3} \frac{1}{24} \frac{1}{48}$ l.

283. When mixt numbers are given to be added, find first of all the sum of the Fractions. (by Art. 279. and 280.) then add the integer or integers (if there be any found) in the sum of the Fractions, to the whole numbers.

So if $3\frac{1}{2}$, $4\frac{1}{2}$, and $16\frac{1}{2}$ were given to be added, their sum will be found $24\frac{1}{2}$, *viz.* the sum of the Fractions $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{2}$, will be found (by Art. 280.) to be $\frac{1}{2}$, and the sum of the whole numbers 3, 4, and 16, is 23, to which adding 1 (the integer found in the sum of the Fractions) the sum is 24; so that $24\frac{1}{2}$ is the sum of the mixt numbers given to be added.

C H A P. XX.

SUBTRACTION of VULGAR FRACTIONS and mixt NUMBERS.

284. WHEN the numbers given are both single Fractions, and have equal Denominators: Subtract the lesser Numerator from the greater, and place the remainder over the common Denominator, so is such new Fraction the difference between the Fractions given.

Thus the difference betwixt the Fractions $\frac{9}{11}$ and $\frac{7}{11}$ is $\frac{2}{11}$, which is found by subtracting the lesser Numerator 7, from the greater Numerator 9, and placing the remainder 2 over the common Denominator 11: Also the difference between the Fractions $\frac{4}{7}$ and $\frac{1}{7}$ is $\frac{3}{7}$; that is, the Fraction $\frac{4}{7}$ exceeds $\frac{1}{7}$ by $\frac{3}{7}$.

285. When the numbers given are both single Fractions, and have not a common Denominator, reduce them to Fractions of the same value which have a common Denominator, (by Art. 274.) and then find their difference by the last Rule.

So the difference between the Fractions $\frac{5}{6}$ and $\frac{7}{8}$, will be found $\frac{1}{24}$, viz. reducing the Fractions given to their equivalent Fractions $\frac{40}{48}$ and $\frac{42}{48}$, which have a common Denominator, the difference sought will be found $\frac{1}{24}$ by Art. 284. Likewise $\frac{7}{12}$ being subtracted from $\frac{11}{15}$ there remains $\frac{1}{20}$.

286. When one of the numbers given is a whole number or a mixt number, also when both of them are mixt numbers; reduce such whole, or mixt numbers to an improper Fraction or Fractions, by Art. 268. and 269. and then the operation will be according to Art 284. or 285.

So $7\frac{2}{3}$ being given to be subtracted from 12, the remainder will be found $4\frac{2}{3}$, viz. first $7\frac{2}{3}$ will be reduced to the improper Fraction $\frac{23}{3}$, also 12 will be reduced to $\frac{12}{1}$, then these two improper Fractions $\frac{23}{3}$, and $\frac{12}{1}$, will be reduced to their equivalent Fractions $\frac{76}{12}$ and $\frac{144}{12}$, (which have a common Denominator.) Lastly, the difference between $\frac{144}{12}$ and $\frac{76}{12}$, is $\frac{68}{12}$, or $4\frac{2}{3}$: In like manner, $9\frac{1}{2}$ being given to be subtracted from $12\frac{1}{3}$, the remainder will be found $2\frac{7}{12}$, as by the subsequent operation is manifest,

$9\frac{1}{2} = \frac{19}{2}$; and $12\frac{1}{2} = \frac{25}{2}$ by Art. 268.

$\frac{9}{2} = \frac{9}{10}$ and $\frac{25}{2} = \frac{125}{10}$ by Art. 274.

Then $\frac{125}{10} - \frac{9}{10} = \frac{116}{10} = 2\frac{7}{5}$ by Art. 284.

Though the three last Rules are sufficient for all cases in Subtraction of Fractions, mixt numbers, or whole and mixt; nevertheless, the following Rules will be more expedient in the Subtraction of mixt numbers, or whole and mixt, especially when the Integers consist of many places, as is manifest by the operation, *viz.*

287. When a whole number is given to be subtracted from a mixt number, subtract the said whole number from the integer or integers of the mixt number, and unto the remainder annex the fractional part of the mixt number given; so is the mixt number thus found, the remainder or difference sought.

As if 7 be given to be subtracted from $24\frac{1}{2}$, the remainder will be $17\frac{1}{2}$; as appears from the operation.

288. When a Fraction is given to be subtracted from unity, subtract the Numerator from the Denominator, and place that which remains over the Denominator, which new Fraction thus found, is the remainder or difference sought.

So $\frac{2}{3}$ being subtracted from unity, or 1, the remainder is $\frac{1}{3}$: Also $\frac{13}{9}$ being subtracted from 1, the remainder is $\frac{6}{9}$.

289. When a Fraction is given to be subtracted from a whole number greater than 1, subtract the said Fraction from one (by the last Article), so the remaining Fraction being annexed to the number of integers lessened by unity or 1, gives the remainder or difference sought.

Thus $\frac{4}{7}$ being subtracted from 17, the remainder is $16\frac{3}{7}$; also $\frac{7}{2}$ being subtracted from 39, the remainder is $38\frac{5}{2}$.

290. When a mixt number is given to be subtracted from a whole number, subtract first of all (by Art. 288.) the fractional part of the mixt number from an unit, and set down the remaining Fraction; then adding the unit borrowed to the integer or integers of the mixt number, subtract the said sum from the whole number given (as is taught in Subtraction of whole numbers); so that which remains, together with the remaining Fraction before found, is the remainder or difference sought.

So if $9\frac{1}{2}$ be subtracted from 50, the remainder is $40\frac{1}{2}$, as by the operation is manifest.

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291. When a Fraction is given to be subtracted from a mixt number, and the said Fraction is less than the fractional part of the mixt number; subtract the lesser Fraction from the greater by Art. 284. or 285. then the remaining Fraction being annexed to the integers of the mixt number, gives the remainder or difference sought for.

$$\text{So } \frac{1}{3} \text{ being subtracted from } 12\frac{1}{3}, \text{ the remainder is } 12\frac{2}{3}, \text{ as by the operation is manifest.}$$

292. When a Fraction is given to be subtracted from a mixt number, and the said Fraction is greater than the fractional part of the mixt number, subtract the said greater Fraction from an unit borrowed from the mixt number (by Art. 288.) and add the remaining Fraction to the fractional part of the mixt number, (by Art. 279. or 280.) so the Fraction found by that addition, being annexed to the integers of the mixt number lessened by an unit, or 1, gives the remainder or difference sought.

Thus $\frac{1}{3}$ being subtracted from $13\frac{1}{3}$; the remainder is $13\frac{1}{3}$ $12\frac{2}{3}$, viz. subtracting $\frac{1}{3}$ from 1, the remainder is $\frac{2}{3}$, which added to $\frac{1}{3}$ gives $\frac{2}{3}$, which being annexed to 12 , (the number of integers in the mixt number lessened by 1 or unity) gives $12\frac{2}{3}$, the remainder sought.

293. When a mixt number is given to be subtracted from a mixt number, and the fractional part of the mixt number to be subtracted, is less than the fractional part of the mixt number from which you are to subtract, subtract the said lesser Fraction from the greater (by Art. 284. or 285.) and set down the remaining Fraction: Also subtract the integers of the lesser mixt number from the integers of the greater (as in Subtraction of whole numbers;) so is the mixt number thus found, the remainder or difference sought.

So if $17\frac{1}{3}$ be given to be subtracted from $20\frac{1}{3}$; the remainder will be found $3\frac{1}{3}$, viz. subtracting $\frac{1}{3}$ from $\frac{1}{3}$, the remainder is $\frac{2}{3}$, also subtracting 17 from 20, the remainder is 3.

294. When a mixt number is given to be subtracted from a mixt number, and the fractional part of the mixt number to be subtracted, is greater than the fractional part of the mixt number from which you are to subtract; subtract the said greater Fraction from an unit borrowed from the greater mixt number (by Art. 288.) and add the remaining Fraction to the fractional part of the greater mixt number (by Art. 279. or 280.) so is the sum to be reserved as the fractional part of the remainder sought;

sought : Then add the unit borrowed to the integer or integers of the lesser mixt number, and subtract the sum from the integers of the greater mixt number (as in Subtraction of whole numbers;) so that which remains, together with the Fraction before referred, is the remainder or difference sought for.

Thus, if $20\frac{1}{4}$ be given to be subtracted from $35\frac{3}{4}$, the remainder will be found $14\frac{2}{4}$, viz. subtracting $\frac{1}{4}$ from an integer, or 1, the remainder is $\frac{1}{4}$, which added to $\frac{3}{4}$, gives $\frac{4}{4}$; then adding the integer borrowed to 20, it will be 21, which subtracted from 35, the remainder is $14\frac{2}{4}$, so the remainder or difference sought, is $14\frac{2}{4}$.

When you cannot clearly discern which is the greater of two Fractions, having unequal denominations, reduce them to Fractions of the same value which have a common Denominator, (by Art. 274.) and then it will be apparent which of the two Fractions is the greater. As if it be desired to know which of these two Fractions $\frac{5}{7}$ and $\frac{11}{17}$ is the greater; after they are reduced to $\frac{85}{119}$ and $\frac{77}{119}$, it is evident that the former exceeds the latter by $\frac{8}{119}$.

C H A P. XXI.

MULTIPLICATION of VULGAR FRACTIONS, and MIXT NUMBERS.

295. **W**HEN the numbers given to be multiplied, are both single Fractions, multiply the Numerators one by the other, and take the product for a new Numerator; also multiply the Denominators one by the other, and the product will be a new Denominator, which new Fraction is the product sought for.

So $\frac{7}{8}$ and $\frac{5}{12}$ being given to be multiplied, the product will be found $\frac{35}{96}$; for 7 multiplied by 5 produces 35 for a new Numerator; and 12 multiplied by 8 gives 96 for a new Denominator; also $\frac{1}{2}$ and $\frac{1}{3}$ being multiplied one by the other, the product will be found $\frac{1}{6}$. Here you may observe that in the Multiplication of proper Fractions, the product is always less than either of the terms given; for in multiplying by an unit, the product is equal to the Multiplicand; therefore when you multiply

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multiply by a proper Fraction, that is, by less than an unit, the product must be less than the Multiplicand.

296. When one of the numbers given is a whole number or a mixt number; also when both of them are mixt numbers, reduce such whole number, or mixt number or numbers to an improper Fraction or Fractions, by Art. 268. and 269. and then the operation will be the same as in the last Article.

So $8\frac{1}{2}$ being given to be multiplied by 5, the product will be found $43\frac{1}{2}$, viz. $8\frac{1}{2}$ being reduced to the improper Fraction $\frac{17}{2}$, also 5 into 1, multiply 26 by 5, the product is 130 for a new Numerator: Also multiply 3 by 1, the product is 3, for a new Denominator, which new Fraction $\frac{130}{3}$ being reduced (according to Art. 271.) will be $43\frac{1}{2}$, the product sought. In like manner, $7\frac{1}{2}$ being multiplied by $\frac{1}{2}$, the product will be found $4\frac{1}{2}$. Here observe, that when either of the terms given is a compound Fraction, it must first of all be reduced to a single Fraction, and then the operation is as before.

297. The directions of Articles 276. and 266. may often be usefully applied here: For Example, if $\frac{5}{7}$ be to be multiplied by $\frac{6}{11}$ write down the product thus $\frac{6 \times 5}{7 \times 11}$, which, contracted by dividing both Numerator and Denominator by 6, becomes $\frac{5}{7 \times 2} = \frac{5}{14}$.

298. To take any part or parts of a number proposed, is nothing else but to multiply the said number by the Fraction, which declares what part is to be taken: So if you desire to know what is $\frac{1}{5}$ of 320, multiply 320^0 by $\frac{1}{5}$, the product will be

200; for $\frac{320 \times 5}{1 \times 8} = 40 \times 5 = 200$. In like manner $\frac{2}{3}$ of $45\frac{1}{2}$ is

$$\frac{2}{3} \times \frac{363}{8} = \frac{2 \times 363}{3 \times 8} = \frac{363}{3 \times 4} = \frac{121}{4} = 30\frac{1}{4}. \text{ And } \frac{1}{4} \text{ of } 120 = 12^0 = 30.$$

299. Hence if it be required to multiply an integer by a Fraction, multiply the integer by the Numerator of the Fraction and divide the product by the Denominator. For Example,

the product of 48 by $\frac{5}{8}$ is $\frac{48 \times 5}{8} = 6 \times 5 = 30$.

That the methods above directed are true, may be proved by Multiplication of whole numbers: Thus, assume 2 Fractions $\frac{2}{3}$ and $\frac{3}{2}$, which are severally equal to 2 and 3, two whole numbers;

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numbers; then it will appear that the product of the Fractions $\frac{8 \times 6}{4 \times 2} = \frac{48}{8}$ is equal to the product of the whole numbers 2×3 for $\frac{48}{8} = 6$.

300. Sometimes the Work of Multiplication, in mixt numbers, may be compendiously performed after the manner of these following Examples, *viz.* if it be required to multiply $120\frac{1}{2}$ by $48\frac{1}{2}$, first multiply the whole numbers mutually, to wit, 120 by 48, and place the particular products orderly one under the other, as in Multiplication of whole numbers; then multiply the said whole numbers first given by the Fractions alternately, *viz.* take $\frac{1}{2}$ of 48, which is 12, also take $\frac{1}{2}$ of 120, which is 60, and place the said 12 and 60 orderly to be added to the former particular products: Lastly, add all together, and to the sum annex the product of the two Fractions; to wit, in this Example, the product of the Multiplication of $\frac{1}{2}$ by $\frac{1}{2}$, which is $\frac{1}{4}$, so the total product required will be $5832\frac{1}{4}$, as you see by the Example in the Margin. In like manner, if $18\frac{1}{3}$ be multiplied by $40\frac{1}{3}$, the product will be $746\frac{1}{3}$; and if $29\frac{1}{2}$ be multiplied by 50, the product is 1475, as you see by the Examples following.

$$\begin{array}{r|l}
 18\frac{1}{2} & 29\frac{1}{2} \\
 40\frac{1}{3} & 50 \\
 \hline
 720 & 1450 \\
 20 & 25 \\
 \hline
 6 & \\
 \hline
 746\frac{1}{3} & 1475
 \end{array}$$

C. H A P. XXII.

DIVISION of VULGAR FRACTIONS, and MIXT NUMBERS.

301. **W**HEN the numbers given are both single Fractions, multiply the Denominator of the Divisor by the Numerator of the Dividend; and take the product for a new Numerator: Also multiply the Numerator of the Divisor by

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by the Denominator of the Dividend, and the product is a new Denominator ; which new Fraction is the quotient sought.

So if $\frac{5}{3}$ be given to be divided by $\frac{3}{4}$, the quotient will be found $\frac{20}{9}$, viz. multiplying 5 by 4 the product is 20 for a new Numerator ; also multiplying 3 by 9 the $\frac{1}{3} \times \frac{5}{3} = \frac{20}{9}$ product is 27 for a new Denominator, so is $\frac{20}{27}$ the quotient sought. In like manner if $\frac{4}{3}$ be given to be divided by $\frac{2}{3}$, the quotient will be found $\frac{12}{5}$, that is $2\frac{2}{5}$, as you see in the Example: Here you may observe that in Division, by proper Fractions, the quotient is always greater than the Dividend; for in Division the less the Divisor is, the greater will the Quotient be ; and when unity is the Divisor the Quotient will be equal to the Dividend; therefore when the Divisor is a proper Fraction, that is less than an unit, the Quotient will be greater than the Dividend.

302. When one of the numbers given is a whole number, or a mixt number ; also when both are mixt numbers, reduce such whole number or mixt number or numbers to an improper Fraction or Fractions, by Art. 268. or 269. and then the operation will be the same as in the last Article.

So if 42 be divided by $7\frac{1}{2}$, the Quotient will be found $5\frac{1}{3}$, for $7\frac{1}{2}$ and 42 will be reduced to these improper Fractions $\frac{15}{2}$ and $\frac{15}{1}$, then multiplying 42 by 2, the product 84 is a new Numerator, also multiplying $\frac{15}{2}$ by 1 produces 15 for a new Denominator ; so is $\frac{15}{2} = \frac{84}{15} = 5\frac{1}{3}$ the Quotient sought.

303. Here also the directions of Art. 276. and 266. will often prove advantegeous : Thus, if $\frac{25}{8}$ be to be divided by $\frac{15}{4}$, the Quotient may be represented by $\frac{4 \times 25}{15 \times 8}$, which reduced by

dividing by 4 becomes $\frac{25}{15 \times 2}$, and that again divided by 5,

becomes $\frac{5}{3 \times 2} = \frac{5}{6}$.

304. If a whole number be given to be divided by a Fraction, multiply the whole number by the Denominator of the Fraction, and divide the Product by the Numerator thereof : For Example, if 48 be to be divided by $1\frac{1}{2} = \frac{3}{2}$, then the Quotient will be $\frac{48 \times 2}{8} = 6 \times 2 = 48$.

305. If two Fractions that have the same Denominator are given for a Divisor and Dividend, then the Quotient of their

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Numerators will be the number required. For Example, if $\frac{1}{2}$ be to be divided by $\frac{1}{3}$, the Quotient will be $\frac{3}{2}$.

$$\text{For } \frac{8 \times 5}{3 \times 8} = \frac{5}{3}.$$

The truth of these methods will appear by assuming 2 Fractions $\frac{16}{4}$ and $\frac{10}{5}$, which are severally equal to 2 whole numbers 4 and 2 for the Quotient of the Fractions $\frac{16}{4}$ divided by $\frac{10}{5}$, $\frac{5 \times 16}{4 \times 10}$ is equal to the Quotient of the whole number 4 divided by 2, for $\frac{5}{2} = \frac{1}{4} = \frac{5}{2} = 2$.

306. Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. The difference of two numbers is $1\frac{1}{2}$, the lesser number is $2\frac{1}{3}$, what is the greater? *Ans.* $3\frac{2}{3}$ (found by Addition.)

Quest. 2. What number is that, which if added to $3\frac{1}{2}$ gives the sum $8\frac{1}{2}$? *Ans.* $4\frac{1}{2}$ (found by Subtraction.)

Quest. 3. There is in 3 bags the sum of $121\frac{2}{5}$ l. viz. in the first bag $50\frac{1}{2}$ l. in the second $40\frac{1}{3}$ l. what is in the third bag? *Ans.* $30\frac{1}{2}$ l. (found by Addition and Subtraction.)

Quest. 4. Two merchants A and B having certain shares in a ship, the share of A is $\frac{7}{8}$ of the ship, that of B $\frac{2}{5}$, what is the difference between their parts? *Ans.* the share of A exceeds the share of B by $\frac{1}{8}$, (found by Subtraction.)

Quest. 5. What is $\frac{1}{2}$ of $130\frac{2}{3}$? *Ans.* $81\frac{2}{3}$ (found by Multiplication.)

Quest. 6. What number is that, which being multiplied by $\frac{1}{3}$, produces $25\frac{1}{3}$? *Ans.* $42\frac{1}{3}$. (found by Division.)

The

The Doctrine of Decimal Fractions.

C H A P. XXIII.

NOTATION of DECIMAL FRACTIONS.

307. IT is hard to determine, who was the first that brought Decimal Arithmetic to light, though it be a late invention; but without doubt it has received much improvement within the compass of a few years by the industry of Artists, and now seems to be arrived at perfection. The excellency thereof is best known to such as can apply it to the practical part of the Mathematics, and to the Construction of Tables, which depend upon standing or constant proportions; such are Trigonometrical Canons, Tables for the computing of Compound Interest, &c. in which cases Decimal operations afford so great help, that, in my opinion, many ages have not produced a more useful invention. But it may be objected, that Decimal Arithmetic often gives an imperfect solution to a question. This I grant, yet the answer so given may be as useful as that which is exactly true; for in common affairs, the loss of $\frac{1}{1000}$ part of a grain, or of an inch, &c. to wit, any quantity which cannot be seen, is inconsiderable: But I would not be mistaken; for in extolling Decimals, I do not cry down Vulgar Fractions; since experience shews, that Decimal Fractions are commonly abused, by being applied to all manner of questions about Money, Weights, &c. when indeed many questions may be resolved with more facility by Vulgar Arithmetick; so that the right use of Decimals depends upon the discretion of the artist.

308. When a single Fraction has for its Denominator, a number consisting of 1 or unity in the extreme place towards the left-hand, and nothing but a cypher or cyphers towards the right, it is more particularly called a Decimal: Of this kind are these that follow, viz. $\frac{5}{10}$, that is, five tenths; $\frac{5}{100}$, five hundredth parts; likewise these are Decimal Fractions, $\frac{1}{1000}, \frac{1}{10000}, \frac{1}{100000}$, &c.

309. A Decimal Fraction may be expressed without the Denominator, by prefixing a point or comma before (to wit, on the left-hand of) the Numerator; so $\frac{1}{10}$ may be writ thus .5, or thus ,5; and $\frac{1}{100}$ thus, .25, or thus ,25.

310. In Decimals, when the Numerator does not consist of so many places as the Denominator has cyphers; fill up the void places, in the Numerator, with cyphers prefixed on the left hand: So $\frac{1}{100}$ is set down thus, ,05; likewise $\frac{1}{1000}$ thus, ,005; and $\frac{1}{10000}$ thus, ,0005; likewise $\frac{1}{100000}$ thus, ,00005.

311. In Decimals thus expressed, the Denominator is discoverable by the places of the Numerator: For if the Numerator consists of one place, the Denominator consists of 10 or unity with one cypher; if of two places, the Denominator consists of 1, with two cyphers annexed; if of three, the Denominator consists of 1 or unity, with three cyphers annexed: So the Denominator of ,25 is 100, the Denominator of .050 is 1000, and the Denominator of .006 is 1000.

312. Cyphers at the end of a Decimal do neither augment nor diminish the value of it: So .2, .20, .200, .2000 are Decimals, which have one and the same value; for $\frac{1}{10}$ being abbreviated by Art. 265. will be made $\frac{1}{100}$, and so will $\frac{1}{1000}$ or $\frac{1}{10000}$. But cyphers placed between the separating comma and the figures of the Decimal diminish its value: So ,5 is $\frac{1}{10}$, but ,05 is $\frac{1}{100}$.

313. Therefore Decimal Fractions are easily reduced to a common Denominator, (which is a troublesome work in Vulgar Fractions;) for if all the Numerators of as many Decimal Fractions as are given, be made to consist of the same number of places, by annexing a cypher or cyphers at the end, (that is on the right-hand) of such Numerators as are defective, they will all be reduced to a common Denominator: So these Decimals, .2, .03, .027, which signify $\frac{1}{5}$, $\frac{1}{33}$, $\frac{27}{1000}$; may be reduced to these, .200, .030, .027, which have 1000 for a common Denominator.

314. The order of places in any Decimal proceeds from the left-hand to the right, contrary to the order of places in Integers, which is from the right-hand to the left: So in this Decimal .247 the figure 2 stands in the first place, (being the outermost towards the left-hand, and next to the point,) the figure 4 stands in the second place, and 7 in the third. Also in this Decimal .0245 a cypher stands in the first place, 2 in the second, 4 in the third, and 5 in the fourth.

315. Every place in the Numerator of a Decimal Fraction has a peculiar Denominator, or proper value, viz. the Denominator

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minator of the first place is 10, of the second 100, of the third 1000, &c. So that the first place of a Decimal, signifies tenth parts of an unit, or integer; the second place, hundredth parts of an integer; the third place, thousandth parts of an integer, &c. Hence it is manifest, that this Decimal .3254 (every place thereof being considered a-part by itself) consists of .3, .02, .005, .0004, (viz. $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$, which being reduced to a common Denominator (by Art. 313.) will give these .3000, .0200, .0050, .0004, viz. $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$, all which collectively make .3254 (or $\frac{3254}{10000}$.)

316. In whole numbers, the first place above (that is on the left-hand of) the place of units signifies tens of units, but in Fractions the first place beneath (that is, on the right-hand of) the place of units denotes tenth parts of 1 or unity, and is called the first place of Decimal parts, or place of primes; likewise the second place above the place of units signifies hundreds of units; but the second place beneath the place of units expresses hundredth parts of 1 or unity, and is called the second place of Decimals, or place of seconds; So that, as the values of the places in integers ascend in a decuple proportion, from the place of units, towards the left-hand; so the values of the places of Decimals descend in a subdecuple proportion, beneath the place of units, towards the right-hand, viz. among the places of integers, every following place towards the left-hand, is ten times the value of the next preceding place: But among the places of Decimal parts, every following place towards the right-hand is one tenth part of the value of the next preceding place: All which will be evident by the following Table,

A TABLE

A TABLE for the NOTATION of Integers
and Decimals.

317. In the foregoing Table, you may observe, that the places of integers or whole numbers are separated from the places of Decimal parts by a point; so the number on the left-hand of the point expresses 73285 integers or units: But the number on the right-hand of the point only shews 8237 parts of 1 (or an integer) supposed to be divided into 10000 equal parts. In like manner, this number 5.8 signifies 5 integers and eight tenth parts of an integer, and this number 285,82 denotes 285 integers (or units) and $\frac{82}{100}$ parts of an integer.

318. Hence it follows, that if the separating comma, in any mix'd or fractional expression, be moved one place towards the left-hand, then every figure, and consequently the whole expression, is but a tenth part of what it was before; that is, it is divided by 10; if it be moved two places, the expression is divided by 100; if three places, by 1000, &c.

But if the separating comma be moved towards the right-hand, then every figure, and the whole expression, is multiplied by 10, 100, 1000, &c. according as it is moved one, two, or three places.

C H A P. XXIV.

Concerning the REDUCTION of VULGAR FRACTIONS to DECIMAL FRACTIONS.

319. If the greatest integer of money, as also of weight, measure, &c. were subdivided decimallly, to wit, a pound of English money into ten equal pieces of coin, and every one of these into ten other equal pieces, &c. and weights, measures, &c. after the same manner; The doctrine of Arithmetic would be taught, with much more ease and expedition than now it is; but it being improbable that such a reformation will ever be brought to pass. I shall proceed in directing a course to the studious, for obtaining the frugal use of such Decimal Fractions as are in their power.

320. Since, in Arithmetical Questions, some of the given numbers for the most part happen to be Fractions, a way must be shewed how to reduce a Vulgar Fraction to a Decimal Fraction; yet in some cases there is no need of this reduction: For example, a foot in length is commonly subdivided into 12 inches, an inch into 4 quarters, and every quarter into 2 half-quarters; but a foot may as easily, and a great deal more commodiously, be divided, first into ten equal parts, and then every of those into ten other equal parts; and each of these into ten other equal parts; (or at least such division may be supposed or imagined when it cannot actually be made.) This foot in length so divided, being applied to the sides of Superficial Figures, or Solids, will at first sight give the quantities of lines in feet and Decimal parts of a foot (as readily as a foot, vulgarly divided, will shew you how many feet, inches, quarters, and half-quarters, are contained in any line) from whence the superficial or solid content may be found in feet, by Multiplication only; and how much this excels the Vulgar way, I shall partly manifest hereafter. The like Subdivision I would have to be made of a yard, perch, &c.

321. A Vulgar Fraction may be reduced to a Decimal of the same value, or infinitely near (for all Vulgar Fractions cannot be exactly reduced to Decimals) by the Rule of Three Direct; for as the Denominator of any Fraction whatsoever, is to the Numerator thereof, so is any other proposed Denominator to its correspondent Numerator: *Example.* Let it be required to reduce $\frac{1}{8}$ to a Decimal, whose Denominator is assigned to be 1000: Say, by the Rule of Three, if the Denominator 8, has 5 for a Numerator; what will the Denominator 1000 require for a Numerator? Multiply and divide as the Rule of Three Direct requires, so will the fourth proportional be found to be 625, which is the Numerator sought; therefore $\frac{625}{1000}$, or ,625, is a Decimal Fraction equal in value to $\frac{1}{8}$. Another *Example.* Let it be required to reduce $\frac{1}{240}$ to a Decimal Fraction, whose Denominator shall be 100000; say by the Rule of Three, if 240 the Denominator give 7 for a Numerator, what will the Denominator 100000 require for a Numerator? *Answ.* 2916, and somewhat more.

322. Hence may be deduced the following Rule to reduce a Vulgar Fraction to a Decimal.

To the Numerator, annex as many cyphers as may be convenient; then divide it by the Denominator, and the Quotient will be the Decimal required. *Note.* There must always be as many figures in the quotient, as there are cyphers annexed to the Numerator; and if so many do not arise, supply the deficiency by placing cyphers after the separating comma.

Examp. 1. What is the Decimal equal to $\frac{1}{2}$?
Denominator 2) 1,0 = the Numerator, with a cypher annexed.
,5 = the Decimal required.

Examp. 2. What is the Decimal equal to $\frac{1}{4}$?
Denominator 4) 1,00 = the Numerator with two cyphers.

,25 = the Decimal required.

Examp. 3. What is the Decimal equal to $\frac{1}{5}$?
Denominator 5) 1,0 = the Numerator with a cypher.
,2 = the Decimal required.

Examp. 4. What is the Decimal equal to $\frac{1}{25}$?
Denominator 25) 1,00 = the Numerator with two cyphers.
,04 = the Decimal required.

Examp. 5. What is the Decimal equal to $\frac{1}{9}$?
Denominator 9) 1,00000 &c. = the Numerator with cyphers.
,11111 &c. = the Decimal required.

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323. If the number of cyphers, annexed to the Numerator of the last Example, had been ever so great, and the division (by the Denominator 9) had been prosecuted till those cyphers had been all exhausted; the Quotient, or Decimal required, would consist of as many ones as there had been cyphers annexed, and there would still be an unit remaining; by annexing cyphers to which, the division might be continued farther.

324. Those Decimals which are produced from Vulgar Fractions, whose Numerators, with cyphers annexed, can be measured by their Denominators, are called finite or terminate Decimals; because they consist of a finite and determinate number of places: Such are the Decimals resulting from the four first Examples.

325. Those Decimals which are produced from Vulgar Fractions, whose Numerators, with cyphers annexed, cannot be measured by their Denominators, are called infinite, interminate, circulating, and repeating Decimals; because they do not consist of a finite and determinate number of places, and because some figure or figures do continually circulate or repeat in them; such is the Decimal resulting from the fifth Example.

326. Hence it follows, that no Vulgar Fraction will produce a finite Decimal, but such, whose Denominators are compounded of the primes 2 and 5; for no other numbers can measure 10, 100, 1000, 10000, &c. or their Multiples, and such every Numerator becomes, by annexing cyphers thereto.

327. The finite Decimals, arising from such Vulgar Fractions, will consist of as many places of figures, as there are tens, two's, or fives, in the composition of their Denominators. Thus $\frac{1}{10}$, $\frac{1}{5}$, and $\frac{1}{2}$, have finite Decimals of 1 place, viz. ,1, ,2, and ,5; $\frac{1}{100}$, $\frac{1}{50}$, and $\frac{1}{20}$, have finite Decimals of 2 places, viz. ,01, ,04 and ,25 &c.

328. In circulating Decimals, if one figure only repeats, it is called a single Repetend: Thus in Examp. 5th. the Decimal, consisting of the repeating figure 1, is a single Repetend.

329. To avoid the trouble of writing down unnecessary figures, a single Repetend is denoted by the repeating digit dashed; that is, the Decimal, 11111 &c. consisting of the single Repetend 1, is expressed by, .1.

330. Since ,1 is the Decimal equivalent to the Vulgar Fraction $\frac{1}{10}$; therefore, $\frac{1}{7}$ will be the Decimal equal to $\frac{1}{7}$; $\frac{1}{3}$, to $(\frac{3}{9} = \frac{1}{3})$; $\frac{1}{4}$, to $\frac{1}{4}$; $\frac{1}{5}$, to $\frac{1}{5}$; $\frac{1}{6}$, to $(\frac{6}{12} = \frac{1}{2})$; $\frac{1}{7}$, to $\frac{1}{7}$; $\frac{1}{8}$, to $\frac{1}{8}$; and $\frac{1}{9}$, to $(\frac{9}{9} = 1)$.

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Therefore in all operations for, $g = 0,9999 \text{ &c. ad infinitum}$, put an unit.

331. *Examp. 6.* What is the Decimal equal to $\frac{1}{6}$?
 Denominator 6) 1,000000 &c. = Numerator with cyphers.
 $\underline{,166666 \text{ &c.}} = ,1\beta$ = the Decimal required.

Examp. 7. What is the Decimal equal to $\frac{1}{12}$?
 Denominator 12) 1,0000000 &c. = Numerator with cyphers.
 $\underline{,083333 \text{ &c.}} = ,08\beta$ = the Decimal requir'd.

Examp. 8. What is the Decimal equal to $\frac{1}{15}$?
 Denominator 15) 1,000000 &c. = Numerator with cyphers.
 $\underline{,066666 \text{ &c.}} = ,06$ = the Decimal requir'd.

Examp. 9. What is the Decimal equal to $\frac{1}{75}$?
 Denominator 75) 1,000000 &c. = Numerator with cyphers.
 $\underline{,013333 \text{ &c.}} = ,01\beta$ = the Decimal requir'd.

The Decimals resulting from the 4 last operations are called mixt repeating Decimals, or mixt single Repetends, because they have a finite number before the single Repetend begins.

332. And, from the four last Examples, it will appear, that the finite part, of mix'd repeating Decimals, arises because the Denominators, of their corresponding Vulgar Fractions, are partly compounded of the primes 2 and 5; for in *Examp. 6.* the Denominator $6 = 2 \times 3$; in *Examp. 7.* $12 = 4 \times 3$; in *Examp. 8.* $15 = 5 \times 3$; and in *Examp. 9.* $75 = 25 \times 3$. And it is farther observable, that the finite parts of the mixt Decimals, in these Examples, consist of as many places, as the finite Decimals before found in Examples 1. 2. 3. and 4. where the Denominators were respectively 2. 4. 5. and 25. which Denominators in these Examples, are severally multiplied by 3.

333. From these Examples, also, may be deduced the manner of expressing the value of mixed Repetends, in the form of Vulgar Fractions, viz. the finite part has a decimal Denominator, as in Art. 311. but the repeating part has 9 for its Denominator, by Art. 330. and should be esteemed only a part of the last finite Denomination.

So in the expression of Example the 6th. $,1\beta = \frac{1}{6}$, the finite part, 1 is $\frac{1}{10}$; and the Repetend, had it stood in the place of primes, would be $\frac{9}{9}$, but because the finite part of the expression is in that place, it must be esteemed only $\frac{9}{9}$ of $\frac{1}{10}$ or $\frac{9}{90}$: Now $\frac{1}{10} + \frac{9}{90} = \frac{9}{90} + \frac{9}{90} = \frac{18}{90} = \frac{1}{5}$.

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In like manner for the expression of the 7th Example, $,08\overline{3} = \frac{1}{12}$ the finite part, $,08 = \frac{8}{100}$; and the Repetend, if it were in the place of primes would be $\frac{3}{9}$; but the last figure of the finite part being in the place of hundredths, therefore the Repetend is $\frac{3}{9}$ of $\frac{1}{100}$ or $\frac{1}{900}$: Now $\frac{8}{100} + \frac{1}{900} = \frac{72}{900} + \frac{1}{900} = \frac{73}{900} = \frac{1}{12}$.

334. Examp. 10. What is the Decimal equal to $\frac{1}{99}$?

99) 1,000000000 &c. (,01010101 &c. the Decimal required.

99

$$\begin{array}{r} 100 \\ 99 \\ \hline 100 \\ 99 \\ \hline 100 \\ 99 \\ \hline 1 & \text{ &c.} \end{array}$$

Tis apparent, that how long soever the Division in the last Example be continued, the two Quotient figures (01) will repeat once, for every two cyphers annexed to the Dividend.

335. And hence we have the generation of Repetends consisting of 2 figures, which (as well as those of 3, 4, 5, &c. figures) are called compound Repetends.

336. Compound Repetends are distinguished by dashing the first and last repeating figure: Thus, $,010101$ &c. ad infinitum, may be wrote $,\overline{01}$, and $,123123123$ &c. ad infinitum, $,\overline{123}$.

337. Since $\frac{1}{99} = ,\overline{01}$; $\frac{2}{99}$ will be $= ,\overline{02}$; $\frac{3}{99} = (\frac{1}{33} =),\overline{03}$; $\frac{4}{99} = ,\overline{04}$, &c. $\frac{2}{99} = (\frac{1}{11} =),\overline{02}$; $\frac{1}{99} = ,\overline{10}$; $\frac{5}{99} = (\frac{1}{3} =),\overline{17}$, &c.

And, in general, a Vulgar Fraction which, being reduced to a Decimal, produces a compound Repetend of 2 figures, either is, or is equal to, a Fraction, whose Numerator is the two repeating figures, and Denominator 99.

338. And the Denominators of those Fractions, which produce mixt compound Repetends of 2 figures, such as, $,1\overline{58}$, $,104\overline{51}$, &c. are compounded of the primes 2 or 5, and the number 99.

339.

339. Examp. 11. What is the Decimal equal to $\frac{1}{999}$?

999) 1,000000000 &c. (,001001001 &c.

999

1000

999

1000

999

1 &c.

Here we have the generation of a compound Repetend consisting of 3 figures, and by reasoning, as in Art. 337, it will appear, that the Vulgar Fractions, which produce such Decimals, are, or are equal to, those, which have the 3 repeating digits for Numerators, and 999 for a common Denominator.

340. Universally, a Decimal Fraction, consisting only of a Repetend, is equal to a Vulgar Fraction whose Numerator is that Repetend, and the Denominator a number consisting of as many nines, as there are places in the Repetend.

341. Examp. 12. What is the Decimal equal to $\frac{7}{3}$?

7) 1,00000000000 (0, 1428571 &c.

7

30

28

20

14

60

56

40

35

50

49

10

7

3 &c.

Since

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Since the number 1, the only significant figure of the Dividend, becomes the remainder, after multiplying by the sixth quotient figure, it is clear that those six figures will continually repeat in the quotient; therefore $\frac{1}{7} = 0.142857$.

342. Hence, if the Denominator of a proper Fraction, in its least terms, be not composed of the primes 2 and 5, that Fraction will be equal to a repeating Decimal, whose Repetend will have fewer figures than there are units in the Denominator.

For, in the 12th Example, where unity, with cyphers annexed, is divided by 7; 'tis obvious, that no remainder, in any part of the division, can be greater than 6: Therefore if, as in that Example, all the possible remainders should happen before any of them return, yet after that, *viz.* after 6 figures (a number less than the Divisor by unity) are obtained in the Quotient, some one of the former remainders must necessarily recur (for by Art. 326. there will be a remainder, because the Divisor cannot measure the Dividend); to which former remainder a cypher being annexed, as in the former part of the Divisor, that figure must repeat in the Quotient, which was placed there when the same Dividual happened before: And therefore the other figures must follow and continue the circulation.

343. Having any proper Fraction given, to find whether the Decimal Fraction, equal thereto, will be finite or infinite; and if infinite, whether it will consist of a pure or mixed Repetend; and how many places that Repetend will consist of.

Rule. Reduce the given Fraction into its least terms, and divide the Denominator by 10, 2, and 5, as often as possible; then make the result a Divisor, and let the Dividend consist of as many nines as may be necessary; continue the Division until there be no remainder; and the number of nines used will shew the number of places in the Repetend; which Repetend will begin after as many places of figures, as there were tens, two's, or fives, divided by.

If the whole Denominator vanishes by repeating your Divisions by 10, 2, or 5, the Decimal will be finite, and consist of as many places as you perform Divisions, by Art. 327. but if not, there will be a Repetend, by Art. 342. Also a Fraction, equal to the given one, will have a number consisting of as many nines in the composition of its Denominator, as there will be figures in the Repetend, by Art. 340. Therefore that number of nines will be a Multiple of the given Denominator, reduced as above.

Exampl.

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Examp. 1. How many places will the Repetend of that Decimal consist of, which is equal to $\frac{1}{7}$?

$$7) \underline{999999} \\ 142857 \quad \text{Ans}w. 6 \text{ places.}$$

Examp. 2. Will the Fraction $\frac{512}{55}$ produce a finite or infinite Decimal; and of how many places will it consist, if finite?

10) 51200	Number of Divisions
10) 5120	1
2) 512	2
2) 256	3
2) 128	4
2) 64	5
2) 32	6
2) 16	7
2) 8	8
2) 4	9
2) 2	10
	11

*Ans*w. The Decimal will be finite, and consist of 11 places.

Proof 512|00) 1,000000000|00 (0,00001953125

$$\begin{array}{r}
 512 \\
 \hline
 4880 \\
 4608 \\
 \hline
 2720 \\
 2560 \\
 \hline
 1600 \\
 1536 \\
 \hline
 640 \\
 512 \\
 \hline
 1280 \\
 1024 \\
 \hline
 2560 \\
 2560 \\
 \hline
 0000
 \end{array}$$

Example

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Example 3. Will the Fraction $\frac{1}{88}$ be finite, or infinite; if infinite, how many figures will its Repetend consist of; and how many finite places will there be, before the Repetend begins?

10) <u>880</u>	Number of Divisions
2) <u>88</u>	1
2) <u>44</u>	2
2) <u>22</u>	3
<u>11</u>	4
11) <u>99 (9</u>	
<u>99</u>	

Ans. The Decimal will be a mix'd compound Repetend of 2 places, beginning after 4 finite places.

Proof $88|0) 1,000000|0 (0,00113\overline{3}$

88

—
120
88

—
320
264

—
560
528

320

344. When a decimal expression is finite, or circulating, it is called compleat; but if it be not finite, and no law of circulation appears, it is called approximate.

345. Approximate Decimals have some places true, and the rest uncertain, and are sometimes wrote with the signs + or —, to denote whether the last figure is less or greater than just: Thus, for 327,645 +, read 327,645 more something; that is, the true Decimal exceeds 327,645 by some uncertain figures; and for 327,646 —, read 327,646 less something; that is, the true Decimal exceeds 327,645, and is less than 327,646.

346. If in reducing a Vulgar Fraction to a Decimal, or in any other process, the Repetend of the result doth not so speedily appear as might be wished for, proceed to a determined number of places, and the result will be an approximate Decimal.

347. Upon the aforesaid grounds, the known or accustomary parts of Money, Weight, Measure, Time, &c. may be reduced to Decimals: For if you desire to know what Decimal Fraction of a pound Sterling is equal in value to one shilling, consider first that a pound is the integer, and that 20 shillings are equal to that integer, therefore 1 shilling is $\frac{1}{20}$ of a pound.

In like manner, forasmuch as 240 pence are equal to a pound of English money, 7 pence are $\frac{7}{240}$ parts of a pound, which Fraction will be reduced to this Decimal ,02916.

After the same method, may the Vulgar Sexagenary Fractions used in Astronomy be reduced to Decimals: For since a degree is usually subdivided into sixty parts, called Minutes, or Primes; a Prime or Minute, into sixty parts, called Seconds; a Second into sixty Thirds; a Third into sixty Fourths, &c. It is evident that 7 Minutes (or Primes) are $\frac{7}{60}$ parts of a degree, which may be reduced to the Decimal ,1.16.

This to the ingenious will be a sufficient light for the finding of Decimals suitable to the Shillings, Pence, and Farthings, which are under a pound Sterling: Also the Decimals of the known parts of Weight, Measure, Time, &c, as they are expressed in the following Table,

THE
TABLE
OF
REDUCTION.

TABLE I. Of English Money, the Integer being a Pound.		Pence with Farthings.	Decimals of a Pound.
Shillings.	Decimals of a Pound.		
19	.95	3	.0489583
18	.9	2	.047916
17	.85	1	.046875
16	.8	11	.04583
15	.75	3	.0447916
14	.7	2	.04375
13	.65	10	.0427083
12	.6	3	.0416
11	.55	2	.040625
10	.5	1	.039583
9	.45	9	.0385416
8	.4	3	.0375
7	.35	2	.0364583
6	.3	1	.035416
5	.25	8	.034375
4	.2	7	.03322916
3	.15	3	.03125
2	.1	2	.0302083
1	.05	1	.02916
		6	.028125
			.027083
			.0260416
			.025

		<i>Decimals of an Ounce.</i>	<i>Grains.</i>	<i>Decimals of an Ounce.</i>
3	.0239583	14.7		
2	.022916	13.65		
1	.021875	12.6		
5	.02083	11.55		
3	.0197916	10.5		
2	.01875	9.45		
1	.0177083	8.4		
4	.016	7.35		
3	.015625	6.3		
2	.014583	5.25		
1	.0135416	4.2		
3	.0125	3.15		
3	.0114583	2.1		
2	.010416	1.05		
1	.009375			
2	.0083			
3	.0072916			
2	.00625			
1 Penny. & 1 Far.	.0052083	23.047916		
1 Penny.	.00416	22.04583		
3 Farth.	.003125	21.04375		
2 Farth.	.002083	20.0416		
1 Farth.	.0010416	19.039583		
		18.0375		
		17.035416		
		16.02		
		15.03125		
		14.02916		
		13.027083		
		12.025		
		11.022916		
		10.02083		
		9.01875		
		8.016		
		7.014583		
		6.0125		

T A B L E T II.
Of Troy-weight, the Integer
being an Ounce.

<i>Penny-Weights.</i>	<i>Decimals of an Ounce.</i>
19	.95
18	.9
17	.85
16	.8
15	.75

5	.010416	11	.0982143
4	.0083	10	.0892857
3	.00625	9	.0803571
2	.00416	8	.0714286
1	.002083	7	.0625
T A B L E T III.			
Of Avoirdupoise great Weight, the Integer being an hundred Weight, to wit, 112 pounds.			
Quarters of 1 hundred.	Decimals of 1 hundred.	Ounces.	Decimals of 1 hundred.
3	.75	15	.0083705
2	.5	14	.0078125
1	.25	13	.0072544
Pounds.	Decimals of 1 hundred.	12	.0066964
27	.2410714	11	.0061383
26	.2321429	10	.0055803
25	.2232143	9	.0050223
24	.2142857	8	.0044643
23	.2053571	7	.0039062
22	.1964286	6	.0033482
21	.1875	5	.0027901
20	.1785714	4	.0022321
19	.1696429	3	.0016744
18	.1607143	2	.0011160
17	.1517857	1	.0005580
16	.1428571	Quarters of 1 Ounce.	Decimals of 1 hundred.
15	.1339286	3	.0004185
14	.125	2	.0002790
13	.1160714	1	.0001395
12	.1071429		

T A B L E T

TABLET IV.			
Of Avoirdupoise little Weight, the Integer being a Pound.		6	.0234375
		5	.01953125
		4	.015625
		3	.01171875
		2	.0078125
		1	.003 .0625
Ounces:	Decimals of a Pound.	Quarters of a Dram.	Decimals of a Pound.
15	.9375		
14	.875		
13	.8125		
12	.75	3	.00292969
11	.6875	2	.00195313
10	.625	1	.00097656
9	.5625		
8	.5		
7	.4375		
6	.375		
5	.3125		
4	.25		
3	.1875		
2	.125		
1	.0625		
Drams:	Decimals of a Pound.	Pints.	Decimals of a Gallon.
15	.05859375		
14	.0546875		
13	.05078125		
12	.046875		
11	.04296875		
10	.0390625		
9	.03515625		
8	.03125		
7	.02734375		
TABLET V.		Quarters of a Pint.	Decimals of a Gallon.
Of Liquid Measures, the Integer being a Gallon.			

T A B L E T VI.		T A B L E T VII.	
<i>Of Dry Measures, the Integer being a Quarter.</i>		<i>Of Long Measures, one Yard being the Integer.</i>	
<i>Bushels.</i>	<i>Decimals of a Quarter.</i>	<i>Quarters of one Yard.</i>	<i>Decimals of one Yard.</i>
7	.875	3	.75
6	.75	2	.5
5	.625	1	.25
4	.5		
3	.375		
2	.25		
1	.125		
<i>Pecks.</i>	<i>Decimals of a Quarter.</i>	<i>Nails.</i>	<i>Decimals of one Yard.</i>
3	.09375	3	.1875
2	.0625	2	.125
1	.03125	1	.0625
<i>Quarters of a Peck.</i>	<i>Decimals of a Quarter.</i>	<i>Quarters of one Nail.</i>	<i>Decimals of one Yard.</i>
3	.0234375	3	.046875
2	.015625	2	.03125
1	.0078125	1	.015625
<i>Pints.</i>	<i>Decimals of a Quarter.</i>		
3	.005859375		
2	.00390625		
1	.001953125		
T A B L E T VIII.		T A B L E T VIII.	
<i>Of the Reduction of Inches, &c. to Decimals, the Integer being a Foot in Length.</i>		<i>Of the Reduction of Inches, &c. to Decimals, the Integer being a Foot in Length.</i>	
<i>Inches</i>		<i>Decimals of a Foot.</i>	
11		.916	
10		.83	
9		.75	

		Parts of a Dozen.	Decimals of a Gross.
8	.6	11	.0763 $\frac{1}{2}$
7	.58 $\frac{1}{2}$	10	.0694
6	.5	9	.0625
5	.41 $\frac{1}{2}$	8	.05
4	.3	7	.0486 $\frac{1}{2}$
3	.25	6	.041 $\frac{1}{2}$
2	.16	5	.0347 $\frac{1}{2}$
1	.08 $\frac{1}{2}$	4	.027
Quarters of an Inch.		3	.0208 $\frac{1}{2}$
3		2	.013 $\frac{1}{2}$
2		1	.0069 $\frac{1}{2}$
Half a Quarter of an Inch.			
T A B L E T IX.			
<i>Of Dozens, the Integer being a Gross.</i>			
Dozens.	Decimals of a Gross.	Hours.	Decimals of a Day.
11	.91 $\frac{1}{2}$	23	.9583
10	.83	22	.91 $\frac{1}{2}$
9	.75	21	.875
8	.6	20	.83
7	.58 $\frac{1}{2}$	19	.791 $\frac{1}{2}$
6	.5	18	.75
5	.41 $\frac{1}{2}$	17	.708 $\frac{1}{2}$
4	.3	16	.6
3	.25	15	.625
2	.16	14	.583
1	.08 $\frac{1}{2}$	13	.541 $\frac{1}{2}$

9	.375	35	.024308
8	.3	34	.02361
7	.2916	33	.022916
6	.25	32	.021527
5	.2083	31	.02083
4	.16	30	.020138
3	.125	29	.020138
2	.083	28	.0194
1	.0416	27	.01875
<hr/>		26	.01808
<i>Minutes.</i>		25	.017367
<i>Decimals of a Day.</i>		24	.016
<hr/>		23	.015977
59	.040977	22	.01527
58	.040277	21	.041583
57	.039583	20	.0138
56	.0388	19	.013194
55	.038194	18	.0125
54	.0375	17	.011808
53	.036808	16	.01
52	.0361	15	.010416
51	.035416	14	.009727
50	.03477	13	.009027
49	.034027	12	.0083
48	.03	11	.007638
47	.032638	10	.00694
46	.03194	9	.00625
45	.03125	8	.003477
44	.0388	7	.004867
43	.029867	6	.00416
42	.02916	5	.003477
41	.028477	4	.0027
40	.0277	3	.002043
39	.027083	2	.00128
38	.02638	1	.000694
37	.025694		
36	.025		

348. This Table aforegoing consists of ten several Tablets, of which the first (intitled *English* money) contains in the first column of it the particular Fractions, (*viz.* the shillings, pence, and farthings) of a pound Sterling; and in the other column the Decimals, to which they may be respectively reduced: So in the same Tablet .65 is the Decimal answerable to 13s. .02083 to 5d. and .003125 to 3f. Likewise, .0489583 is the Decimal of 11d. together with 3 farthings: Also .03125 is the Decimal of 7 pence half-penny.

349. The next Tablet (intitled *Troy*-weight) contains in the first column the particular Fractions (*viz.* the penny-weights and grains) of an ounce *Troy*, and in the other their respective Decimals: So .6 is the correspondent Decimal of 12 penny-weight, and .002083 of 1 grain. Likewise .025 is the Decimal of 12 grains.

350. The third Tablet (intitled Avoirdupoise great weight,) contains in the first column thereof the Fractions, (*viz.* the quarters, pounds, ounces, and the quarters of an ounce of an hundred, according to Avoirdupoise weight,) and in the other their proper Decimals: So .5 is the Decimal of two quarters or half a hundred, .1517857 of 17 pounds, .0033482 of 6. ounces, and .0004185, the Decimal of three quarters of an ounce.

351. The fourth (intitled Avoirdupoise little weight) shews the Fractions (*viz.* the ounces, drams, and quarters of a dram) of a pound Avoirdupoise, together with their respective Decimals: So the Decimal of 3 ounces is .1875, the Decimal of 9. drams is .03515625, and the Decimal of one quarter of a dram is :00097656.

352. The fifth (intitled Liquid Measures) has the Fractions (*viz.* the pints and quarters of a pint) of a gallon, and likewise their several Decimals: So the Decimal of five pints is .625, and the Decimal of two quarters or half a pint is .0625.

353. The sixth (intitled Dry Measures) gives the Fractions (*viz.* the bushels, pecks, quarters of pecks, and pints,) of a quarter, together with their peculiar Decimals: So .375 is the Decimal of 3 bushels, .03125 of one peck, .0234375 of $\frac{1}{4}$ of a peck, and .00390625 of two pints.

354. The seventh (intitled Yards) offers you the Fractions (*viz.* the quarters, nails, and quarters of nails) and their respective Decimals: So .25 is the Decimal of one quarter of a yard, .125 of two nails, and .046875 of three quarters of a nail.

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355. The eighth (intitled Reduction of inches, &c. to Decimals of a foot) presents you with the Fractions (*viz.* the inches, quarters, and half-quarters of an inch) of a foot, together with their correspondent Decimals: So .41 $\frac{1}{2}$ is the Decimal of 5 inches, .0625 of $\frac{1}{4}$ of an inch, and .01041 $\frac{1}{2}$ of $\frac{1}{8}$, or half a quarter of an inch.

356. The ninth Tablet (intitled Dozens) yields you the Fractions (*viz.* the dozens and particulars) of a gross, as also their respective Decimals: So .25 is the Decimal of 3 dozen, and .0486 $\frac{1}{2}$ of 7 particulars.

357. The tenth and last Tablet (intitled Time) gives you the Fractions, (*viz.* the hours and minutes) of a day: So .625 is the Decimal of 15 hours, .0375 of 54 minutes, and .00069 $\frac{1}{2}$ of one minute.

358. When a single Fraction of any of the premised Tablets is proposed to be reduced to a Decimal, find it in the first column of the Tablet to which it belongs; this done, just against that Fraction so found, you'll have the Decimal required. So $1\frac{3}{5}$ s. being propounded, taking the first premised Tablet, I find $1\frac{3}{5}$ s. in the first column of the Tablet of Money, and just against the same $1\frac{3}{5}$ s. I observe 65, before which having prefixed a point, and by that means signified it for a Decimal, I conclude the same .65 so ordered, to be the correspondent Decimal of $1\frac{3}{5}$ s. the Fraction propounded. In like manner, .02291 $\frac{1}{2}$ is the Decimal of 11 grains in the Tablet of Troy-weight; and .0357143 the Decimal of 4*l.* in the Tablet of Avoirdupoise great weight, &c.

359. When two or more Fractions are propounded, and it is required to find a Decimal equivalent to the sum of them, find the Decimal of each of the Fractions given according to the last Rule; then adding together the Decimals so found, that entire sum is the Decimal sought: So $1\frac{3}{5}$ s. 5*d.* being reduced to a Decimal, is .67083; for the Decimal of $1\frac{3}{5}$ s. is .65, and the Decimal of 5*d.* .02083; which being added together (by Art. 382.) amount to .67083, *viz.* the Decimal which represents $1\frac{3}{5}$ s. 5*d.* the Fraction proposed: In like manner, the Decimal of 9 penny-weight and $1\frac{3}{5}$ grains, is .477083, and the Decimal of $\frac{1}{2} C.$ 19*lb.* 7*oz.* is .6735491, &c.

$1\frac{3}{5}$ s.	$.65$
$5d.$	$.020833$
$.670833$	

9 p.wt.

9 p.wt.	.45
13 gr.	.027083
	<hr/>
	.477083
i C.	
19 lb.	.5
7 o	.1696429
	<hr/>
	.5039062
	<hr/>
	.6735491

And here as you see proper Fractions reduced, so likewise may the Fractions of mixt numbers be reduced to Decimals; for example, these numbers 97 lb. 7 ounces, 13 $\frac{1}{2}$ drams; 67 gallons 5 $\frac{1}{2}$ pints; 28 quarters, 0 bushels, and 0 $\frac{1}{2}$ pecks, after Reduction are severally 97.4891, 67.7187, and 28.0781.

97.4375	67.625	28.0625
.0507	.0937	.0156
.0009		<hr/>
	67.7187	28.0781
97.4891		

360. When a Decimal is proposed, to know what Fraction it represents, search the same Decimal in the second column of the Tablet, to which it belongs; where if you find it expressly, the number just against it in the first column is the Fraction you look for: So .65 (representing the Fraction of a pound Sterling,) being given, I find it in the second column of the Tablet of money, and over-against it in the first column I find 13*s.* which is the Fraction represented by .65, the Decimal propounded. In like manner, 3.025, (representing 3 ounces and .025 of an ounce *Troy*) being given, the number represented by it, is 3 ounces, 0 p.w. 12 grains.

361. When in the second column of the Tablet to which you are directed, you cannot precisely find the Decimal proposed, search that which being less, comes nearest to it, and take the number that answers it in the first column, for the greatest Fraction of the number required: Then deducting the Decimal so found, out of the Decimal given, find likewise the remainder as another Decimal, and take its correspondent number for the next Fraction of the number required; and so proceed in that order, till you have discovered the entire number represented by the Decimal propounded.

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Example., .6739 being given, I demand the Fraction of a pound Sterling represented by it: The Decimal in the Tablet of Money, which being less, comes nearest to .6739 is .65, whose correspondent number in that Tablet is 13, which are the shillings of the number required; then subtracting .65 out of .6739, the remainder is .0239, and the nearest Decimal in the same Tablet to .0239 is .0208, whose correspondent number is 5, which are the pence of the number required. Last of all, deducting .0208 out of .0239, the remainder is .0031, which gives you in the first column 3, being the farthings of the number required: So that I conclude the entire Fraction represented by the Decimal .6739, is $\frac{13}{16}$.

5 d. 3 f.

	.6739 l. Sterling.
Subtract 13 s.	<u>.65</u>
	.0239
Subtract 5 d.	<u>.0208</u>
	.0031

3 f.

In like manner, 7.359 C. being reduced by the Tablet of Avoirdupoise great weight, is $7\frac{1}{4}$ C. 12 l. 4 ounces: and 94.58 lb. reduced by the Tablet of Avoirdupoise little weight, is 94 lb. 9 ounces, and 6 drams.

	7. 359 C.
Subtract 1 quarter	<u>.25</u>
	.109
Subtract 12 lb.	<u>.107</u>
	.002
4 oz.	<u>.002</u>
Subtract 9 oz.	<u>.56</u>
6 Drams	.02

362. But when you have not such Tables as the above at hand then, to reduce the different Denominations of Money, Weight, and Measure, &c. into Decimals, use the following

Rule. 1. Write the given denominations, or parts, orderly under each other; the inferior or least parts, being uppermost: Let these be Dividends.

2. Against each part, on the left-hand, write the number thereof, contained in one of its next superior: Let these be Divisors.

3. Then

3. Then beginning with the upper one, write the quotient of each division as Decimal parts, on the right-hand of the Dividend, next below it; and let this mixed number be divided by its Divisor.

Examp. 1. Reduce 10*s.* 8*d.* to its equivalent Decimal of a pound Sterling.

1. The given shillings and pence wrote under each other, will stand thus :

$$\begin{array}{r} 8 \\ 10 \end{array}$$

2. These with their respective Divisors annexed, will be

$$\begin{array}{r} 12 \\ 20 \end{array} \mid \begin{array}{r} 8 \\ 10 \end{array}$$

3. The division by 12 being performed, by supposing cyphers to be annexed to the Dividend 8, the work will appear as below :

$$\begin{array}{r} 12 \\ 20 \end{array} \mid \begin{array}{r} 8,000, \&c. \\ 10,666, \&c. \end{array} \quad \text{Or} \quad \begin{array}{r} 12 \\ 20 \end{array} \mid \begin{array}{r} 8, \\ 10,\$ \end{array}$$

4. Then, dividing by 20 gives the answer ,53.

$$\begin{array}{r} 12 \\ 20 \end{array} \mid \begin{array}{r} 8, \\ 10,\$ \end{array} \quad ,53$$

Examp. 2. What Decimal of a pound *Troy* is equivalent to 10*oz.* 18*p.w.* 16*gr.*

$$\begin{array}{r} 24 \\ 20 \\ 12 \end{array} \mid \begin{array}{r} 16, \\ 18,\$ \\ 10,9\$ \\ ,9\$ \end{array}$$

363. But the Decimals of Sterling money may be wrote in one line by the following

Rule. Write half of the greatest even number, in the given shillings, for the place of primes.

Let the number of farthings, contained in the given pence and farthings, possess the places of seconds and thirds; observing, if the given shillings are odd, to increase the place of seconds by 5. And to increase the thirds by as many units as there are times 24 in the pence and farthings.

Divide half the number of farthings, in the pence and farthings (rejecting 24 or 6 pence, if there is one) by 12, the quotient written after the three places before found, will give the Decimal required,

E X A M P L E S.

1.	.	10s. 8d.	is equal to	,53l.
2.	.	13 : 10 $\frac{1}{4}$:	,69375l.
3.	.	15 : 9 $\frac{1}{4}$:	,790625l.
4.	.	19 : 11 $\frac{1}{4}$:	,996875l.
5.	.	1 : 10 $\frac{1}{4}$:	,0927083l.
6.	.	0 : 8 $\frac{1}{4}$:	,0364583l.
7.	.	0 : 2 $\frac{1}{2}$:	,010416l.
8.	.	0 : 0 $\frac{1}{4}$:	,003125l.

One of these Examples explained, will make the Rule familiar in the 5th, viz. 1s. 10 $\frac{1}{4}$ d. Half of one shilling is 0, write 0 in the place of primes; 10 $\frac{1}{4}$ is 41 farthings, and 1 added (for the 24 contained in 41) makes 42, and 50 added (for the odd shilling) makes 92, therefore the three first places of the Decimal are ,092: Now 24 taken from 41 leaves 17, its half is 8,5; which divided by 12 gives 708 $\frac{1}{3}$; these wrote as they arise, after the former three places, make ,0927083 for the Decimal required.

364. A Decimal of a superior Denomination being given, to find its value in the known parts of inferior Denominations

Rule. Multiply the given Decimal by the number of parts in the next lesser Denomination; from the product cut off as many places to the right-hand, as there are in the given Decimal.

Multiply the figures above cut off to the right-hand by the parts in the next lesser Denomination, and from this product cut off as before.

And thus proceed until the least Denomination is arrived at, then the several parts cut off on the left-hand, are equivalent to the given Decimal.

Example 1. What is the value of ,53334l.?

20

10,66680

12

8,00160

Answer 10s. 8d.

Example,

Example 2. What is the value of ,91112 lb. Troy.

$$\begin{array}{r}
 12 \\
 \hline
 10,93344 \\
 -20 \\
 \hline
 78,66880 \\
 -24 \\
 \hline
 2,67520 \\
 -13,3760 \\
 \hline
 16,05120
 \end{array}$$

Answer 10 oz. 18 p. wt. 16 gr.

365. But the value of the decimal part of a pound Sterling may be expressed in one line, thus;

Double the place of primes for shillings, and if the second place be 5, or exceed 5, reckon 1 shilling more; the figures in the second and third places (rejecting 5 in the second place) are so many farthings, abating 1 for every 24.

E X A M P L E S,

1. The value of 0,92763*l.* is 18*s.* 6*½*.
 2. 0,87638*l.* is 17 : 6*½*.
 3. 0,09937*l.* is 1 : 11*½*.
 4. 0,0428*l.* is 0 : 10*½*.
 5. 0,0095*l.* is 0 : 2*½*.
-

C H A P. XXV.

Of the Management of CIRCULATING DECIMALS.

366. Those Repetends, which consist of the same number of places, are called similar; thus, 409, and, 73514 are similar Repetends.

367. Similar Repetends that begin at the same place, *viz.*, at the place of units, primes, seconds, &c. are said to be co-terminous.

368. A finite expression may put on the form of an infinite, by making cyphers the Repetend. And,

A single Repetend may put on the form of a compound Repetend, without altering its value; thus, $\frac{4}{9}$ may be wrote $\frac{44}{99}$; or, $\frac{444}{999}$; or, $\frac{4444}{9999}$. For $\frac{4}{9} = \frac{44}{99} = \frac{444}{999} = \frac{4444}{9999}$, &c.

369. Hence any given Repetend may put on the form of another Repetend, if the number of repeating figures in the latter be a multiple of the number of repeating figures in the former.

Thus the Repetend $0.\overline{45}$ consisting of two figures, may put on the forms of Repetends consisting of 4, 6, 8, 10, &c. figures; that is, $0.\overline{45} = 0.\overline{4545} = 0.\overline{454545} = 0.\overline{45454545}$, &c.

370. Any two or more dissimilar Repetends may be made similar, by transforming them into other Repetends, which shall consist of as many places of figures, as the least common multiple of the several numbers of places found in all the Repetends, contains units.

EXAMPLE 1.

Diffimilar,	Made similar.
$0.\overline{47}$	$0.\overline{474747}$
$0.\overline{34}$	$0.\overline{343434}$

EXAMPLE 2.

$0.\overline{478}$	$0.\overline{475478}$
$0.\overline{34}$	$0.\overline{342424}$
$0.\overline{59}$	$0.\overline{595959}$
$0.\overline{327}$	$0.\overline{32777777}$
$0.\overline{1}$	$0.\overline{111111}$

In the last Example 6 is the least common Multiple, and therefore the similar-Repetends consist each of 6 places.

371. A pure Repetend may put on the form of a mixt Repetend, without altering its value; thus $0.\overline{45}$ may be wrote $0.\overline{454}$; or $0.\overline{4545}$; or $0.\overline{45454}$, &c.

For $0.\overline{454} = \frac{4}{9} + \frac{4}{99}$ of $\frac{1}{10} = \frac{4}{90} + \frac{4}{990}$, by Art. 333.

And $\frac{4}{9} + \frac{4}{99} = \frac{44}{99} + \frac{4}{99} = \frac{48}{99} = \frac{4}{9} = 0.\overline{45}$.

Again, $0.\overline{4545} = \frac{4}{9} + \frac{4}{99}$ of $\frac{1}{100} = \frac{4}{900} + \frac{4}{9900}$, by Art. 333.

And $\frac{4}{900} + \frac{4}{9900} = \frac{44}{9900} + \frac{4}{9900} = \frac{48}{9900} = \frac{4}{990} = 0.\overline{454}$.

372. Hence any two or more similar Repetends may be made conterminous, that is, may begin at the same place.

EXAMPLE 1.

Make $0,40\overline{0}$ and $0,73\overline{514}$ conterminous. This will be performed by making $0,40\overline{0}$ put on the same form with $0,73\overline{514}$, viz. that of a mixed Repetend having 2 finite places; thus, $0,40\overline{040}$.

EXAMPLE 2.

Similar Repetends	Made conterminous.
$0,\overline{475478}$	$0,47\overline{847547}$
$0,\overline{37424247}$	$0,32\overline{424247}$
$0,\overline{59595958}$	$0,59\overline{595958}$
$0,\overline{32777771}$	$0,32\overline{777771}$
$0,\overline{11111111}$	$0,11\overline{111111}$

373. The expression $\frac{14,2}{9}$ may be called a mixed Fraction; its Numerator consists of a mixed decimal number equal in value to $14 \frac{2}{10}$, and the above mixed Fraction is designed to represent the quotient, made by dividing the said mixed decimal Numerator by 9 the Denominator; which Fraction, if reduced to a Decimal by Art. 322. will produce a mixed Repetend: See the Work.

$$9) \underline{14,2000} \text{ &c.} \\ 1,5777 \text{ &c.} = 1,5\overline{7}$$

374. Hence every mixed Repetend may be reduced to a mixed Fraction, whose Denominator shall be 9, 99, 999, or 9999, &c. by multiplying the finite part of the mixt Repetend, by as many nines as there are circulating figures in the Repetend, and adding the said Repetend to the product for a Numerator.

For the mixed Repetend $1,5\overline{7} = 1,5 \frac{7}{10}$ of $\frac{1}{10}$, by Art. 333. which being multiplied by 10 will become $15,\overline{7}$, by Art. 318.

Now to reduce $15,\overline{7}$ to an improper Fraction, by Art. 268. the finite number 15 must be multiplied by 9 the Denominator, and the Repetend 7 must be added thereto to make the Numerator of the equivalent improper Fraction $\frac{142}{9}$; which Numerator being divided by 10 to destroy the former Multiplication becomes $\frac{14,2}{9}$ by Art. 318. the same as will be found from the above Rule.

375. But if we make use of the method delivered in Art. 79, the above process will be rendered much shorter.

$$\begin{array}{r} \text{Thus, From } 1,5 \times 10 = 15,0 \\ \text{Take . . .} \quad \underline{1,5} \\ \text{Remains } 15 \times 9 = \underline{13,5} \\ \text{Add the Repetend} \quad \underline{7} \\ \text{Numerator . . } \quad \underline{14,2} \end{array}$$

376. And since the Repetend 7 is to be added, it will be done with less trouble by annexing it to the finite part 15, instead of the cypher. Thus,

$$\begin{array}{r} \text{From } 1,5 \times 10 + ,7 = 15,7 \\ \text{Take . . .} \quad \underline{1,5} \\ \text{Remains the Numerator} = \underline{14,2} \end{array}$$

377. Therefore in order to reduce any mixt Repetend to a mixt Fraction; write the mixt Repetend down without dash-ing its repeating figures, and remove the separating comma as many places toward the right-hand as there are figures in the Repetend; write the finite part of the mixed Repetend under the former, placing the right-hand figure and separating comma of this, under the right-hand figure and separating comma of that; then subtract the latter from the former, and the remain-der will be the Numerator of the mixed Fraction required; and its Denominator will be as many nines, as there are re-peating figures in the Repetend.

Examp. 1. Reduce $0,\overline{475}$ to a mixed Fraction.

$$\begin{array}{r} \text{From } 0,475 \times 100 = 47,5 \\ \text{Take the finite part} = \underline{0,4} \\ \text{Remains . . .} \quad \underline{47,1} \\ \text{Therefore } 0,\overline{475} = \frac{47,1}{99} \end{array}$$

Examp. 2. Reduce $0,\overline{3485734}$ to a mixed Fraction.

$$\begin{array}{r} \text{From } 0,3485734 \times 10000 = 3485,734 \\ \text{Take the finite part} = \underline{0,348} \\ \text{Remains . . .} \quad \underline{3485,386} \\ \text{Therefore } 0,\overline{3485734} = \frac{3485,386}{9999} \end{array}$$

378. If all the decimal part of the expression be a Repetend, the Fraction arising will be wholly vulgar, and not mixed.

Examp.

Examp. 3. Reduce $36.\overline{7}$ to a Vulgar Fraction.

$$\text{From } 36.\overline{7} \times 10 = 367,0$$

$$\text{Take the finite part } = \underline{\underline{36,0}}$$

$$\text{Remains } \dots \dots \dots 33,0$$

$$\text{Therefore } 36.\overline{7} = \frac{331}{9}.$$

Examp. 4. Reduce $3,\overline{847}$ to a Vulgar Fraction.

$$\text{From } 3,842 \times 1000 = 3842,0$$

$$\text{Take the finite part } = \underline{\underline{3,0}}$$

$$\text{Remains } \dots \dots \dots 3839,0$$

$$\text{Therefore } 3,\overline{847} = \frac{3839}{999}.$$

379. If the Repetend begins among the places of integers, then as many cyphers must be annexed to the mixed Repetend and finite part, as will be sufficient for the removing the separating comma, the required number of places.

Examp. 5. Reduce $57.\overline{1}$ to a Vulgar Fraction.

$$\text{From } 57 \times 10 = 570,0$$

$$\text{Take the finite part } = \underline{\underline{50,0}}$$

$$\text{Remains } \dots \dots \dots 520,0$$

$$\text{Therefore } 57.\overline{1} = \frac{520}{9}.$$

Examp. 6. Reduce $4275,\overline{84}$ to a Vulgar Fraction.

$$\text{From } 4275,\overline{84} \times 10000 = 42758400$$

$$\text{Take the finite part } . = \underline{\underline{4200}}$$

$$\text{Remains } \dots \dots \dots , 42754200$$

$$\text{Therefore, } 4275,\overline{84} = \frac{42754200}{9999}.$$

Examp. 7. Reduce $5794\overline{67},946, \&c.$ to a Vulgar Fraction.

$$\text{From } 579460 \times 10000 = 5794600000$$

$$\text{Take the finite part } \underline{\underline{500000}}$$

$$\text{Remains } \dots \dots \dots 5794100000$$

$$\text{Therefore } 5794\overline{67},946, \&c. = \frac{5794100000}{9999}.$$

380. If the expression be a pure Repetend, the above Rule will give its equivalent Vulgar or mixed Fraction.

Examp.

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Exampl. 8. Reduce $0,0075$ to its equivalent mixed Fraction.

$$\text{From } 0,0075 \times 100 = 0,75$$

$$\text{Take the finite part} = \underline{0,00}$$

$$\text{Remains} \dots \dots \cdot \underline{0,75}$$

$$\text{Therefore } 0,0075 = \frac{0,75}{99}.$$

Exampl. 9. Reduce $0,75$ to a Vulgar Fraction.

$$\text{From } 0,75 \times 100 = 75,0$$

$$\text{Take} \dots \dots \cdot \underline{0,0}$$

$$\text{Remains} \dots \dots \cdot \underline{75,0}$$

$$\text{Therefore } 0,75 = \frac{75}{99}. \text{ See Art. 337.}$$

Exampl. 10. Reduce $7,5$ to a Vulgar Fraction.

$$\text{From } 7,5 \times 100 = 750$$

$$\text{Take} \dots \dots \cdot \underline{00}$$

$$\text{Remains} \dots \dots \cdot \underline{750}$$

$$\text{Therefore } 7,5 = \frac{750}{99}.$$

C H A P. XXVI.

A D D I T I O N o f D E C I M A L F R A C T I O N S:

381. **T**O such as well understand the Notation of Decimal Fractions, all the varieties of their Numeration, to wit, Addition, Subtraction, &c. will be as easy as the operations by whole numbers; therefore he that would be a good proficient in Decimal Arithmetic, must thoroughly understand the Chapters foregoing.

382. When divers Decimal Fractions are given to be added together, they must first of all be orderly placed one under another, according to the doctrine of their Notation. So if these Decimal Fractions, to wit, .125, .39, and .7 were given to be added, they must be written thus:

.125.

.39

.7

Or

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Or if you will have the same number of places to be in all the Decimals given, without altering their values, they may be writ thus:

$$\begin{array}{r} .125 \\ .390 \\ .700 \end{array}$$

Not thus,

$$\begin{array}{r} .125 \\ .39 \\ \hline .7 \end{array}$$

For the figures or cyphers, which are of like degrees or places, must be subscribed directly one under another, *viz.* tenth parts or primes must be set down exactly under tenths; also hundredth parts or seconds are to be placed under hundredth parts: As you see in the first Example, where 3 or three tenth parts in the second Decimal, stands directly under 1 or one tenth part in the first Decimal; likewise 7 or seven tenths in the third Decimal, stands directly under the tenths in the former; and so of the rest.

383. In like manner, when mixt numbers, which consist of Integers and Decimal parts, are given to be added, due respect must be had to their subscription one under another: So if these mixt numbers, to wit, 32.056, 7.07, and 1.9 were given to be added, they are to be writ down thus:

$$\begin{array}{r} 32.056 \\ 7.07 \\ \hline 1.9 \end{array}$$

384. To add finite Decimals: Having placed the Decimals, and drawn a line underneath in manner aforesaid, add them together, beginning with the outermost rank towards the right-hand (as has been taught in Addition of whole numbers of one Denomination:) So if the Decimals in the first Example of Art. 382. were given to be added, first subscribe 5, which is all that stands in the first rank towards the right-hand; then proceeding to the second rank, say 9 and 2 make 11; wherefore set down 1, which is the excess of 11 above 10; and for the 10 carry 1 in mind to the next rank, saying 1 in mind added to 7 makes 8, which added to 3 and 1 make 12; wherefore write 2, which is the excess of 12 above

$$\begin{array}{r} .125 \\ .39 \\ .7 \\ \hline 1.215 \end{array}$$

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10, under the line, reserving 1 in mind for the 10; then prefix a point before 2, which stands in the first place of Decimals; and on the left-hand of the point, to wit, in the place of units, or first place of integers, write 1 (being the 1 in mind); which done, the sum of the Decimals given, is, 1.215, that is, one integer (whether it be a perch, yard, foot, &c.) and $\frac{215}{1000}$ parts of an integer, as you see in the Example. In like manner, these mixt numbers 32,056, 7.07, and 1.9 being given to be added, their sum will be found to be 41.026, that is 41 integers, and $\frac{26}{1000}$ parts of an integer, as you see in the Margin.

32.056

7.07

1.9

41.026

385. To add Decimals that have single Repetends.

Rule. Make the Repetends conterminous; then add up the right-hand column, and add thereto as many units as there are nines contained in the sum, so shall the figure, to be placed under that column, be a Repetend, and the rest of the process will be as in Art. 384.

EXAMPLE 1.

Numbers proposed.	Made conterminous.
2,3	2,33
2,7	2,77
4,78	4,78
0,3	0,33
5,8	5,80
4,78	4,78
	20,74

EXAMPLE 2.

Numbers proposed.	Made conterminous.
4,724	4,724
28,	28,888
3,	3,000
25,268	25,268
18,771	18,771
8,	5,558
	<u>86,213</u>

In the first Example, the sum of the right-hand column is 22, which contains 2 nines; therefore say $22 + 2 = 24$; set down 4 for a Repetend, and carry 2. The rest of the operation being as common.

In like manner, the sum of the right-hand column (in Example 2.) being 30, contains 3 nines; therefore $30 + 3 = 33$: Therefore 3 is wrote for the Repetend, and 3 carried to the next column.

386. To add Decimals having compound Repetends.

Rule. Make the Repetends similar and conterminous, and add as in Art. 384. then to the right-hand figure of the sum add as many units; as are carried from that column of figures, wherein all the Repetends begin together, to the column next above it: Lastly, dash off for a Repetend, as many places as were so in the numbers added together.

EXAMPLE

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EXAMPLE 1.

Numbers Made similar and
proposed. conterminous.

$$\begin{array}{r}
 16\frac{1}{7}, \quad 162, 162\overline{16} \\
 2.\overline{83} \quad 2,\overline{83}9393 \\
 17\frac{1}{7}, \quad 172,\overline{722227} \\
 3,\overline{169238} \quad 3,\overline{169238} \\
 5, \quad 5,\overline{000008} \\
 \hline
 & 346,093007 \\
 & \quad 2
 \end{array}$$

Answer 1 346,~~0~~93008

EXAMPLE 2.

Numbers Made similar and
proposed. conterminous.

$$\begin{array}{r}
 134,\overline{58} \quad 134,09\overline{090908} \\
 97,\overline{28} \quad 97,26\overline{666668} \\
 99,\overline{083} \quad 9,\overline{08333333} \\
 1,\overline{5} \quad 1,50\overline{000008} \\
 0,\overline{814} \quad 0,81\overline{481481} \\
 \hline
 & 242,75572389
 \end{array}$$

242,75572398

In the first Example, the number 2, which is added to the right-hand place, is the same which is carried from the place of primes, where the Repetends begin together, to the place next above it.

And in the second Example, the number 1, which is so added, is the same with that carried from the place of thirds.

387. To add approximate Decimals.

Rule. Place and work as in Art. 382. and 384. only observe, that the certain places of the Decimal are, for the most part, fewer by one than the Decimal places in any one of the Approximates given.

$$\begin{array}{r}
 12.34518 \\
 + 10.5 \\
 + .28 \\
 + 3.45455 \\
 + 1.41421 \\
 + 1.239284 \\
 \hline
 39.2532
 \end{array}$$

This Rule is such, that we shall scarce ever err more than unity in the last place; but by two operations, one made with more than just, the other made with less than just, we shall always be able to judge how far is certain, ~~and~~, as far as they agree.

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$$\begin{array}{r}
 12.34518 \\
 10.5 \\
 .28 \\
 3.45455 \\
 1.41422 \\
 1.25929 \\
 \hline
 29.25324
 \end{array}
 \qquad
 \begin{array}{r}
 12.34517 \\
 10.5 \\
 .28 \\
 3.45454 \\
 1.41421 \\
 1.25928 \\
 \hline
 29.25320
 \end{array}$$

C H A P. XXVII.
SUBTRACTION of DECIMAL FRACTIONS.

388. To subtract finite Decimals: Having first set down the greater of the two numbers given (whether it be a whole number, mixt number, or Decimal) and the lesser under the greater, according to the directions in Art. 382. proceed as you are taught in Subtraction of whole numbers: So if this Decimal Fraction .784 were given to be subtracted from this Decimal .837, the remainder will be .053, that is, $\frac{53}{1000}$ parts of an integer: In like-manner, if this mixt number 78.919 were given to be subtracted from 295.094, the remainder will be 216 $\frac{175}{1000}$. In each of which Examples you may observe that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole numbers of one Denomination: Note also, when the Decimals in both the numbers given do not consist of the same number of places, that Decimal which is defective in places towards the right-hand, should have the void places filled up with cyphers, or at least cyphers must be supposed to be annexed: So if this Decimal .04338 be given to be subtracted from this .65, the remainder will be found to be .60662, and the Work will stand as in the Margin; where you see the three void places are supplied with cyphers, and then the operation is as in whole numbers, by borrowing 10 as often as the lowest figure cannot be subtracted from the upper.

$$\begin{array}{r}
 .837 \\
 .784 \\
 \hline
 .053
 \end{array}$$

$$\begin{array}{r}
 295.094 \\
 78.919 \\
 \hline
 216.175
 \end{array}$$

$$\begin{array}{r}
 .65000 \\
 .04338 \\
 \hline
 .60662
 \end{array}$$

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389. To subtract Decimals that have Repetends.

Rule. Make the Repetends similar and conterminous, and subtract as in Art. 388. observing only, if the Repetend of the number to be subtracted, be greater than the Repetend of the number it is to be taken from; that then, the right-hand figure of the remainder must be less by unity than it would be if the expressions were finite. *Note,* The Repetend in the remainder will consist of as many places as those of the other two numbers.

EXAMPLE 1.

	Made conter-		Made conter-	
	minous.		minous.	
From 110,8	110,66668	From 5,03	5,0333	
Take 94,14583	94,14583	Take 3,0418	3,0418	
Remains	16,52083			

EXAMPLE 2.

	Made conter-		Made conter-	
	minous.		minous.	
From 5,03	5,0333	From 1,9917	1,9917	
Take 3,0418	3,0418			
Deduct				
Remains	1,9917			

EXAMPLE 3.

	Made similar		Made similar	
	and conter-		and conter-	
	minous.		minous.	
From 6,571428	6,571428	From 10,5	10,500	
Take 3,6428	3,6428	Take 3,45	3,454	
Remains	2,9785857			
Deduct				
Remains	7,046			

EXAMPLE 4.

	Made similar		Made similar	
	and conter-		and conter-	
	minous.		minous.	
From 10,5	10,500	From 7,046	7,046	
Take 3,45	3,454			
Deduct				
Remains	7,045			

390. To subtract approximate Decimals.

Rule. Place as in Addition, and subtract as in Art. 388. and the last place will never err more than an unit, if both are made more than just, or, both less than just.

$$\begin{array}{r} 19.5 \\ - 3.45455 \\ \hline 7.04545 \end{array}$$

$$\begin{array}{r} 84.3275 + \\ - 2.1847 + \\ \hline 82.1428 \end{array}$$

C H A P. XXVIII.

MULTIPLICATION of DECIMAL FRACTIONS.

391. WHEN two numbers are given to be multiplied, and are both mixt numbers, or both Decimal Fractions, or one of them a whole number, and the other a Decimal or mixt number, (which are all the cases that can happen) there is no necessity of writing them precisely one under the other, as in Addition and Subtraction; for the product or number sought in Multiplication depends not upon any regular placing of the two numbers given: So if this mixt number 56.3 were given to be added to this mixt number 1.30526, they ought to be set down one under the other, as you see (according to Art. 282.) but if they are to be multiplied one by the other, they may be writ thus:

$$\begin{array}{r} 1.30526 \\ \times 56.3 \\ \hline \end{array}$$

392. To multiply finite Decimals: Multiply the numbers given as if they were whole numbers; then cut off always from the product by a point or comma, so many places towards the right-hand, as there are places of Decimal parts in both the numbers given to be multiplied; that done, the figure or figures, if any happen to be, on the left-hand of the said point or line of separation, declare the integer or integers in the product, and those on the right-hand of the point are decimal parts of an integer: So if this mixt number 1.30526 be given to be multiplied by this mixt number 56.3 (that is, 56 integers and $\frac{1}{10}$ of an integer) the product will be found 73,486138, that is, 73 integers, and $\frac{486138}{100000}$ parts of an integer; for having chose that to be the Multiplier, which will cause least work, and subscribed it under the Multiplicand (to wit, 56.3 under the 1.30526) proceed according to the Rules of Multiplication of whole numbers, viz. having drawn a line under the numbers given, multiply all the Multiplicand, to wit, 1.30526, as if it were a whole number, by 3 the first multiplying figure, and subscribe the product thereof which is 391578 under the line; and proceeding in like manner with the

the other multiplying figures 6 and 5, at last find the total of the particular products, viz. 73,486138; and because there are six places or decimal parts in both the numbers given (to wit, 5 places of decimal parts in the Multiplicand, and 1 place in the Multiplier) cut off 6 places to the right-hand from the total before produced, so will it stand thus 73,486138: Wherefore the true product is 73 $\frac{486138}{1000000}$ or 73.486138, that is, 73 integers and almost one half of an integer.

In like manner, if this mixt number 246,25 (that is 246 $\frac{1}{2}$) were given to be multiplied by 35 integers, the true product will be found 8618.75, that is 8618 integers, and $\frac{75}{100}$ parts of an integer, as you see by the operation in the Margin, where you may observe that two places are cut off from the number produced by the Multiplication, towards the right-hand, because there are two places of Decimals in the Multiplicand (the Multiplier consisting of integers only); but if there had been decimal parts also in the Multiplier, so many more places should have been cut off, as was shewed in the first Example.

Again, if these two Decimals .87 and .9 (to wit, $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true product will be found to be .783, that is $\frac{783}{1000}$ parts of an integer, as you see in the Example; where you may observe that the product is a Fraction only; for after 3 places (being the number of places of Decimals in both the numbers given to be multiplied) are cut off to the right-hand, there remains no integer on the left-hand.

393. When the Multiplication is finished, if there arise not so many places in all as ought to be cut off by Art. 392. (which may often happen when the product is a Fraction;) in such case, as many places as are wanting, so many cyphers must be prefixed to the product on the left-hand thereof, and then a point or comma must be prefixed to sign the product so increased for a Decimal: So these Decimals .0375. and .05 being given to be multiplied one by the other, multiply 375 by 5, and there arises 1875: Now, according to Art. 392. I should cut off 6 places to the right-hand, and here are but 4 in all; therefore I prefix two cyphers, to wit, as many as there are places wanting, and then prefixing a point,

$$\begin{array}{r}
 1.30526 \\
 56.3 \\
 \hline
 391578 \\
 783156 \\
 652630 \\
 \hline
 73,486138
 \end{array}$$

$$\begin{array}{r}
 246.25 \\
 35 \\
 \hline
 123125 \\
 73875 \\
 \hline
 8618.75
 \end{array}$$

$$\begin{array}{r}
 .87 \\
 .9 \\
 \hline
 .783
 \end{array}$$

$$\begin{array}{r}
 .0375 \\
 .05 \\
 \hline
 .001875 \\
 5.525 \\
 .0026 \\
 \hline
 33150 \\
 11050 \\
 \hline
 .0143650
 \end{array}$$

the true product will be .001875, or $\frac{1875}{10000}$. In like manner, if this mixt number 5.525 be multiplied by this Decimal .0026, the true product will be found to be .0143650 (or $\frac{143650}{100000}$) as you may see by the operation in the Margin, where one cypher is prefixed to the numbers arising from the Multiplication to discover the true product,

394. To multiply a finite Decimal by a Decimal which has a Repetend.

Rule. Reduce the repeating Decimal to its equivalent Vulgar or mixt Fraction, by Art. 377, multiply the finite Decimal by the Numerator of such Fraction, as in Art. 392, and divide by its Denominator, according to the method taught in Art. 108. so shall the result be the product required.

The reason of this operation will appear from Art. 299,

Example 1. Multiply 48,734 by .04.

$$.04 = \frac{0,4}{9} \text{ by Art. 380.}$$

$$48,734 \times 0,4 = 19,4936 \text{ by Art. 392.}$$

$$\text{And } \frac{19,4936}{9} = 2,1659\overline{3} \text{ the Answer.}$$

Note. When the Divisor is a single 9, it seems easier to use the common method of Division, than to proceed by Art. 108.

Example 2. Multiply 48,75 by 4.

$$48,75 = \frac{438,8}{9} \text{ by Art. 377.}$$

$$438,8 \times 4 = 1755,2 \text{ by Art. 392.}$$

$$\text{And } \frac{1755,2}{9} = 195,0\overline{7}, \text{ the Answer.}$$

Example 3. Multiply 58,7648 by 7.

$$58,7648 = \frac{58705,8}{999} \text{ by Art. 377.}$$

$$\text{And. } \frac{58,7}{58705,8} \times 7 = 410940,6 \text{ by Art. 391.}$$

$$\text{Also } \frac{410940,600}{410,350} \text{ divided by } 999 = 411,3 \overline{518} \text{ of } \frac{1}{999}$$

$$\begin{array}{r} 58,7 \\ - 410,350 \\ \hline 11,3518 \end{array}$$

Where observe, that because there is one Decimal place in the Dividend 410940,6, therefore there is one Decimal place in the Quotient 411,3, also that the remainder (518) will always be a Repetend.

Example

Example 4. Multiply 48.76 by 0.1345.

$$0.1345 = \frac{1345}{999} \text{ by Art. 397.}$$

$$\begin{array}{r} ,1 \\ 1345 \times 48.76 = 6553.344 \text{ by Art. 392.} \\ 48.76 \\ \hline 8064 \\ 9408 \\ 10752 \\ 5376 \\ \hline 6553.34400 \\ - 655989 \\ \hline 6.559.003 \end{array}$$

$$6553.34400 \text{ divided by } 999 = 6.559 \frac{903}{999} \text{ of } \frac{1}{1000}$$

395. To multiply Decimals when both Factors have Repetends.

Rule. Reduce them to Vulgar or mixt Fractions, by Art. 377. then multiply their Numerators together, as in Art. 392. Lastly divide by each of their Denominators separately, according to Art. 108.

This process is the same as in Multiplication of Vulgar Fractions.

Example 1. Multiply 2.8 by 5.8.

$$2.8 = \frac{28}{9} \text{ and } 5.8 = \frac{58}{9} \text{ by Art. 378.}$$

$$28 \times 58 = 1071 \text{ by Art. 78.}$$

$$\begin{array}{r} 105 \\ \hline 9)1071 \\ \hline 9)119 \\ \hline \end{array}$$

$$13.\overline{7} \text{ And } \frac{1071}{9 \times 9} = 13.\overline{7}$$

Example

Example 2. Multiply 1,1 by 1,1.

$$\begin{array}{r} 1,1 = \frac{10}{9} \text{ by Art. 378.} \\ 10 \times 10 = 100 \\ 9)100 \\ \hline 9)11,1\overline{11111} \\ \hline 1,23456798 \text{ the Answer.} \end{array}$$

Example 3. Multiply 3,143 by 4,797

$$\begin{array}{r} 3,143 = \frac{311,4}{99} \text{ and } 4,797 = \frac{4293}{999} \text{ by Art. 377. and 378.} \\ 4293 \times 311,4 = 1336840,2 \text{ by Art. 392.} \\ 311,4 \\ \hline 17172 \\ 4293 \\ 4293 \\ 12879 \\ \hline 1336840,20 \quad \text{and} \quad \frac{1338,1788}{99} = 1338,1788 \text{ by Art. 108.} \\ 133817 \\ \hline 13381783783783783 \text{ &c.} \\ 135168512966492764 \\ 1223445 \\ \hline 13,516953316953316952 \text{ &c.} \end{array}$$

The operation of the Division by 99 is continued by writing (783) the Repetend of the former Quotient as often as was convenient, that the reader might be the better satisfied that 169533 will repeat in the latter Quotient.

396. To multiply finite or approximate Decimals, so that the product shall consist of no more than a determinate number of places.

Rule. Under that place in the Multiplicand, thought necessary to be retain'd in the Product, write the unit's place of the Multiplier, and invert the order of all its other places; that is, write the Decimals on the left, and the integers (if any) on the right,

In multiplying, omit those places in the Multiplicand which stand to the right of the digit multiplying by, and let the right-hand place of every line stand under each other.

In each line, let the lowest place be increased by the carriage which would arise from the omitted places, carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. instead of carrying 1 for every 10; and the sum of these lines will give the product, generally exact.

Note. In multiplying approximate Decimals by the method of Art. 392. many figures in the product will be uncertain, the finding of which will be avoided by using this Rule.

Example 1. Multiply 384,672158, by 36,8345.

Now seeing there would be 10 decimal places in the product, whereof the greatest part are unnecessary; therefore keep only 4 decimal places in the product.

384,672158	
5438,63	
	384,672158
115,401647	36,8345
23080329	
3077377	1923 360790
115402	15386 88632
15387	115401 6474
1923	3077377 264
	23080329 48
14169,2065	115401647 4
	14169,2066 038510

Here the Example is wrought both ways, by which may be easily seen what is fav'd by the Rule.

In this Example, because it is intended to keep 4 decimal places in the product, set 6, the unit's place of the Multiplier under 1, the 4th place in Decimals of the Multiplicand, and invert the order of all the rest of the figures : Then say three times 8 is 24, and carry 2 ; three times 5 is 15, and 2 is 17 ; now set down the 7 and carry 1, because this is the product arising by multiplying the 5 that stands over the 3.

Again, 6 times 8 is 48, and carry 5; 6 times 5 is 30, and 5 is 35, and carry 3; 6 times 1 is 6, and 3 is 9: Now being come to the figure over the 6, set down 9, &c.

Again, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17: Now being come to the figure over the 8, set down 7, and carry 1, &c., proceeding in like manner till all is done.

Example

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Example 2. Multiply 3,141592 by 52,7438, so as to have only 4 Decimal places in the product.

$$\begin{array}{r} 3,141592 \\ \times 52,7438 \\ \hline 1570796 \\ 62832 \\ 21991 \\ 1256 \\ 94 \\ 25 \\ \hline 165,6994 \end{array}$$

C H A P. XXIX. DIVISION of DECIMAL FRACTIONS.

397. In any of the cases which may happen in Division, if the Dividend be greater than the Divisor, the Quotient will be either a whole number, or else a mixt number: But when the Dividend is less than the Divisor, the Quotient must necessarily be a Fraction; for a lesser number is contained in a greater once at the least, but a greater is not contained once in a lesser.

398. Sometimes the Dividend, whether it be a whole number, mixt number, or decimal Fraction, is to be prepared by annexing a competent number of cyphers thereto, to make room for the Divisor: So if 32.5 were given to be divided by 17.325, the Dividend 32.5 must be increased with cyphers at pleasure after this manner 32.50000, &c. Likewise if 1 were given to be divided by 360, the Division cannot be made till the Dividend 1 be increased with cyphers, which being annexed, the Dividend will stand thus 1.000000, &c. Here note, that the cyphers annexed in manner aforesaid, supply places of decimal parts, and will be useful in discovering the quality of the Quotient, according to Art. 400.

399. After the Dividend is prepared by annexing cyphers, when occasion requires, (as in the last Rule,) all the places thereof

thereof must be esteemed as one whole number, (to wit, consisting of units, or integers;) and so is the Divisor to be esteemed, whether it be a Decimal Fraction, or mixt number; for in all cases the Division must be performed in every respect according to the Rules of Division of whole numbers. So if this mixt number 326.25 were given to be divided by this mixt number 12.3 , you must divide in the same manner, as when you divide 32625 integers by 123 integers. Also if this Decimal $.8356$ were given to be divided by this Decimal $.05$, you are to divide in the same manner, as when you divide 8356 integers by 5 integers; and after the Quotient is found, the degree or place of the first figure which arises in the Quotient, is to be enquired after, *viz.* you must know how far such figure is distant from the place of units, to the end, that the point or comma which is used to separate between the place of units (or first place of integers) and the first place of Decimals, may be duly fixed: This is the only knot in Decimal Division, and may be resolved by the following Rule, *viz.*

400. In any of the cases which may happen in Division of Decimals, the first figure which arises in the Quotient will be always of the same place or degree with that figure or cypher of the Dividend, which at the first question stands over, or at least belongs to the place of units in the Divisor. To illustrate this Rule, I shall give Examples in all the principal cases; and first, let a mixt number be given to be divided by a mixt number, *viz.* Let it be required to divide 172.5 by 3.746 , here (according to Art. 398.) the Dividend must be increased with cyphers at pleasure, so will it stand thus 172.500000 , &c. then Division being made according to the Rules of Division of whole numbers, the Quotient arising will be 46049 , &c.

$$3.746) \overline{172.500000} (46.049, \text{ &c.}$$

Now it remains to separate the integers in this Quotient from the Decimal parts; to perform which, subscribe the Divisor 3.746 , orderly under the first Dividual 172.50 (being

$$3.746) \overline{172.500000} (46.049, \text{ &c.}$$

$$\begin{array}{r} 3.746 \\ \times 172.500000 \\ \hline \end{array}$$

that part of the Dividend, whereof the first question must be asked) or at least imagine the Divisor to be so subscribed, and the figure 3 which stands in the place of units in the Divisor will be placed under 7 , which is the place of tens, (or second place of integers) in the Dividend; therefore by the Rule

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Rule before given, conclude that the first figure arising in the Quotient must likewise stand in the place of tens (or second place of integers) and consequently the next place on the right-hand must be the place of units; so it is evident that the separating point or comma must be placed between the figure 6 and 0 in the Quotient; that done, the true Quotient is found to be $46.\overline{049}$ &c. to wit, 46 integers and $\frac{49}{1000}$ parts of an integer, and somewhat more; for $46\frac{49}{1000}$ is less than the true Quotient; but $46\frac{50}{1000}$ is greater than it; and therefore, tho', after the aforesaid Division of 172,500000 by 3.746 is ended, there will be a remainder, to wit, .446, which seems to be greater, yet here it is less in value than $\frac{1}{1000}$ part of an unit or integer; and if to that remainder you annex another cypher and continue the Division, you will proceed nearer the truth, and not miss $\frac{1}{1000}$ part of an unit of the true Quotient, and in that order you may proceed until you find a Repetend.

Example 2. Suppose this mixt number 2.34 be given to be divided by this mixt number 52.125 (where you may observe that the Dividend is less than the Divisor;) first (as before) annex cyphers at pleasure to the Dividend, to make room for the Divisor, then the Division being prosecuted as in whole numbers, at length these figures will arise in the Quotient, to wit.

$$52.125) \ 2.3400000 (.0448, \&c.$$

52.125

448; and to the end the degree or quality of the first figure 4 may be discovered, subscribe the Divisor 52.125 under the Dividend 2.34000 (for so far the first question did extend in the Division;) and thereby find that the figure 2, which stands in the place of units in the Divisor, will be seated under 4, which is in the second place of Decimals; therefore the first figure arising in the Quotient must also stand in the second place of Decimals, and consequently the first place of Decimals (which is next on the left-hand to the second) must be supplied with a cypher; so that if a cypher be prefixed on the left-hand of 4, and then a point placed before that cypher, the Quotient will at length be discovered to be .0448, &c. or $\frac{448}{1000}$ and somewhat more, that is to say, $\frac{448}{1000}$ is less than the true Quotient, but $\frac{450}{1000}$ is greater than it; and if you would proceed nearer the truth, you may continue the division, as is directed in the first Example of this Rule.

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Example 3. Where a whole number is divided by a Decimal Fraction, viz. suppose 82 integers were given to be divided by this Decimal .056: After cyphers are annexed to the Dividend at pleasure, and the Division prosecuted as in

$$.056) \ 82.00000 \ (146428 \text{ &c.}$$

whole numbers (to wit, 8200000 being divided by 56) these figures 146428 will arise in the Quotient; now to the end the degree or seat of 1, the first figure in the Quotient, may be known, subscribe the Divisor .056 under the first Dividend 82 (for so far did the first question in the Division extend;) and because the Divisor is less than unity, supply the place of units by a cypher or 0 prefixed on the left-hand of the point of separation

$$.056) \ 0082.00000 \ (1464.28 \text{ &c.}$$

$$0.056$$

in the Divisor; also prefix cyphers before, (to wit, on the left-hand of) the integers in the Dividend, to represent a succession of places of integers (for the order of places in integers is from the right-hand towards the left;) then the cypher or 0 which represents the place of units in the Divisor, stands under that cypher, which represents the fourth place of integers in the Dividend (as you see by the Example;) therefore the first figure arising in the Quotient must also be seated in the fourth place of integers, and consequently the 4 first places in the Quotient will be Integers, and the rest a Decimal: So that the true Quotient is 1464 Integers, and $\frac{28}{56}$ parts of an Integer, and somewhat more, viz. 1464.28 is less than the true Quotient, but 1464.29 is greater than it.

Example 4. Suppose this Decimal .0125 be given to be divided by this Decimal .5; after Division is finished according to the Rules of Division of .5) .0125 (25 whole numbers (to wit, after 125 is divided by .5) these figures 25 will arise in the Quotient; now to discover the degree or seat of 2 the first figure in the Quotient, subscribe the Divisor 5 under the first Dividend .012, and having (as in the last Example) prefixed .5) .0125 (.025 a cypher on the left-hand of the point of separation in the Divisor, to denote, or represent the place of units, such cypher or place of units stands under the figure which is seated in the second place of Decimals in the Dividend, therefore the first figure which arises in the Quotient must also be in the second place of Decimals, and therefore prefixing a cypher to supply the first place

$$0.5$$

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place of Decimals, and putting a point before that cypher, the Quotient is at length discovered to be .025 or $\frac{25}{1000}$.

Example 5. Suppose this Decimal .8564 be given to be divided by this .008. First, annex cyphers to the Dividend at pleasure, then prosecuting the Division as in whole numbers, to wit, dividing .856400 by 8, the

Quotient arising is 107050 : Now .008) 856400 (107050 to discover the degree or place of

1, the first figure in the Quotient, subscribe the Divisor .008 under the first Dividual .8, then

prefix a cypher to set forth, or .008) 000.85640 (107.050 supply the place of units in the

Divisor; also prefix cyphers to 0.008 represent places of integers in

the Dividend; that done, the cypher or 0, which supplies the place of units in the Divisor, stands under the cypher which is seated in the third place of integers in the Dividend: Therefore the first figure arising in the Quotient must be also in the

third place of integers, and consequently the three first places in the Quotient will be Integers, and the rest a Decimal; so

that the true Quotient is 107.050 or $107 \frac{5}{8}$.

Example 6. Let it be required to divide this Decimal Fraction .73952 by this .32; first, dividing 73952 by 32, as if they were whole numbers, the figures arising in the Quotient will

be 2311. Now, to discover the quality or value of the said figures, subscribe the Divisor .32 under the first Dividual 73, then prefixing a cypher as well on the

left-hand of the Dividend, as of the .32) 0.73952 (2.311 Divisor so subscribed (or imagined to

be subscribed) as aforesaid, to represent 0.32

the place of units in each of them, the cypher or 0, which supplies the place of units in the Divisor, stands under the 0, which represents the place of units in the Dividend; wherefore the first figure arising in the Quotient will stand in the place of units, and consequently the following places of the Quotient will be a Decimal Fraction, so that the true Quotient is 2.311 or $2 \frac{311}{1000}$.

401. Or the number of Decimal places in the Quotient of any such Division may be found thus:

Rule. From the number of Decimal places used in the Dividend, take the number of Decimal places in the Divisor, and the remainder will be the number of Decimal places in the Quotient, which number of places mark off by a point or comma from the right-hand in the Quotient; but if there be

not

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not so many places therein, as this Rule requires, supply the defect with cyphers on the left-hand.

In the first Example, Art. 400. viz.

$$3,746) \overline{172,500000} (46049.$$

From (6) the number of Decimal places in the Dividend, take (3) the number of Decimal places in the Divisor, the remainder is 3; therefore in the Quotient let 3 places be mark'd off from the right-hand, thus 46,049.

In the second Example; Art. 400. viz.

$$52,125) \overline{2,3400000} (448.$$

From (7) the number of Decimal places in the Dividend, take (3) their number in the Divisor, there remains 4; now since there are but 3 places in the Quotient therefore 1 cypher must be placed on the left-hand of them to make their number equal with the above remainder 4, and then prefix the separating comma; thus ,0448.

From the operation of these two Examples it follows, that this Rule gives the same result with the former in Art. 400.

This Rule is the converse of that given for cutting off the proper number of Decimal places in Multiplication, Art. 392. For, since the product of the Divisor and Quotient will be equal to the Dividend, it will follow from Art. 392. that the Decimal places in the Dividend, must be equal to the sum of those in the Divisor and Quotient.

402. If the Dividend be a circulating Decimal, and the Divisor finite; use the repeating figure or figures in the same manner as cyphers are directed to be used in Articles 398, 399, and 400.

Exampl. 1. Divide 195,07 by 4.

$$4) \overline{195,022 \text{ &c.}}$$

$$\underline{48,75} \quad \text{&c..} = 48,75$$

O

Exampl.

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Exampl. 2. Divide 6,559903 by 48,76.

$$48,76) 6,559903903 \text{ Fr. } (.1345848 \text{ Fr.} = .1345848$$

4876

$$\begin{array}{r} 16839 \\ 14628 \\ \hline \end{array}$$

$$\begin{array}{r} 22110 \\ 19504 \\ \hline \end{array}$$

$$\begin{array}{r} 26063 \\ 24380 \\ \hline \end{array}$$

$$\begin{array}{r} 16839 \\ 14628 \\ \hline \end{array}$$

$$\begin{array}{r} 22110 \\ 19504 \\ \hline \end{array}$$

$$\begin{array}{r} 26063 \\ 24380 \\ \hline \end{array}$$

$$\begin{array}{r} 1683 \\ \hline \end{array}$$

This Division is continued till the Repetend occurs twice in the Quotient for the reader's satisfaction.

403. If the Divisor be a circulating Decimal, make the Repetends of the Divisor and Dividend similar by Art. 370. Then, instead of them, use the Numerators of their equivalent vulgar or mixed Fractions (found by Art. 377.) in the same manner as finite Decimals in Art. 398, 399, and 400.

Note, If the Dividend be finite, make it a similar Repetend by annexing cyphers.

Example 1. Divide 2,16593 by 0,04.

$$\begin{array}{r} 0,4) 21,6593 \\ 0,0) 2,1659 \\ \hline 0,4) 19,4936 \\ \hline 48,734 \end{array}$$

Example

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Example 2. Divide 54 by .17:

$$\begin{array}{r}
 1.7) 540 \\
 \underline{-} \\
 54 \\
 \underline{-} \\
 1.6) 486,000 (303.75 \\
 \underline{-} \\
 48 \\
 \underline{-} \\
 60 \\
 \underline{-} \\
 48 \\
 \underline{-} \\
 120 \\
 \underline{-} \\
 112 \\
 \underline{-} \\
 80 \\
 \underline{-} \\
 80 \\
 \underline{-} \\
 00
 \end{array}$$

Example 3. Divide 411,351.9 by 58,764.5.

$$\begin{array}{r}
 58764.5) 411351.9 \\
 \underline{-} \\
 58.7) 4113 \\
 \underline{-} \\
 58705.8) 410940.6 \\
 \underline{-} \\
 410940.6 \\
 \underline{-} \\
 0000000
 \end{array}$$

Example 4. Divide 9 by .45.

$$\begin{array}{r}
 45) 900 \\
 \underline{-} \\
 0) 9 \\
 \underline{-} \\
 45) 891.0 (19.8 \\
 \underline{-} \\
 45 \\
 \underline{-} \\
 44 \\
 \underline{-} \\
 405 \\
 \underline{-} \\
 360 \\
 \underline{-} \\
 360 \\
 \underline{-} \\
 000
 \end{array}$$

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Example 5. Divide 13,7 by 5,8.

$$\begin{array}{r}
 56 \\
 5 \\
 \hline
 51 \\
) \\
 51 \\
 \hline
 0 \\
 \hline
 132 \\
 13 \\
 \hline
 19,0 \\
 102 \\
 \hline
 170 \\
 153 \\
 \hline
 17
 \end{array}
 \quad (2,3)$$

Example 6. Divide $x^23456798$ by $1,x$.

1111111111, 1, <hr/> 1111111110,	1234567901, 1, <hr/> 1234567900,0 1111111110 <hr/> 1234567900
---	--

Example 7. Divide 13,5169533 by 4,79%.

$\begin{array}{r} 4297297 \\ - 42 \\ \hline 4297293 \end{array}$	$\begin{array}{r} 13516953,3 \\ - 13,5 \\ \hline 13516939,800 \end{array}$
	$\begin{array}{r} 12891879 \\ \hline \end{array}$
	$\begin{array}{r} 6250608 \\ - 4297293 \\ \hline \end{array}$
	$\begin{array}{r} 19533150 \\ - 17189172 \\ \hline \end{array}$
	$\begin{array}{r} 23439780 \\ - 21486465 \\ \hline \end{array}$
	1953315

404. Division by finite or approximate Decimals may be contracted as follows.

Rule. Let each remainder be a new Dividend, and for each such new Dividend, point off one figure from the right-hand of the Divisor; observing at each Multiplication to have regard

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gard to the increase of the figures so cut off, as in Multiplication of Approximates, Art. 396.

EXAMPLE I.

$$384,672158) \underline{14169,2066038510} (36,8345$$

11540,16474

262904188

230803295

.32100893

30773772

.1327121

1154016

.173105

153869

.19236

19234

EXAMPLE 2.

$$9,365407) \underline{87,076326} (9,297655$$

84,288663

2,787663

1,873081

914582

842886

71696

65558

6138

5619

519

468

51

47

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405. Having explained all necessary Rules in Division, concerning Decimal Fractions, I shall give a table of their excellent use by the two following Questions, and then conclude this Chapter.

Ques. 1. A merchant bought of gold plate 356 ounces, 13 penny-weights, and 15 grains, for 1160 pounds Sterling; the question is, what he paid for an ounce? *Ans.* 31. 5s. $\frac{1}{2}$ d. very near. The operation by Decimals may be after this manner, *viz.*

By the second Table of Reduction the Decimal of 13 penny-weight is } .65

The Decimal of 15 grains is } .03125

The sum of these two Decimals is } .68125

Therefore the quantity of plate in ounces and } 356.68125
Decimal parts of an ounce is }

Then by the Rule of Three, say, if 356.68125 ounces cost 1160 pounds, what will 1 ounce cost? Here 'tis evident, that if I divide 1160 by 356.68125, the Quotient will give the value of an ounce, to wit, 3.252 pounds, or 3 pounds, 5 shillings, and $\frac{1}{2}$ d. very near.

356.68125) 1160.0000000 (3.252, &c.

Ques. 2. Suppose the length of the Tropical year, (or the space of time in which the sun running through the whole Ecliptic circle, consisting of 360 degrees, is returned to the same Equinoctial or Solstitial point from whence he departed) to consist of 365 days, 5 hours, and 49 minutes, the question is to know the sun's mean or equal motion for 1 day, to wit, what part of 360 degrees the sun moves thro' in a whole day? The operation by Decimals, thus:

By the tenth Table of Reduction the Decimal } 2083333
correspondent to 5 hours is }

The Decimal of 49 minutes is } 0340274

The sum of those Decimals is } .242361x

Therefore the time given in days and Decimal parts of a day is } 365.242361x

Then by the Rule of Three, if 365.242361x days give 360 degrees (or a total circumference) what will 1 day give? Here divide 360 by 365.242361x, the Quotient will give the diurnal motion required; which will be found very near .98564, &c. or $\frac{59}{60}$ parts of a degree, which Decimal being reduced to the common Sexagenary parts (by Art. 364.) will give 59' 8", &c. and such is the sun's diurnal motion very near, according to the aforesaid supposition of the length of the Tropical year.

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I shall here add the vulgar Sexagesimal resolution of this question, that by comparing both ways together, the excellency of Decimal Arithmetic in calculations of this nature may be the more perspicuous.

The aforesaid questions being stated according to the Rule of Three, will stand thus :

Days. Hours. Min. Degrees. Day.

If $365 \cdot 5 \cdot 49 : 360 :: 1$

The first term in the Rule must be reduced into minutes (by Art. 121.) so there will be found 525649 minutes.

$$\begin{array}{r}
 D. \quad H. \quad M. \\
 357 : 5 : 49 \\
 -\frac{24}{1465} \\
 -\frac{720}{745} \\
 -\frac{60}{8765} \\
 -\frac{60}{8705} \\
 \hline
 \end{array}$$

Hours.

525949 Minutes.

Likewise the third term 1 Day must be reduced to Minutes, which will be found to be 1440, as you see by the following operation.

$$\begin{array}{r}
 1 \text{ Day, or } 24 \text{ Hours.} \\
 -\frac{60}{1440} \\
 \hline
 \end{array}$$

Then multiply the third term by the second, to wit, 1440 by 360, the product is 518400, which being divided by the first term 525949 the quotient will give $\frac{1}{11}\frac{12}{13}$ parts of a degree; by Art. 235. which Fraction being reduced to the accustomed Sexagesimal parts (by Art. 267.) will give as before $59^{\circ} 8'$, &c. for the sun's mean diurnal motion: Now which of these two ways is the more expeditious, I leave to him who is skilled in both to determine.

C H A P. XXX.

The UNIVERSAL RULE of PROPORTION.

406. **B**Y producing terms, in the following Rule, are meant whatsoever necessarily and jointly produce any effect; as the cause and the time; length, breadth, and depth; the buyer and his money; the seller and his goods, &c. and by produced terms, such as are the effects of the former, as purchase, produce, expence, gain, loss, interest, value, &c.

R U L E.

1. Set down the terms expressing the condition of the question in one line, and in any order.
2. Under each conditional term, set its corresponding one, in another line.
3. Multiply the producing terms of one line, and the produced term of the other line, continually, and take the result for a Dividend.
4. Multiply the remaining terms continually, and let their product be the Divisor.
5. The Quotient of this Division will be the term required.

This Rule is so general, as to comprehend all cases that come under the common Rule of Three, whether direct or inverse; whether single, or any how compounded; in integers or fractions; so that questions in any of the above Rules, where proportion is used, may be readily solved thereby.

Example 1.: (Being the first proposed in Art. 145.) If 4 students spend 19 pounds in 3 months, how much will serve 8 students 9 months?

Students.	£.	Mon.
The terms expressing the condition of the question _____	4	19
The terms corresponding to them putting A to signify the answer—	8	A

Here the students and months are producing terms, and the money or expence is produced.

And therefore the producing terms of the lower line, viz. 8 students and 9 months are to be multiplied by 19. the produced term of the upper line for a Dividend: And the product of the remaining terms 4 students and 3 months, is to be

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be the Divisor: This Dividend and Divisor expressed in the manner of a vulgar Fraction, may by Art. 276. stand as below.

$$\frac{8 \times 9 \times 19}{4 \times 3} = A, \text{ or the answer.}$$

Which expression being abbreviated by Art. 266. will be reduced as follows;

$$\frac{8 \times 9 \times 19}{4 \times 3} = \frac{2 \times 9 \times 19}{1 \times 3} = \frac{2 \times 3 \times 19}{1 \times 1} = 6 \times 19 = 114, \text{ the answer.}$$

Examp. 2. (Being the last Example proposed in Art. 143.) If 9 bushels of provender serve 8 horses 12 days; how many days will 24 bushels last 16 horses?

	Bushels.	Horses.	Days.
Conditional terms	—	9	8
Corresponding	—	24	16

Where the horses and days are producing and the consumption of provender produced.

$$\text{Therefore } A = \frac{8 \times 12 \times 24}{9 \times 19} = \frac{8 \times 4 \times 24}{3 \times 16} = \frac{8 \times 4 \times 8}{4 \times 16} = \frac{2 \times 8}{1 \times 1} = 2 \times 8 = 16 \text{ the answer.}$$

Examp. 3. (Being the Example in Art. 152.) If the carriage of 8 C. wt. 128 miles, cost 48 shillings; for how much may I have 4 C. wt. carried 32 miles?

	C. wt.	Miles.	Shill.
Conditional terms	—	8	128
Corresponding	—	4	32

Where the weight and distance are producing, and the expense produced.

$$\text{Therefore } A = \frac{4 \times 32 \times 48}{8 \times 128} = \frac{4 \times 32 \times 6}{1 \times 128} = 6,$$

Examp. 4. A regiment of 136 soldiers eat up 391 quarters of wheat in 108 days, I demand how many quarters of wheat 11232 soldiers will eat in 56 days?

	Sold.	Quarters.	Days.
Conditional terms	—	136	391
Corresponding	—	11232	A.

$$\text{Therefore } A = \frac{11232 \times 56 \times 391}{136 \times 108} = 104 \times 7 \times 23 = 16744 \text{ quarters.}$$

Examp.

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Examp. 5. If 50 acres of grass be mowed by 24 men in 28 days, how many men will do the same work in 24 days?

	Acres.	Men.	Days.
Conditional terms	50	24	28
Corresponding	50	A.	24

$$\text{Therefore } A = \frac{24 \times 28 \times 50}{50 \times 24} = 28 \text{ men.}$$

Examp. 6. If 48 bushels of corn, or other seed, yield 576 bushels in a year, how much will 240 bushels yield in 6 years at that rate? That is to say, if there were sowed 240 bushels every one of the 6 years.

	Bushels.	Years.	Produce.
Conditional terms	48	1	576
Corresponding	240	6	A.

$$\text{Therefore } A = \frac{240 \times 6 \times 576}{48 \times 1} = 30 \times 576 = 17280 \text{ Bush.}$$

Examp. 7. If 40 shillings is the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days?

	Shil.	Men.	Days.
Conditional terms	40	8	5
Corresponding	A.	32	24

$$\text{Therefore } A = \frac{32 \times 24 \times 40}{8 \times 5} = 32 \times 24 = 768 \text{ Shillings.}$$

Examp. 8. If, in a family consisting of 7 persons, there are drank out 2 kilderkins of beer in 12 days, how many kilderkins will there be drank out in 8 days, by another family consisting of 14 persons?

	Persons.	Kilderkin.	Days.
Conditional terms	7	2	12
Corresponding	14	A.	8

$$\text{Therefore } A = \frac{14 \times 8 \times 2}{7 \times 12} = \frac{8}{3} = 2\frac{2}{3} \text{ Kilderkins.}$$

Examp. 9. If when wine is worth 30/- per tun, 20/- worth is sufficient for the ordinary of 100 men, how many men will 4/- worth suffice, when it is worth 24/- per tun? Answer 25 men.

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	Ton.	£.	£.	Men.
Conditional terms	—	1	30	20
Corresponding	—	1	24	4 A.

$$\text{Therefore } A = \frac{30 \times 100 \times 4}{24 \times 20} = 5 \times 5 = 25.$$

Examp. 10. If 48 pioneers in 12 days, cast a trench 14 yards long, how many pioneers will cast a trench 168 yards long in 16 days?

	Pioneers.	Days.	Long.
Conditional terms	—	48	12
Corresponding	—	A.	16, 168

$$\text{Therefore, } A = \frac{48 \times 12 \times 168}{14 \times 16} = 3 \times 12 \times 12 = 432 \text{ Pioneers.}$$

Example 11. If 12 C. weight being carried 100 miles, cost £l. 6s. I desire to know how many C. weight may be carried 150 miles for £2l. 12s. 8d.

$$6s. = 12s. \text{ and } 12l. 12s. = 252s.$$

	C. wt.	Miles.	Shillings.
Conditional terms	—	12	100
Corresponding	—	A.	150

$$\text{Therefore } A = \frac{12 \times 100 \times 252}{126 \times 150} = 4 \times 2 \times 2 = 16 \text{ C. wt.}$$

Examp. 12. If 60 ells English be equal to 100 ells Flemish, and each ell English contains 20 nails of a yard, how many such nails doth the Flemish ell contain?

	Ells Eng.	Nails.	Ells Flem.
Conditional terms	—	60	20
Corresponding	—	A.	100

$$\text{Therefore } A = \frac{60 \times 20 \times 1}{100 \times 1} = 6 \times 2 = 12.$$

Exam. 13. If 14 men, in 15 days, build 16 rods of wall. How many men must added be, to do it in 2 days, that's all?

	Men.	Days.	Rods.
Conditional terms	—	14	15
Corresponding	—	A.	2, 16

$$\text{Therefore } A = \frac{14 \times 15 \times 16}{2 \times 16} = 7 \times 15 = 105.$$

That is 105 men will build it in 2 days; therefore (to the 14 men) 91 must be added.

Examp.

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Exam. 14. If 8 cannons in one day spend 48 barrels of powder, I demand how many barrels 24 cannons will spend in 22 days?

	Cannons.	Days.	Barrels.
Conditional Terms	8	1	48
Corresponding	24	22	A.

$$\text{Therefore } A = \frac{24 \times 22 \times 48}{8 \times 1} = 24 \times 22 \times 6 = 3168 \text{ Barrels.}$$

Exam. 15. If 15 hundred British lead cost 16 pounds in gold, How much of that Saturnian ore for 40 pounds is sold?

	C.	£.
Conditional terms	15	16
Corresponding	A.	40

$$\text{Therefore } A = \frac{15 \times 40}{16} = \frac{15 \times 5}{2} = \frac{75}{2} = 37\frac{1}{2} C.$$

Exam. 16. If 30 pence and 30 groats, buy 50 pints of wine, What is the cost of 60 quarts in current English coin?

30 groats = $30 \times 4 = 120$ pence, $120 + 30 = 150$ pence; and 50 pints = 25 quarts.

	Quarts.	Pence.
Conditional terms	25	150
Corresponding	60	A.

$$\text{Therefore } A = \frac{60 \times 150}{25} = 60 \times 6 = 360d. = 1l. 10s.$$

Exam. 17. If 9 pounds of our English wool, doth 90 pence require, The value of 1 C. weight of you I do desire?

	Ib.	Pence.
Conditional terms	9	90
Corresponding	112	A.

$$\text{Therefore } A = \frac{112 \times 90}{9} = 112 \times 10 = 1120d. = 4l. 13s. 4d.$$

Exam. 18. If 16s. and a crown, for 13 weeks supply For meat and drink; that is my board; how much a day gave I?

16s. + 1 crown = 21s; and 13 weeks = $13 \times 7 = 91$ days.

	Man.	Days.	Shillings.
Conditional terms	1	91	21
Corresponding	1	1	A.

$$\text{Therefore } A = \frac{1 \times 1 \times 21}{1 \times 91} = \frac{21}{91}s. = 2\frac{1}{4}d. \text{ nearly.}$$

407. When this Rule is to be applied to Fractions, after reducing mixed numbers to improper fractions, and compound Fractions to single ones, observe,

1°. That to multiply by a Fraction is to multiply by the Numerator; and to divide by the Denominator thereof: Therefore having prepared a line to separate the Numerator of the answer from the Denominator, as below.

A = —

Write the Numerator of a Fraction, which is to be multiplied by, above that line; and the Denominator below it.

2°. To divide by a Fraction is to multiply by its Denominator, and divide by its Numerator: Therefore write the Denominator of a Fraction which is directed to be a Divisor above the line, and its Numerator below it.

Examp. 19. (Being the same as in the old editions of this work is given as the first in the Rule of Three Direct in Fractions) If $\frac{5}{6}$ of a yard of velvet be sold for $\frac{2}{3}$ of a pound, what will $\frac{5}{6}$ of a yard cost?

Conditional terms ————— $\frac{2}{3}$ Yard. $\frac{2}{3}$ £.

Corresponding ————— $\frac{5}{6}$ Yard.

Here the goods are producing terms, and the money produced.

Therefore the product of $\frac{5}{6}$ by $\frac{2}{3}$ is to be divided by $\frac{2}{3}$: Now the product of $\frac{5}{6}$ by $\frac{2}{3}$ will be represented by $\frac{5 \times 2}{6 \times 3}$ by the first direction above, and therefore

A = $\frac{5 \times 2 \times 4}{6 \times 3 \times 3}$ by the second direction.

$$\text{Whence } A = \frac{5 \times 2 \times 4}{6 \times 3 \times 3} = \frac{5 \times 4}{3 \times 3 \times 3} = \frac{20}{27} l.$$

$$\text{And } \frac{20}{27} l. = 14s. 9d. \frac{21}{27} = 14s. 9d. \frac{7}{9} \text{ by Art. 267.}$$

Examp. 20. When $\frac{2}{5}$ of $\frac{3}{4}$ of a ship is valued at 197l. 11s. 3d. how much is the whole ship worth?

$$\frac{2}{5} \text{ of } \frac{3}{4} = \frac{2 \times 3}{5 \times 4}; \text{ and } 197l. 11s. 3d. = \frac{15805}{80} l.$$

Conditional Terms	—————
	$\frac{2 \times 3}{5 \times 4}$
	$\frac{15805}{80}$

Corresponding	—————
	I
	A

Therefore

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$$\text{Therefore } A = \frac{1 \times 15805 \times 5 \times 4}{80 \times 2 \times 3} = \frac{15805}{4 \times 2 \times 3} l.$$

$$\text{And } \frac{15805}{4 \times 2 \times 3} = \frac{15805}{24} = 658 l. 10 s. 10 d.$$

Examp. 21. (Being the first Example in the Rule of Three Inverse in Fractions in the old edition.) If cloth which is $1\frac{1}{2}$ yards in breadth and $3\frac{1}{2}$ yards long will make a cloak, how much in length of stuff which is $\frac{1}{2}$ of a yard in breadth, will make a cloak of the same bigness with the former?

$$1\frac{1}{2} = \frac{3}{2}; \text{ and } 3\frac{1}{2} = \frac{7}{2}.$$

	<i>Yards long.</i>	<i>Yards Breadth.</i>	<i>Cloth.</i>
Conditional terms	$\frac{3}{2}$	$\frac{7}{2}$	$\frac{1}{2}$
Corresponding	$A.$	$\frac{1}{2}$	$\frac{1}{2}$

Here the length and breadth are producing, and the garment produced.

$$\text{Therefore } A = \frac{7 \times 7 \times 1 \times 8}{2 \times 4 \times 1 \times 5} = \frac{7 \times 7}{5} = \frac{49}{5} = 9\frac{4}{5}.$$

Examp. 22. If $18\frac{1}{4}$ yards of serge cost $2l. 5s.$ how many yards of the same may I have for 300 guineas at $2l. 6d.$ each?

$$18\frac{1}{4} = \frac{75}{4} \text{ yards}, 2l. 5s. = 45s., \text{ and } 2l. 6d. = 43s.$$

	<i>Yards.</i>	<i>Pes. Mon.</i>	<i>Shillings.</i>
Conditional terms	$\frac{75}{4}$	1	45
Corresponding	$A.$	300	43

$$\text{Therefore } A = \frac{75 \times 300 \times 43}{4 \times 2 \times 45 \times 1} = \frac{25 \times 5 \times 43}{2} = 268\frac{1}{2} \text{ yards.}$$

Examp. 23. Bought a cask of wine for $62l. 8s.$ how many gallons were in the same, when the gallon was valued at $5s. 4d.$?

$$62l. 8s. = 1248s. l. \text{ and } 5s. 4d. = \frac{64}{240} l.$$

	<i>Gall.</i>	<i>s.</i>
Conditional terms	1	$\frac{64}{240}$
Corresponding	$A.$	1248

$$\text{Therefore } A = \frac{1 \times 1248 \times 240}{20 \times 64} = 78 \times 3 = 234 \text{ gallons.}$$

Examp. 24. If 15 pounds cost 16 pence, of bread that's made of rye,

How many loaves, of 6 pounds each, for four-score pounds had I?

Loaf.

	Loaf.	lb.	Pence.
Conditional terms	1	15	15
Corresponding	A.	6	80

$$\text{Therefore } A = \frac{1 \times 15 \times 80 \times 240}{6 \times 16} = 15 \times 5 \times 40 = 3000.$$

Examp. 25. (Being the first Example in the Double Rule of Three in Fractions in the old Edition) If I pay 28 shillings for the carriage of 3*C. wt.* 50 miles; how much ought I to pay for the carriage of 17*C. wt.* 8*4* miles?

	Shil.	C. wt.	Miles.
Conditional terms	28	3	50
Corresponding	A.	17	84

$$\text{Therefore } A = \frac{17 \times 84 \times 28}{3 \times 50} = \frac{17 \times 28 \times 14}{3 \times 25} = 132 \text{ sh. } 6\text{d. } \frac{17}{25}.$$

Examp. 26. (Being the second Example in the last-mentioned Rule) If 40*3* l. in $\frac{2}{3}$ of a year gain 2*1*/*2* l. what will 100*l.* gain in $\frac{7}{2}$ of a year?

$$\text{Now } 40\frac{3}{4} = \frac{163}{4}; \text{ and } 2\frac{1}{2} = \frac{7}{2}.$$

	£.	Years.	£.
Conditional terms	$\frac{2}{3}$	$\frac{2}{3}$	
Corresponding	100	$\frac{7}{2}$	A.

$$\text{Therefore } A = \frac{100 \times 7 \times 5 \times 3}{12 \times 2 \times 203 \times 2} = \frac{25 \times 7 \times 5 \times 5}{4 \times 203}.$$

$$\text{And } \frac{25 \times 7 \times 5 \times 5}{4 \times 203} = \frac{4375}{812} = 5l. 7s. 9\frac{17}{25}d.$$

Examp. 27. An usurer put 75*l.* out to receive interest for the same, and when it had continued 9 months, he received for principal and interest 78*l.* 7*s.* 6*d.* I demand at what rate per cent. per annum he received interest?

The amount 78*l.* 7*s.* 6*d.* less the principal 75*l.* leaves the interest 3*l.* 7*s.* 6*d.* = 3*1*/*2* l. = $\frac{7}{2}$ l.

	£.	Months.	£.
Conditional terms	75	9	$\frac{7}{2}$
Corresponding	100	12	A.

$$\text{Therefore } A = \frac{100 \times 12 \times 27}{75 \times 9 \times 8} = 2 \times 3 = 6l. \text{ the answer.}$$

Examp.

Exampl. 28. If $17\frac{1}{2}$ ells of diaper, $\frac{1}{2}$ of a yard wide, cost 5 guineas; what cost $32\frac{1}{2}$ yards $\frac{3}{4}$ of an ell wide, of the same sort and fineness?

$$17\frac{1}{2} = \frac{35}{2} \text{ ells} = \frac{35 \times 5}{2 \times 4} \text{ yards}; \quad \frac{2}{3} \text{ ells} = \frac{2 \times 5}{3 \times 4} \text{ yards.}$$

$$5 \text{ guineas} = 5l. 5s. = \frac{105}{48} l. 32\frac{1}{4} s. = 1\frac{29}{4} s.$$

	Yards long.	Yards wide.	£.
Conditional terms	$\frac{35 \times 5}{2 \times 4}$	$\frac{3}{4}$	$\frac{105}{20}$
Corresponding	$\frac{129}{4}$	$\frac{2 \times 5}{3 \times 4}$	A.

$$\begin{aligned} \text{Therefore } A &= \frac{129 \times 2 \times 5 \times 105 \times 2 \times \frac{3}{4} \times \frac{5}{4}}{4 \times 3 \times \frac{5}{4} \times 20 \times 35 \times \frac{3}{4} \times 3} \\ &= \frac{\cancel{129} \times 2 \times \cancel{105} \times 2}{3 \times 20 \times \cancel{35} \times 3} = \frac{\cancel{43} \times \cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{48} \times \cancel{7} \times 3} \\ &= \frac{43 \times 3}{5 \times 3} = \frac{43}{5} = 8l. 12s. \end{aligned}$$

C H A P. XXXI.

Of POWERS, and their INDICES, or LOGARITHMS.

408. If unity be continually multiplied by any number whatsoever (suppose 2) the products arising are called powers of that number; thus 2. 4. 8. 16. &c. are powers of the given number 2.

409. The given number (2) is called the first power, being produced by the first Multiplication; the next number (4) is called the second power (or square) being the product of the second Multiplication; the next number (8) is called the third power (or cube) being the result of the third Multiplication, &c.

410. Again, with respect to any of the superior products 4, 8, 16, &c. the given number (2) is called the root; thus 2 is the square root of 4; the cube root of 8; the biquadrate root of 16, &c.

411. The number of Multiplications used, in producing any power, is called its Index; thus, the index of the root 2 is 1; the index of the square 4, is 2; of the cube 8 is 3, &c.

412. If a rank of powers be wrote, with their proper indexes over them as below:

Indexes 0. 1. 2. 3. 4. 5. 6. 7, &c.

Powers 1. 2. 4. 8. 16. 32. 64. 128, &c., Then,

First., The sum of the indexes of any two powers, will stand over the product of those powers; thus 5, the sum of 2 and 3, stands in the rank of indexes, over 32, the product of 4 and 8 in the rank of powers; and 6, the sum of 3 and 3, stands over 64, the product of 8 and 8.

413. *Secondly.*, The difference of the indexes of any two powers, will be the index of the quotient arising by dividing one of those powers by the other; thus 2, the difference of the indexes 7 and 5, is the index of 4, the quotient of 128 divided by 32.

414. Hence having a few of the initial powers given, any other power may be found, without finding all the intermediate powers.

Rule. Compose the index of the required power by the addition of the indexes of the given powers; then multiply the powers corresponding to those indices, and the product will be the power required.

Exampl. 1. Let it be required by the help of the first 7 powers of 2, given in Art. 412. to form the 20th power thereof.

This may be performed divers ways, because (20) the index of the required power can be diversely composed of the given indexes.

For $7+7+6 = 20$ the index of the required power.

$$5+5+5+5 = 20$$

$$3+4+6+7 = 20$$

&c.

And $128 \times 128 \times 64 = 1048576$ the power required.

$$32 \times 32 \times 32 \times 32 = 1048576$$

$$8 \times 16 \times 64 \times 128 = 1048576$$

415. If unity be continually divided by any number (2) the quotients arising $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \text{ &c.}$ may be also called powers of 2.

416. But, because the formation of these last powers is by a process directly contrary to the former, the indexes of these powers are generally distinguished, from the indexes of those, by placing the sign ($-$) either before or under the figure thus (-1 , or $\underline{1}$) and are called negative indexes.

417. A rank of these powers, being placed under their indexes, have the same properties as the former.

Indexes $0 \cdot \underline{1} \cdot \underline{2} \cdot \underline{3} \cdot \underline{4} \cdot \underline{5} \cdot \underline{6} \cdot \underline{7}, \text{ &c.}$

Powers $1 \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{16} \cdot \frac{1}{32} \cdot \frac{1}{64} \cdot \frac{1}{128}, \text{ &c.}$

For $\underline{5}$ the sum of $\underline{2}$ and $\underline{3}$, stands over $\frac{1}{2}$, the product of $\frac{1}{2}$ and $\frac{1}{4}$.

And $\underline{2}$, the difference of $\underline{7}$ and $\underline{5}$, stands over $\frac{1}{4}$ the quotient of $\frac{1}{128}$ divided by $\frac{1}{32}$.

418. But when the negative powers are joined in the same rank with the affirmative as below,

$\text{&c. } \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \text{ &c.}$

$\text{&c. } \frac{1}{8} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{1} \cdot 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \text{ &c. Then,}$

The negative indexes will, in all operations, have effects contrary to those of the affirmative indexes, viz.

If a negative and an affirmative power be to be multiplied together, take the difference of their indexes for the index of the product required.

Thus $16 \times \frac{1}{2} = 8$

And $4 - \underline{1} = 3$

Also $\frac{1}{8} \times 2 = \frac{1}{4}$

And $\underline{3} - 1 = 2$

Again $\frac{1}{4} \times 4 = 1$

And $\underline{2} - 2 = 0$

419. Hence, if any affirmative power be multiplied by a negative power, whose index is the same number, the product will be unity.

Note, Numbers that have the above property are called reciprocals; and their indexes, complements of each other.

420. If the reciprocal of any number be decimals expressed, then, if you multiply thereby, the same result will be produced as would arise from dividing by the number; and if you divide

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vide thereby, the same figures will appear as by multiplying by the number.

Example. $\frac{1}{4} = .25$ is the reciprocal of the number 4: Now let it be proposed to divide 56 by 4; or to multiply it by .25, the result will be the same as you see.

$$\begin{array}{r} 4) \quad 56 \\ \hline 14 \\ \hline 280 \\ 112 \\ \hline 14,00 \end{array}$$

Again, let 9 be multiplied by 4, or divided by .25.

$$\begin{array}{r} 9 \\ 4 \\ \hline 36 \\ \hline \end{array} \quad ,25) \quad 9,00 \quad (36$$

$$\begin{array}{r} 75 \\ \hline 150 \\ 150 \\ \hline \end{array}$$

421. If a rank of numbers be calculated, having the same properties with those of indexes, mentioned in Art. 412, 413, 417, and 418, and be adapted not only to a rank of powers as those, but, to all numbers whatsoever within certain limits, the same is called a Table of Logarithms.

422. Instead of the rank of powers 1, 2, 4, 8, &c. assume 1. 10. 100. 1000; &c. and adapt the indexes to them, viz.

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 \\ 1 & 10 & 100 & 1000 & 10000, & \text{&c.} \end{array}$$

Then, since the Index or Logarithm of 1 is 0, and the Index or Logarithm of 10 is 1; it follows, that the Logarithms of all numbers, that are greater than 1 and less than 10, such as 2. 3. 4. 4. 1. 4. 2. 4. 3. &c. must be Decimal Fractions.

423. Also, since the Index or Logarithm of 10 is 1, and the Index or Logarithm of 100 is 2; therefore the Logarithms of all numbers that are greater than 10, and less than 100, such as 20, 30; 40, 41, 42, 43, &c. will be mixed decimal expressions, having 1 in the place of integers.

424. In like manner, the Logarithms of all numbers greater than 100, and less than 1000, will be mixed decimal expressions, having 2 in the place of integers, &c.

425. Hence it follows, that the decimal part of the Logarithms of all numbers that have the same significant figures

will be the same; for the number 4,1 stands with regard to 1 and 10, in the same place that 41 does with regard to 10 and 100, and that 410 does with regard to 100 and 1000.

Therefore supposing that ,6127839 be the Decimal part of the Logarithm belonging to the significant figures 41: Then

$$\begin{aligned}
 \text{The Logarithm of } 4100, &= 3,6127839 \\
 410 &= 2,6127839 \\
 41 &= 1,6127839 \\
 4,1 &= 0,6127839 \\
 ,41 &= \underline{1,6127839} \\
 ,041 &= \underline{2,6127839} \\
 &\quad \ddots
 \end{aligned}$$

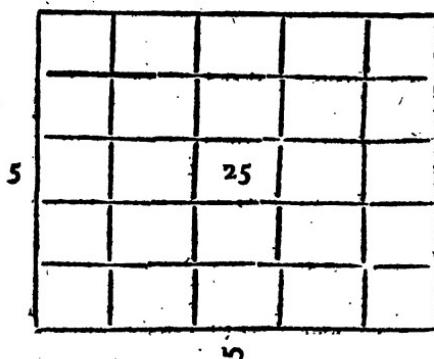
426. Of this kind are the Logarithms commonly put in Tables, which if rightly calculated will perform the work of Multiplication and Division by Addition and Subtraction, &c. in the same manner as the Indices did.

427. But as the operations in which Logarithms are chiefly useful in common Arithmetic, viz. extracting roots, and computing questions relating to compound interest and annuities, are in this treatise supplied by Tables, calculated for each purpose; and as the methods of calculating them depend upon algebraical and other superior principles; the farther prosecution of the subject here is declined, it being sufficient to have given the reader an idea of their nature and properties.

C H A P. XXXII.

The EXTRACTION of the \$QUARE (or QUADRAT) Root.

428. **T**HE extraction of the Square-root is that, by which having a number given, we find out another number, which being multiplied by itself, produces the number given.



429. In the extraction of the Square-root, the number proposed is always conceived to be a square number, that is, a certain number of little squares comprehended within one entire great square; and the root or number required is the side of that great square; as will readily appear by this Diagram, where you see the 25 little squares contained within one great square; now if the said content 25 be given, and the side or root of the square containing the said 25 little squares is required, the invention of such side or root is called the Extraction of the Square-root; which root must be such, that if it be squared, that is, multiplied by itself, the product will be equal to the square content first given: So 5 is the Square-root of 25, for 5 times 5 is 25. Likewise this square number 49 being propounded, its root is 7.

430. Square numbers are either single or compound.

431. A single square number is that, which being produced by the Multiplication of one single figure by itself, is always less than 100: So 25 is a single square number produced by 5; likewise 4 is a square number produced by 2.

432. All the single square numbers, together with their respective roots are expressed in the following Table.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

433. Here in the uppermost rank of the Table are placed the single square numbers of every particular figure, and in the other their respective roots; and therefore if it were demanded, what is the Square-root of 36? The answer will be 6. So the Square-root of 16 is 4; the Square-root of 9 is 3, &c. And contrarily, the square of the root 6 is 36: Also the square of 3 is 9.

434. When a square number is given, that exceeds not 100, and yet is none of the square numbers mentioned in the Table, for its root you are to take the root of the square number that being less yet comes nearest to it: So 45 being given, the root that belongs unto it is 6, and 10 being given, its correspondent root is 3.

435. A compound square number is that which being produced by a number (that consists of more places than one) multiplied by itself, is never less than 100: So 1024 is a compound square number produced by the Multiplication of 32 by itself.

436. To prepare any square number given for Extraction, put a point over the first place thereof on the right-hand (being the place of units;) then proceeding towards the left-hand, pass over the second place; and put another point over the third place; also passing over the fourth place put another point over the fifth, and so forward in such manner that between every two points which are next to the other, one place will be intermitted: So if the Square-root of 1024 be required, the first point is to be placed over 1, and the second over 0, and so many points as are in that manner placed, of so many figures the root demanded will consist.

437. Having thus prepared the number; it is distributed by the points into several squares: So in the last Example, 10 is the first square, and 24 the second; likewise if this number 144 were propounded for Extraction, after points are duly placed according to the last Rule, 1 is the first square, and 44 the second.

438. Having drawn a crooked line on the right-hand of the number given for Extraction, (after the same manner as is usually done in Division to denote the place of the Quotient,) find the root of the first square, and place it in the Quotient: Now by Art. 433. 3 is the correspondent root of 10; wherefore write 3 in the Quotient, and then the Work will stand as in the Margin.

$$\begin{array}{r} 1024 \\ \times 3 \\ \hline 9 \end{array}$$

439. Subscribe the square of the figure placed in the Quotient under the first square of the number given.

440. Having drawn a line under the square (of the figure placed in the Quotient, subscribed as aforesaid,) subtract the same out of the first square of the number proposed, and place the remainder orderly under the line; so the square of 3 which is 9, being subtracted from 10, the remainder is 1, and the Work will appear as in the Margin.

$$\begin{array}{r} 1024 \\ - 9 \\ \hline 1 \end{array}$$

441. To the said remainder bring down the next square of the given number, that is, write down the figures or cyphers standing in the two following places of the number propounded on the right-hand of the said remainder, so the square 24 being placed next to the remainder 1, there will be found this number 124, which may be called the Resolvend.

$$\begin{array}{r} 1024 \\ - 9 \\ \hline 124 \end{array}$$

442. Double the root, being the number placed in the Quotient, and place the said double on the left-hand of the Resolvend, like a Divisor: So the double of 3 is 6, which being placed before a crooked line on the left-hand of the Resolvend 124, the work will stand as in the Margin.

$$\begin{array}{r} 1024 \\ \times 6 \\ \hline 6 \end{array}$$

443. Let the whole Resolvend, except the first place thereof on the right-hand (being the place of units) be always esteemed as a Dividend, then demanding how often the Divisor before found, is contained in the said Dividend, and observing in that behalf the Rules before taught in Division, write the answer in the Quotient, and also on the right hand of the Divisor, to wit, between the Divisor and the crooked line: So if you ask how often the Divisor 6 is found in the Dividend 12, the answer is 2; therefore write 2 in the Quotient, and also after the Divisor 6; as in the Margin.

$$\begin{array}{r} 1024 \\ \times 6 \\ \hline 62 \end{array}$$

444. Multiply all the figures which stand on the left-hand of the Resolvend (to wit, before the crooked line) by the figure last placed in the Quotient, and write the product orderly under the Resolvend, to wit, units under units tens under tens, &c.) then having drawn a line under the said product, subtract it from the Resolvend, and subscribe the remainder under the line: So 62 being multiplied by 2, the product is 124, which subtract out of the Resolvend 124, the remainder is 0; and thus the whole Work being finished, the Square-root of 1024 (the number proposed) is found to be 32.

$$\begin{array}{r} 1024(32 \\ - 9 \\ \hline \end{array}$$

$$\begin{array}{r} 62) 124 \\ - 124 \\ \hline \end{array}$$

Note 1. When the product before-mentioned exceeds the Resolvend placed above it, the Work is erroneous, and then you are to reform it by placing a lesser figure in the Quotient.

Note 2. For every one of the particular squares (distinguished by the points) except the first on the left-hand, a Resolvend is to be set a-part, by bringing down to the remainder the congruent particular square, as is directed in Art. 441. and as often as a Resolvend is set a-part, so often a new Divisor is to be found by doubling or multiplying by 2 all the root in the Quotient (consisting of what number of places soever.)

Note 3. The Work of Art. 438, 439, and 440. for finding out the first figure in the root, is but once used in the Extraction of the Square-root of a number consisting of what number of places soever; but the Work of Art. 441, 442, 443, and 444. is to be repeated for the finding of every place in the root, except the first.

The practice of these three Notes will be seen in the following Examples.

Example 1. Let it be required to extract the Square-root of 43623.

Having distributed the number proposed into several squares by points, as is directed in Art. 436. I demand the Square-root of 4 the first square, which by Art. 432. is 2; therefore placing 2 in the Quotient, and the square thereof, which is 4, under the first square 4, draw a line, and subtracting 4 from 4, the remainder is 0, which subscribe under the line. This is always the first Work, which is no more repeated in the whole Extraction, (as was intimated in the third Note aforesoing.)

$$\begin{array}{r} 43623(2 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \hline \end{array}$$

Then bringing down the next square, which is 36, and placing it next after the remainder 0, the Resolvend is 36; and

$$\begin{array}{r} 0 \\ \hline \end{array}$$

and doubling the root 2 in the Quotient, the product is 4 for a Divisor, (by Art. 441, 442.) and the Dividend will be 3, (by Art. 443.) demand therefore how often the Divisor 4 is contained in the Dividend 3, and not finding it once contained in it, place 0 in the Quotient, and also next after the Divisor 4; and because the product of 40 multiplied by 0 (the last character in the Quotient) is 0, the Resolvend 36, from which the said product ought to be deducted, remains the same without alteration; therefore bring down 23 the next square, and place it after the remainder 36, so will 3623 be a new Resolvend; then doubling the whole root in the Quotient, which is 20, the Divisor will be 40, (according to the second Note before-mentioned,) and the Dividend will be 362 (to wit, all the Resolvend except the first place on the right-hand by Art. 443.) therefore demand how often the Divisor 40 is contained in the Dividend 362, or how often 4 in 36, and though it be 9 times in it, yet (according to the first Note foregoing) I can take but 8, (for if I should take 9, and proceed according to Art. 443, 444. a number would arise greater than the Resolvend, from which such number arising ought to be subtracted:) Therefore write 8 in the Quotient, and also after the Divisor 40; this done, multiply 408 (the number on the left-hand of the Resolvend) by 8 the figure last placed in the Quotient, and the product, to wit, 3264, subscribe underneath, and subtract from the Resolvend 3623, so there will remain 359; thus the Work being finished, 208 is the number of units contained in the root sought; and because after the Extraction is ended there happens to be a remainder, to wit, 359, I conclude that the root is greater than the said 208, but less than 209; yet how much it is greater than 208, no rules of art hitherto known will exactly discover, tho' we may proceed infinitely near, as in the next Article will be manifest.

445. To find the fractional part of the root very near, a competent number of pairs of cyphers, to wit, 00, 0000, 000000, or 00000000, &c. are to be annexed to the number first proposed; then esteeming the said given number with the cyphers annexed, to be but one entire number, the extraction is to be made according to the preceding Rules, and look how many

43623 (20

4

40) 036

43623 (20

4

40) 03623

43623 (208

4

408) 03623

3264

359

many points were placed over the number first given, so many places of integers will be in the root, the rest of the root towards the right-hand will be the Numerator of a Decimal Fraction, which Numerator consists of so many places as there were points over the cyphers annexed: So if 43623 were given as before, to find the root thereof (according to this Rule,) annex cyphers in this manner, and then if you extract it according to the Rules aforesaid, you'll find the root arising in

$$43623.000000 \text{ (208.861. &c.)}$$

the Quotient to be 208.861, that is $208\frac{861}{10000}$; and because after the Extraction is finished there happens to be a remainder, I conclude that $208\frac{861}{10000}$ is less than the true or exact root, but $208\frac{862}{10000}$ is greater than it; so that by annexing three pairs of cyphers to the number propounded, you will not miss $\frac{1}{1000}$ part of an unit of the true root; also by annexing four pairs of cyphers, you will not miss $\frac{1}{10000}$ part of an unit; and in that order you may proceed infinitely near, when you cannot obtain the true root. The whole operation of the said Example here follows:

$$43623.000000 \text{ (208.861, &c.,}$$

4
the Root,

$$\begin{array}{r} 408) 03623 \\ \underline{3264} \end{array}$$

$$\begin{array}{r} 4168) 35900 \\ \underline{33344} \end{array}$$

$$\begin{array}{r} 41766) 255600 \\ \underline{250596} \end{array}$$

$$\begin{array}{r} 417721) 500400 \\ \underline{417721} \end{array}$$

$$\begin{array}{r} 82679 \end{array}$$

Again, if 10 were proposed to be extracted, you must prepare it thus:

$$10.00000000000000.$$

And

And then the Root thereof being extracted will be $\frac{3\overline{)8888888}}{3}$.

Which (according to Art. 309. may be } 3,162,277,686.
writ thus _____ }

See the Work.

30.000000000000000 (3.16227, 5°,

$$\begin{array}{r}
 9 \\
 \hline
 61) \quad 100 \\
 \quad \quad 61 \\
 \hline
 626) \quad 3900 \\
 \quad \quad 3756 \\
 \hline
 6322) \quad 14400 \\
 \quad \quad 12644 \\
 \hline
 63242) \quad 175600 \\
 \quad \quad 126484 \\
 \hline
 632447) \quad 4911600 \\
 \quad \quad 4427129 \\
 \hline
 \end{array}$$

446. The number of the places in the root may be nearly doubled, if, after the above Work, you proceed with the last Divisor and remainder, according to the manner of contracted Division, in Art. 404.

Thus assuming the last Example.

632447)

$$\begin{array}{r}
 632447) 484471 (76602, &c. \\
 \cdot \cdot \cdot \cdot \cdot \\
 442713 \\
 \hline
 41758 \\
 37947 \\
 \hline
 3811 \\
 3795 \\
 \hline
 16 \\
 13 \\
 \hline
 3
 \end{array}$$

If the figures thus found (76602) be placed on the right-hand of (3,16227) those before obtained, the Root will be 3,1622776602, uncertain only in the last figure 2.

447. Hence after it is determined to what exactness the Square-root of any surd number is to be found, the Rule for extracting the Square-root must be used, to one place of figures more than half the number of places required, and then the rest may be found by Division as above.

448. The Extraction of the Square-root is proved by multiplying the root by itself, for that done, the product (in such case where there is no remainder after the Extraction is finished) will be equal to the root whose square was enquired; so in the first Example of this Chapter the root 32 being multiplied by itself, produces 1024 the number propounded: But when, after the Extraction is finished, there happens to be a remainder, and that the root is found as near as you please, in a mixt number of Integers and Decimal parts (by annexing cyphers, as in Art. 445.) then such mixt number being multiplied by itself must produce a mixt number less than the number first given for Extraction; yet so near it, that if the figure standing in the last place of the Numerator of the Decimal Fraction in the Root, be made greater by 1, and then the mixt number so increased be multiplied by itself, the product must be greater than the number first proposed: So in the Example of Art. 445. if 208.861 be multiplied by itself, it produces 43622.917, &c. which is less than the given number 43623; -but if 208.862 be multiplied by itself, the product will be 43623.335, &c. which is greater than 43623.

449. But if the method of inverted Multiplication, delivered in Art. 396. be applied to the multiplying the Root by itself, the product will approximate to the square very nearly.

Proof

Proof of the Example in Art. 446.

$$\begin{array}{r}
 31622776602 \\
 206672261,3 \\
 \hline
 94868329806 \\
 3162277660 \\
 1897366596 \\
 63245553 \\
 6324555 \\
 2213594 \\
 221359 \\
 18974 \\
 1897 \\
 6 \\
 \hline
 10,0000000000
 \end{array}$$

450. The Square-root of a Fraction is found in this manner, viz. extract the root of the Numerator (by the preceding Rules of this Chapter) which root shall be a new Numerator. Also the root of the Denominator is to be taken for a new Denominator: So the new Fraction will be the Square-root of the Fraction first proposed. Thus the Square-root of $\frac{1}{18}$ is $\frac{1}{3}$, viz. the root of 9 is 3 for a new Numerator, also the root of 16 is 4 for a new Denominator. In like manner the Square-root of $\frac{1}{4}$ is $\frac{1}{2}$. But here note diligently, that if the Fraction, whose Square-root is required, be not in its least terms, it must first of all be reduced by Art. 261, &c. before any Extraction be made; for it frequently happens that the Fraction first given has not a perfect root, but when such Fraction is reduced to its least terms, the root thereof may be extracted: So in this Fraction $\frac{8}{18}$ each term is incommensurable to its Square-root, viz. neither 8 nor 18 has a Square-root expressible by any true or rational number; but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the root of this may be extracted; for the root of 4 is 2 for a new Numerator; also the root of 9 is 3 for a new Denominator; so that $\frac{2}{3}$ is found to be the Square-root of $\frac{8}{18}$ (equivalent to $\frac{4}{9}$).

451. When either the Numerator or Denominator of a Fraction has not a perfect Square-root, such root is usually expressed by prefixing this character, $\sqrt{}$ before the Fraction given: So the Square-root of $\frac{1}{18}$ is signified thus, $\sqrt{\frac{1}{18}}$, because the root of $\frac{1}{18}$ cannot be express'd by any true or rational number whatsoever, yet it may be found very near, as in the next Rule.

452. The Square-root of a Fraction which is incommensurable to its root, may be found near, in this manner, viz. reduce the Fraction proposed to a Decimal, by Art. 322. The more places are in the Decimal, the nearer will the root be found; but the Decimal must consist of an even number of places, viz. either of two, four, six, eight, or ten, &c. places; then extract the Square-root of that Decimal, as if it were a whole number, according to the Rules aforescored, which root found shall be a Decimal expressing near the Square-root of the given Fraction.

So if the Square-root of $\frac{13}{16}$ be required near, reduce the said $\frac{13}{16}$ to a Decimal, which is found .81250000, &c. then extracting the Square-root thereof as if it were a whole number, it will be found .9013 very near.

453. The Square-root of a mixt number commensurable to its root is found in the same manner, as in Art. 450. the mixt number being reduced to an improper Fraction, by Art. 268.

So the Square-root of $34\frac{1}{4}$ will be found $5\frac{1}{2}$, viz. $34\frac{1}{4}$ being reduced to the improper Fraction $\frac{137}{16}$, the Square-root of the Numerator 2209 will be 47 for a new Numerator; also the Square-root of the Denominator 64 is 8, for a new Denominator; so we find $\frac{47}{8}$ which (by Art. 271.) is $5\frac{1}{2}$ the Square-root sought. And here the same caution is to be observed, as in Art. 450. viz. the fractional part of the mixt number, or the improper Fraction equivalent to the mixt number, must be in the least terms before any Extraction be made..

454. When the mixt number given is incommensurable to its Square-root, prefix this character before it, viz. $\sqrt{\cdot}$. So the Square-root of $7\frac{2}{3}$ will be thus expressed, $\sqrt{7\frac{2}{3}}$: But if you desire to find the Square-root near of a mixt number incommensurable to its root, reduce the fractional part of the mixt number to a Decimal of an even number of places, as in Art. 452. and annex the Decimal so found to the whole part of the mixt number; then esteeming the said whole number and Decimal as one entire number, extract the Square-root thereof according to the preceding Rules of this Chapter, and from the root found cut off always to the right-hand, so many places as there are points over the Decimal annexed, which number so cut off shall be a Decimal, shewing the fractional part of the root, and that on the left-hand will be the whole part of the root; so the Square-root of $7\frac{2}{3}$ is found 2.7688 very near.

C H A P. XXXIII.

The EXTRACTION of the CUBE Root.

455. THE Extraction of the Cube-root is that, by which having a number given, we find another number, which being first multiplied by itself, and then by the product, produces the number given.

456. In the Extraction of the Cube-root, the number proposed is always conceived to be a Cube number, that is, a certain number of small Cubes, comprehended within one entire great Cube, and the root or number required is the side of that great cube: What a Cube is may be well expressed by a die, which indeed is a little Cube itself; therefore if you place four dice in a square form, that is laying two and two in a rank, you'll have a square containing four dice, upon which if you yet erect such another square of dice, you shall have another great entire Cube comprehending two times four, that is 8 dice or small cubes; and here 8 is the Cube number given, and 2 is the root or number required: In like manner, if you rank 25 dice in a square form, viz. laying 5 in a rank, you have a square containing 25 dice; now upon this square of dice if you erect four other such squares one upon another, you'll have a great entire Cube comprehending 5 times 25, that is 125 little Cubes, and in this case 125 is the Cube number propounded, and 5 the root or number required.

457. A Cube-number is either single or compound.

458. A single Cube-number is that, which being produced by the Multiplication of one single figure, first by itself, and then by the product, is always less than 1000. So 125 is a single Cube-number produced by 5 multiplied first by itself, and then by 25 the product; for 5 times 5 is 25, and 5 times 25 is 125.

459. All the single Cube-numbers, and Square-numbers, together with their respective roots, are expressed in the following Table.

Cubes.	1	8	27	64	125	216	343	512	729
Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed the single Cube-numbers of the particular figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; in the next, the squares of those figures, and in the lowest rank the figures themselves being the respective roots of the Cubes and Squares in the uppermost rank; and therefore the Cube-root of 125 being demanded, the answer is 5, and the Cube-root of 216 being required, the Table will give 6, and so of the rest.

460. When a Cube-number is given, that exceeds not 1000, and yet is none of the Cube-numbers mentioned in the Table, for its Root you are to take the root of the Cube-number, that being less, yet comes nearest to it: So 157 being given, the root that belongs to it is 5.

461. A compound Cube-number is that, which being produced by a number (that consists of more places than one) first multiplied by itself, and then by the product, is never less than 1000. So 157464 is a compound Cube-number, produced by 54 multiplied first by itself, and then by 2916 the product, for 54 times 54 is 2916, and then 54 times 2916 is 157464, the compound Cube-number proposed.

462. To prepare a Cube-number for Extraction, put a point over the first place thereof towards the right-hand (to wit, the place of units) then passing over the second and third places, put another point over the fourth, and passing over the fifth and sixth, put another point over the seventh, and in that order (to wit, two places being intermitted between every two adjacent points,) place as many points as the number will permit: So 157464 being given, you are to place the points as in the margin, and so many points 157464 as are in that manner fixed, of so many figures the root demanded will consist.

463. Having thus prepared your number, you may see it distributed by the points into several Cubes, or periods.

So in the same Example 157 is the first Cube, and 464 the second. In like manner, if this number 157464 were proposed for Extraction, after points are duly placed as before, you'll see 7 to be the first 7464 Cube, and 464 the second.

464. Having drawn a crooked line on the right-hand of the number propounded to signify a Quotient, seek for the Cube-root of the first Cube and set it in the Quotient: So finding (by Art. 460.) 5 to be the correspondent root of 157, write 5 in the Quotient, and then the Work 157464 (5 will stand as you see in the Margin.

465. Subscribe the Cube of the root placed in the Quotient under the first Cube of the number given:

So 125 being the Cube of 5, the root, (by Art.

459.) write it under 157 the first Cube of the number given, as you see in the Example.

157464 (5
125

466. Draw a line under the Cube, subscribed as aforesaid, (to wit, the Cube of the root set in the Quotient) and subtract this Cube from the first Cube of the number proposed, placing the remainder orderly under the line: So 125 the Cube of 5 being subtracted from 157, the remainder is 32, and the Work will stand as you see.

$$\begin{array}{r} 157464 (5 \\ 125 \\ \hline 32 \end{array}$$

467. To the said remainder bring down the remaining periods of the number propounded, placing these next after, to wit, on the right-hand of the remainder; so 464 being set after the remainder 32, there will be found this number 32464, which may be called the Resolvend.

157464 (5
125

32464 Ref.

468. To (5) the root, or figure placed in the Quotient, annex as many cyphers as there are Cubes or periods remaining unused in (157464) the number whose root is to be extracted, which in this case is 1, and therefore the root will be 50.

469. Divide the Resolvend (32464) by the root (50); so will the Quotient be 649.

470. To (649) the above found Quotient, add three times the square of the root (50), viz. 7500; so will the sum be 8149. Let this number be a Divisor.

471. Divide the Resolvend (32464) by the Divisor (8149); and the Quotient figure, or figures, being annexed to (5) the figure first placed in the root, instead of the cypher or cyphers, will be the required root nearly. See the whole process below.

$$\begin{array}{r} 157464 (50 \text{ Root} \quad 5.0) \quad 3246.4 \\ 125 \\ \hline \text{Resolvend} \quad 32464. \quad 50 \times 50 \times 3 = 7500 \\ \hline \text{Divisor} = 8149 \\ 8149) \quad 32464 \text{ (4 Quotient.} \end{array}$$

Therefore $50 + 4 = 54$ is the root required, as will appear by cubing thereof.

Q

Note,

Note, Altho' the Divisor 8149 cannot be had quite 4 times in the Resolvend 32464, yet as it is very near thereto, it will be safe to take that for the Quotient.

472. The Rule above given will, if the second figure of the root be 1 or 0, generally give three or four figures true in the root (if it consists of so many); but if the second figure be 2, or more, it will be best to find no more places than it, until the process be renewed.

473. To renew the process, cube the figures found in the root, that is, multiply them by themselves, and then by the product; and place the result under the given number, in such sort, that it may stand under as many periods therein, as you have cubed figures. Subtract the lower number from the upper, and bring down the remaining periods for a Resolvend, as in Art. 467. Find a new Divisor, by Art. 468, 469, and 470. Divide the Resolvend thereby, and annex the Quotient to the former part of the root, as in Art. 471. so shall the root be found true to 5 or 6 places of figures: For each operation commonly triples the figures found in the former.

Example 2. Let it be required to extract the Cube-root of 9302348?

The given number pointed <i>per Art. 462</i>	9302348 (2)
The Cube of the root 2 is (<i>Art. 465.</i>)	8
The Resolvend, <i>per Art. 466, and 467.</i>	1302348
The root with 2 cyphers annexed by <i>Art. 468.</i>	200
The Resolvend divided thereby } = $\frac{1302348}{200}$ = 6511 (<i>Art. 469.</i>)—————	6511
Three times the square of the root (200) " "	120000
The Divisor = sum of the 2 last, by <i>Art. 470.</i>	126511
The Resolvend divided thereby } = $\frac{1302348}{126511}$ = 10 per <i>Art. 471.</i> —————	10

Therefore 210 is the root nearly.

But,

But, to renew the operation,

From the given number = 9302348 (210)

Take $210 \times 210 \times 210$ = 9261000

Resolvend : = 41348

The Resolvend divided by the root = $\frac{41348}{210} = 196$

Three times the square of 210: : = 132300

Divisor : = 132496

The Resolvend with cyphers annexed
divided thereby, quotes _____ ; ,31207

See the Operation.

$$132496) \ 41348.00 : (; 31207$$

397488

$$\begin{array}{r} 159920 \\ 132496 \\ \hline \end{array}$$

$$\begin{array}{r} 27424 \\ 26499 \\ \hline \end{array}$$

$$\begin{array}{r} 925 \\ 927 \\ \hline \end{array}$$

Therefore the true Root is 210,31207.

474. In the last Example, when the process is renewed, and 210 the first found part of the root is cubed, it is evident that the given number 9302348 has no true root in whole numbers; because there is a remainder 41348; but, by proceeding according to the Rules before given, the remaining part of the root ,31207 appears to be a Decimal Fraction.

475. If it should be thought necessary to find more places in the root, the same may be done by cubing the root 210,31207, and subtracting the cube thereof from the given number 9302348, with 3 times as many cyphers annexed as there are Decimal places in the root, and then proceeding as before.

476. The Extraction of the Cube-root is proved by multiplying the root cubically, to wit, the root being first multiplied by itself, and then the product multiplied by the root, the number arising, or last product, will be equal to the num-

ber proposed: So in the first Example of this Chapter, the Cube-root 54 being multiplied first by itself produces 2916, which being multiplied again by 54 produces 153432, to wit, the number whose Cube-root was required. But if, as in the second Example, the given number have not a perfect root in whole numbers; then the proof will be most expeditiously made by inverted Multiplication, as before in the Square-root,
Art. 449.

477. The Cube-root of a Fraction is found in this manner, *viz.* extract the Cube-root of the Numerator (according to the foregoing Rules,) which root reserve for a new Numerator; also the Cube-root of the Denominator shall be a new Denominator: Lastly, this new Fraction will be the Cube-root of the Fraction first proposed: So the Cube-root of $\frac{8}{27}$ is $\frac{2}{3}$; for the Cube-root of 8 is 2, for a new Numerator; also the Cube-root of 27 is 3, for a new Denominator. In like manner, the Cube-root of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently, that the Fraction whose Cube-root is required, must be in its least terms before any Extraction be made; for it frequently happens that the Fraction first given has not a perfect root, though, when such Fraction is reduced to its least terms, the root thereof may be extracted: So in this Fraction $\frac{16}{27}$ neither the Numerator nor Denominator has a perfect Cube-root, yet the said $\frac{16}{27}$ being reduced to its least terms $\frac{8}{27}$, the Cube-root of this may be extracted; for the Cube-root of 8 is 2 for a new Numerator, also the Cube-root of 27 is 3 for a new Denominator; so that the Cube-root of $\frac{8}{27}$ (which is equal to $\frac{16}{27}$) is found to be $\frac{2}{3}$.

478. The Cube-root of a Fraction, which has not a perfect Cube-root, may be found nearly in this manner, *viz.* reduce the Fraction given to a Decimal Fraction, by *Art. 322.* the more places are in the Decimal, the nearer will the root be found: But the Decimal must consist of ternaries of places, to wit, either of three, six, nine, or twelve, &c. places; then extract the Cube-root of the Numerator of that Decimal, as if it were a whole number, (according to the Rule before given,) which root found, shall be a Decimal expressing nearly the Cube-root of the Fraction proposed.

So if the Cube-root of $\frac{2}{3}$ were required, reduce the said $\frac{2}{3}$ to a Decimal, whose Numerator may consist of ternaries of places, to wit, into this, 666666666666, &c. then extracting the Cube-root thereof, I find .8735, which is very near the Cube-root of $\frac{2}{3}$.

479. The Cube-root of a mixt number commensurable to its root may be found in the same manner as in *Art. 477.* the mixt

mixt number being first reduced to an improper Fraction (by Art. 268.)

So the Cube-root of $12\frac{1}{27}$ is discovered to be $2\frac{1}{3}$, viz. reducing $12\frac{1}{27}$ to this improper Fraction $\frac{343}{27}$, the Cube-root of $\frac{343}{27}$ will be found $\frac{7}{3}$ or $2\frac{1}{3}$. And here the same caution is to be observed, as in Art. 477. viz. the fractional part of the mixt number, or the improper Fraction equivalent to the mixt number, must be expressed by a Numerator and Denominator in the least terms, before any Extraction is made.

480. When the mixt number, whose Cube-root is required, has not a perfect Cube-root, this character, $\sqrt[3]{}$ is usually prefixed before such mixt number; so the Cube-root of $2\frac{1}{3}$ is thus expressed $\sqrt[3]{2\frac{1}{3}}$. Likewise $\sqrt[3]{\frac{1}{3}}$ denotes the Cube-root of $\frac{1}{3}$, which is a Fraction whose Cube-root is inexpressible by any true or rational number: But if you desire to know the Cube-root near, of a mixt number which has not a perfect Cube-root, reduce the fractional part of the mixt number to a Decimal, (as in Art. 478.) and annex the Decimal so found to the Integers of the mixt number; then esteem the said Integers with the Decimal so annexed as one entire number, extract the Cube-root thereof, and from the root found, cut off always to the right-hand so many places as there were points over the said Decimal annexed, which places so cut off will be the fractional part of the root, and those remaining on the left-hand shall be the Integers of the root: So the Cube-root of $2\frac{1}{3}$ is 1.334, and somewhat more.

481. A quadrate or square number multiplied by itself produces a biquadrate number: So 4 multiplied by itself produces the biquadrate 16. Therefore if a number be proposed, and the biquadrate-root of it is required, first extract the quadrate or square-root of the said given number, and then extract the Square-root of that root for the biquadrate-root sought. Thus if 20736 be a number propounded, the biquadrate-root thereof will be found 12: For the Square-root of 20736 is 144, and the Square-root of 144 is 12. When the given number has not a perfect biquadrate-root, you are to annex quaternaries of cyphers, to wit, either 4, 8, 12, or 16, &c. cyphers, and then proceed as before; so you'll find the root near, whose fractional part will be a Decimal. Thus the biquadrate-root of 7 is near 1.62.

482. A TABLE of the Square and Cube-Roots of Numbers not exceeding 144.

N°	Squ. Root.	Cub. Root.	N°	Squ. Root.	Cub. Root.
1	1,000000	1,000000	37	6,082763	3,332222
2	1,414214	1,259921	38	6,164414	3,361975
3	1,732051	1,442250	39	6,244998	3,391211
4	2,000000	1,587401	40	6,324555	3,419952
5	2,236068	1,709976	41	6,403124	3,448217
6	2,449490	1,817121	42	6,480741	3,476027
7	2,645751	1,912933	43	6,557439	3,503398
8	2,828427	2,000000	44	6,633250	3,530348
9	3,000000	2,080084	45	6,708204	3,556893
10	3,162278	2,154435	46	6,782330	3,583048
11	3,316625	2,223980	47	6,855655	3,608826
12	3,464102	2,289428	48	6,928203	3,634241
13	3,605551	2,351335	49	7,000000	3,659306
14	3,741657	2,410142	50	7,071068	3,684031
15	3,872983	2,466212	51	7,141428	3,708430
16	4,000000	2,519842	52	7,211103	3,732511
17	4,123106	2,571282	53	7,280110	3,756286
18	4,242641	2,620741	54	7,348469	3,779763
19	4,358899	2,668402	55	7,416198	3,802953
20	4,472136	2,714418	56	7,483315	3,825862
21	4,582576	2,758923	57	7,549834	3,848501
22	4,690416	2,802039	58	7,615773	3,870877
23	4,795832	2,843867	59	7,681146	3,892996
24	4,898979	2,884490	60	7,745967	3,914867
25	5,000000	2,924018	61	7,810250	3,936497
26	5,099020	2,962496	62	7,874008	3,957892
27	5,196152	3,000000	63	7,937254	3,979057
28	5,291503	3,036589	64	8,000000	4,000000
29	5,385165	3,072317	65	8,062258	4,020726
30	5,477226	3,107232	66	8,124038	4,041240
31	5,567764	3,141381	67	8,185353	4,061548
32	5,656854	3,174802	68	8,246211	4,081656
33	5,744563	3,207534	69	8,306624	4,101566
34	5,830952	3,239612	70	8,366600	4,121285
35	5,916080	3,271066	71	8,426150	4,140818
36	6,000000	3,301927	72	8,485281	4,160168

The TABLE of Square and Cube Roots continued.

N°	Squ. Root.	Cub. Root.	N°	Squ. Root.	Cub. Root.
73	8,544004	4,179339	109	10,44031	4,776856
74	8,602325	4,198336	110	10,48809	4,791420
75	8,660254	4,217163	111	10,53565	4,805896
76	8,717798	4,235824	112	10,58301	4,820284
77	8,774964	4,254321	113	10,63015	4,834583
78	8,831761	4,272659	114	10,67708	4,848808
79	8,888194	4,290841	115	10,72381	4,862944
80	8,944272	4,308870	116	10,77033	4,876999
81	9,000000	4,326749	117	10,81665	4,890973
82	9,055385	4,344481	118	10,86278	4,904868
83	9,110434	4,362071	119	10,90871	4,918685
84	9,165151	4,379519	120	10,95445	4,932424
85	9,219544	4,396830	121	11,00000	4,946088
86	9,273618	4,414005	122	11,04536	4,959675
87	9,327379	4,431047	123	11,09054	4,973190
88	9,380832	4,447960	124	11,13553	4,986631
89	9,433981	4,464745	125	11,18034	5,000000
90	9,486833	4,481405	126	11,22497	5,013298
91	9,539392	4,497942	127	11,26943	5,026526
92	9,591663	4,514357	128	11,31371	5,039684
93	9,643651	4,530655	129	11,35782	5,052774
94	9,695360	4,546836	130	11,40175	5,065797
95	9,746794	4,562903	131	11,44552	5,078753
96	9,797959	4,578857	132	11,48912	5,091643
97	9,848858	4,594701	133	11,53256	5,104469
98	9,899495	4,610436	134	11,57584	5,117230
99	9,949874	4,626065	135	11,61895	5,129928
100	10,000000	4,641589	136	11,66190	5,142563
101	10,04988	4,657010	137	11,70470	5,155137
102	10,09950	4,672330	138	11,74734	5,167649
103	10,14889	4,687548	139	11,78983	5,180101
104	10,19804	4,702669	140	11,83216	5,192494
105	10,24695	4,717694	141	11,87434	5,204828
106	10,29563	4,732624	142	11,91638	5,217103
107	10,34408	4,747459	143	11,95826	5,229321
108	10,39230	4,762203	144	12,00000	5,241482

C H A P. XXXIV.

The RELATION of NUMBERS in QUANTITY.

483. Thus far single Arithmetic: Comparative Arithmetic ensues, which is worked by numbers, as they are considered to have relation one to another.

484. This Relation consists in quantity, or quality.

485. Relation in quantity is the reference or respect, that the numbers themselves have one to another: As when the comparison is made between 6 and 2, or 2 and 6; 5 and 3, or 3 and 5.

486. Here the terms or numbers proposed are always two, of which the first is called the Antecedent, and the other the Consequent: So in the first Example, 6 is the Antecedent, and 2 the Consequent: And in the second, 2 is the Antecedent, and 6 the Consequent.

487. Relation in quantity consists either in the difference, or else in the ratio or reason that is found between the terms propounded.

488. The Difference of two numbers is the remainder left after Subtraction of the less out of the greater: So 6 and 2 being the terms given, 4 is the difference between them; for if you subtract 2 from 6, the remainder is 4.

489. The Ratio or reason between two numbers is the Quotient of the Antecedent divided by the Consequent: So if it be demanded what ratio or reason 6 has to 2, I answer triple reason: For if you divide 6 the Antecedent, by 2 the Consequent, the Quotient is 3; 2 being contained just 3 times in 6. In like manner, there is a subtriple reason between 2 and 6; for if you divide 2 by 6, the Quotient is $\frac{1}{3}$, or (which is all one) $\frac{1}{3}$; because 6 being not once found in 2, there remains 2 for the Numerator, 6 the Divisor being the Denominator of the Fraction produced in the Quotient, by Art. 255.

490. This Ratio or reason of numbers is either equal or unequal.

491. Equal reason is the relation that equal numbers have to one another: As 5 to 5, 6 to 6, 7 to 7, &c.

492. Here the one being divided by the other, the Quotient is always an unit: For if it be demanded how often 5 is in 5, the answer is 1.

493. Unequal reason is the relation that unequal numbers have one to another; and this either of the greater to the less, or of the less to the greater.

494. Unequal reason of the greater to the less, is when the greater term is Antecedent: As of 6 to 2, 5 to 3, and the like.

495. Here the Quotient of the Antecedent divided by the Consequent is always greater than an unit: So 6 divided by 2, the Quotient is 3, and 5 divided by 3, the Quotient is $1\frac{2}{3}$.

496. Unequal reason of the less to the greater, is when the less term is Antecedent: As of 2 to 6, 3 to 5, &c.

497. Here the Quotient of the Antecedent divided by the Consequent is always less than an unit: So 2 divided by 6, the Quotient is $\frac{1}{3}$, or $\frac{1}{2}$; and 3 divided by 5, the Quotient is $\frac{3}{5}$.

498. Each of these kinds of unequal reason is again subdivided into five other sorts, or varieties, of which the three first are simple, and the other two are mixt.

499. The simple kinds of unequal reason are, 1. Manifold.
2. Superparticular. 3. Superpartient.

500. Manifold Reason of the greater to the less, is, when the Consequent is contained in the Antecedent divers times without any part remaining: As 4 to 2, 8 to 4, 16 to 8, which is also called double reason, because the less is contained twice in the greater; so 6 to 2 is triple reason, 8 to 2 is fourfold reason, &c.

501. Here the Quotient of the Antecedent divided by the Consequent is always a whole number: So 8 divided by 2, the Quotient is 4.

502. The opposite of this kind, *viz.* of the less to the greater, is termed Submanifold: Examples hereof are 2 to 4, 4 to 8, 8 to 16, &c. Likewise 2 to 6, 2 to 8, 2 to 10, &c.

503. Superparticular is, when the Antecedent contains the Consequent once, and besides an Aliquot part of the Consequent; that is, an half, a third, a fourth, or a fifth, part, &c. of the Consequent; as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here 3 divided by 2, the Quotient is $1\frac{1}{2}$, and 4 being divided by 3, the Quotient is $1\frac{1}{3}$. In like manner, 5 divided by 4, the Quotient is $1\frac{1}{4}$, and 6 divided by 5, the Quotient is $1\frac{1}{5}$; therefore I say 2 and half 2 (that is 1) constitute 3: So likewise 3 and one third part of 3 (*viz.* 1.) make 4, and so of the rest.

504. Here the Quotient of the Antecedent divided by the Consequent is a mixt number, whose whole part, as also the

Numerator of the Fraction annexed, is always an unit : As is observable in the Examples last-mentioned.

505. The opposite reason of this kind is Subsuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, &c.

506. Superpartient is, when the Antecedent contains the Consequent once, and besides several parts of the Consequent : As 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, &c. here 5 divided by 3, the Quotient is $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3, (viz. 2.) constitute 5.

507. Here the Quotient of the Antecedent divided by the Consequent is a mixt number, whose whole part being an unit, has always for the Numerator of the Fraction annexed to it a number composed of more units than 1 : So the comparison being made between 5 and 3, and 5 the Antecedent being divided by 3 the Consequent, the Quotient is $1\frac{2}{3}$.

508. The opposite of this reason is Subsuperpartient : Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like.

509. The mixt kind of unequal reasons are Manifold Superparticular, and Manifold Superpartient.

510. Manifold Superparticular reason is, when the Antecedent contains the Consequent divers times, and besides an aliquot part of the Consequent ; as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

511. Here the Quotient of the Antecedent divided by the Consequent, is a mixt number, whose whole part consisting of more units than one, has always an unit for the Numerator of the Fraction annexed to it ; so 5 divided by 2, the Quotient is $2\frac{1}{2}$, and 21 divided by 5, the Quotient is $4\frac{1}{5}$.

512. The opposite of this reason, is Submanifold Superparticular ; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, &c.

513. Manifold Superpartient is, when the Antecedent contains the Consequent several times, and besides, divers parts of the Consequent ; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, &c.

514. Here the Quotient of the Antecedent, divided by the Consequent, is a mixt number, whose whole part, as also the Numerator of the Fraction annexed to it, is always a number composed of more units than one : So 8 divided by 3, the Quotient is $2\frac{2}{3}$, and 28 divided by 5, the Quotient is $5\frac{3}{5}$.

515. The opposite here is Submanifold Superpartient ; as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

These are the several kinds or varieties of the ratios, or reasons that are found amongst numbers ; so that no two numbers

whatsoever can be named, but the ratio or reason between them is comprehended under one of these five kinds.

C H A P. XXXV.

The Relation of Numbers in Quality, viz. Arithmetical and Geometrical Proportion.

516. **R**EALTION in Quality (otherwise called Proportion) is either the reference or respect that the reasons of numbers have one to another; or else, which the differences of numbers have one to another.

517. Therefore here the terms proposed ought always to be more than two; for otherwise there cannot be a comparison of reasons or differences in the plural number.

518. This proportion is either Arithmetical, or Geometrical.

519. Arithmetical Proportion is, when several numbers differ according to an equal difference, as 2, 4, 6, 8; 10, &c. here 2 is the common difference between 2 and 4, 4 and 6, 6 and 8, 8 and 10, &c. also 1, 2, 3, 4, 5, 6, 7, 8, &c. differ by Arithmetical proportion, 1 being the common difference between them.

520. Arithmetical proportion is either continued or interrupted.

521. Arithmetical proportion continued, is, when divers numbers are linked together by a continual progression of equal differences. Such are the Examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, &c. And 100000, 200000, 300000, 400000, &c. This is also called Arithmetical progression.

522. Arithmetical proportion interrupted, is, when the progression is discontinued: As in these numbers, 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10, differ according to Arithmetical proportion; but so do not 4 and 8 differ; for 2 is the common difference betwixt 2 and 4, 8 and 10; whereas the difference betwixt 4 and 8 is 4. In like manner 8, 14, 17, 23, differ by Arithmetical proportion interrupted.

523. Arithmetical proportion may be continued either upwards or downwards.

.524. Upwards, when the terms of the progression increase, as these, 2, 4, 6, 8, 10, 12, &c. or those 1, 2, 3, 4, 5, 6, &c. And this last rank is more particularly termed Natural Progression.

.525. Here, when the first term is also the common difference of the terms, the last term being divided by the number of the terms, the Quotient will give you the first term of the rank : Again, in this case, the first term multiplied by the number of the terms produces the last term : So this rank 3, 6, 9, 12, 15, 18, 21, being proposed, in which 3 is both the first term, and also the common difference of the terms ; I say 21, the last term being divided by 7 the number of the terms, the Quotient is 3 the first term ; contrariwise the first term multiplied by 7 produces 21 the last term.

.526. Arithmetical proportion continued downwards, is, when the terms of the progression decrease : Such as are 35, 32, 29, 26, 23, 20 : And 40, 35, 30, 25, 20, 15, 10, 5.

.527. Here, when the last term is also the common difference of the terms, the first term being divided by the number of the terms, the Quotient will give you the last term : Again, the last term, multiplied by the number of the terms, produces the first term of the rank.

For Example, This rank 40, 35, 30, 25, 20, 15, 10, 5, being proposed, in which 5 is both the last term, and likewise the common difference of the terms ; I say 40, the first term being divided by 8 the number of the terms, the Quotient is 5 the last term : On the other side, 5 the last term being multiplied by 8, the product is 40 the first term.

.528. Three numbers being proposed that differ by Arithmetical proportion continued, the mean being doubled, is equal to the sum of the extremes : So if 5, 6, 7, be given, 6 being doubled, is equal to the sum of 5 and 7, the two extremes.

.529. Four numbers being given, that differ by Arithmetical proportion either continued or interrupted, the sum of the two means is equal to the sum of the two extremes : So 5, 6, 7, 8, being given, the sum of 6 and 7, the two mean numbers, is equal to the sum of 5 and 8, the two extremes ; and 8, 14, 17, and 23, being propounded, the sum of 14 and 17 being added together is equal to the sum of 8 and 23.

.530. In an Arithmetical progression, whose number of terms is odd, the mean term, being doubled, is equal to the sum of any two terms that are equally distant therefrom ; so if 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. be proposed, 6 the mean term being doubled, that is 12, is equal to 11 + 1 = 10 + 2 = 9 + 3, &c.

531. Hence the sum of an Arithmetical progression, whose number of terms is odd, will be equal to the mean term multiplied by the number of the terms : In the above Example, the mean term (6) multiplied by (11) the number of terms produces (66) the sum of the progression.

532. Otherwise, because half the sum of the greatest and least terms is equal to the mean : Therefore, the progression will be equal to the product of the number of the terms, multiplied by half the sum of the greatest and least terms.

533. In an Arithmetical progression, whose number of terms is even, the sum of the two mean terms, is equal to the sum of any two terms equally distant from them ; so if 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be proposed, $(5+6=)$ 11 the sum of the two mean terms is equal to $10+1=9+2=8+3$, &c.

534. Hence the sum of an Arithmetical progression, whose number of terms is even, will be equal to the product arising from multiplying the sum of the two mean terms by half the number of the terms : In the above Example 11 the sum of the two means being multiplied by 5, half the number of the terms, produces 55 the sum of the progression.

535. Otherwise, because the sum of the greatest and least term is equal to the sum of the two mean terms ; therefore the progression will be equal to the product of half the number of the terms, multiplied by the sum of the greatest and least terms.

536. Therefore, by comparing Art. 532. and 535. it will appear, that the sum of any Arithmetical progression will be equal to half the product, made by multiplying the sum of the greatest and least terms, by the number of the terms : For Example, 9 terms of the progression 1, 3, 5, 7, 9, 11, 13, 15,

$$17, \text{ is equal to } \frac{1+17 \times 9}{2} = \frac{18 \times 9}{2} = 9 \times 9 = 81.$$

537. If from the number of the terms of an Arithmetical progression, unity be subtracted, and the remainder be multiplied by the common difference of the terms, the product will be the difference between the greatest and least term.

This will appear from the generation of the progression ; supposing 6 were the first term, and 2 the common difference,

Then,

Then, to the first term	6
Adding the common difference	<u>2</u>
Makes the second term	8
To which add the common difference 2	<u>10</u>
Makes the third term	10
To which add the common difference 2	<u>12</u>
Makes the fourth term	12 &c.

Where it is evident, that the common difference is added as often as the number of terms less 1.

538. Hence, if to the least term be added the product of the common difference by the number of terms less 1; the sum arising will be the greatest term; and, on the contrary, if the said product be subtracted from the greatest term, the remainder will be the least term.

For Example, In the progression 3, 5, 7, 9, 11, 13, consisting of 6 terms; 5, the number of the terms less 1, being multiplied by 2, the common difference of the terms, produces 10, the difference between 3 and 13, the least and greatest terms; and therefore $3 + 10 = 13$, and $13 - 10 = 3$.

539. Hence, the sum of an Arithmetical progression will be equal to half the product of the common difference, number of the terms, and number of the terms less 1, multiplied continually; and added to the product of the least term and number of the terms; or subtracted from the product of the greatest term, and number of the terms.

For by Art. 536. the sum of the progression is equal to half the product, made by multiplying the greatest and least term, by the number of the terms; and the above will appear to be half the said product from Art. 538.

For Example, The sum of the Arithmetical progression whose least term is 3, common difference 2, and number of terms 6, will be $\frac{1}{2} \times 2 \times 6 \times 5 + 3 \times 6 = \frac{60}{2} + 18 = 48$. And the sum of the Arithmetical progression whose greatest term is 13, common difference 2, and number of terms 6, will be $6 \times 13 - \frac{1}{2} \times 2 \times 6 \times 5 = 78 - \frac{60}{2} = 48$.

540. When numbers have the same or an equal ratio or reason, they are said to be in Geometrical proportion: Thus 3, 6, 12, 24, 48, &c. are numbers in Geometrical proportion; each Antecedent being to its Consequent in proportion as the two first terms 3 and 6, viz. as 1 to 2.

541. Geometrical proportion is either continued or interrupted.

542. Geometrical proportion continued, otherwise called Geometrical progression, is when every Antecedent and its Consequent have the same ratio or reason as in the Example quoted in Art. 540. 3, 6, 12, 24, 48, &c. each Antecedent containing its Consequent twice. Note, The number (2) by which the progression is continued, is called the ratio of the progression.

543. Hence, in a Geometrical progression, each Antecedent is to its Consequent, as the first term is to the second, or as unity is to the ratio of the progression.

Note, All the numbers, comprehended between the first and last, are called mean proportionals; so 6, 12, 24, are three mean proportionals between 3 and 48.

544. Geometrical proportion interrupted, is, when the progression of like reason is discontinued, in such sort, that the four numbers being given, the like reason is not found between the second and third, that is between the first and second, and the third and fourth; of this sort are these numbers 2, 4, 16, 32; here, as 2 is to 4, so is 16 to 32; for they differ by double reason; but as 2 is to 4, so is not 4 to 16, for 4 and 16 differ by fourfold reason, 4 being contained 4 times in 16.

545. The numbers of Multiplication and Division are proportional; for in Multiplication, as 1 is to the Multiplier, so is the Multiplicand to the product, or as 1 is to the Multipli-cand, so is the Multiplier to the product: Again, in Division, as the Divisor is to 1, so is the Dividend to the Quotient: Or, as the Divisor is to the Dividend, so is 1 to the Quo-tient.

546. If the rank of powers in Art. 412. viz. 1, 2, 4, 8, 16, &c. be severally multiplied by the number 3, the products 3, 6, 12, 24, 48, &c. are a Geometrical progression, whose ratio is equal to the root of those powers.

547. Hence a rank of powers is a Geometrical progression, and if a Geometrical progression be divided by the first term, it quotes a rank of powers.

548. Hence also the least term and ratio of any Geometrical progression being given, any term (whose distance from the least is known) may be found; by raising the ratio of the progression to a power, whose index is the given distance, and multiplying that power by the first term of the progression.

For Example, The fifth term (48) of the above progression (whose distance from the first term is 4) is equal to (16) the fourth

fourth power of the ratio (2) multiplied by (3) the first term of the progression, for $3 \times 16 = 48$.

549. But if the greatest term of the progression be given; then divide it by the said power of the ratio, and the Quotient will be the least term.

For Example, If it be required to find the fifth term of a Geometrical progression decreasing, whose greatest term is 162, and ratio 3.

The distance of the terms being 4, raise 3 to its 4th power 81, then divide 162 the greatest term, by 81 the power, and the Quotient 2 will be the term required.

550. Any number of mean proportionals may be found between two given numbers, thus :

Divide the greater of the two numbers by the lesser, and extract that root of the Quotient whose Index is greater by unity than the number of means required, so shall the result be the ratio of the progression; from which the terms may be made by continual Multiplication. *Example*, Let it be required to find 3 mean proportionals between 2 and 162.

Then will $\sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = \sqrt[2]{9} = 3$ be the ratio.

And $2 \times 3 = 6$; $6 \times 3 = 18$; $18 \times 3 = 54$; the means required.

551. If three numbers be in Geometrical progression, the square of the mean term is equal to the product of the extremum terms : For Example, in the three numbers 3. 6. 12; the square of 6, viz. 36, is equal to the product of 3 and 12.

552. If four numbers be in Geometrical proportion, either continued or interrupted, the product of the two extremes will be equal to the product of the two means.

Example 1. The numbers 3. 6. 12. 24. are in Geometrical proportion continued, and $3 \times 24 = 72 = 6 \times 12$.

Example 2. The numbers 2. 4 :: 16. 32. are in Geometrical proportion interrupted, and $2 \times 32 = 64 = 4 \times 16$.

Hence, if, in any 4 numbers, the product of the two extremes, is equal to the product of the two means, those numbers are proportionals.

553. If 4 numbers 2 : 6 :: 7 : 21 are proportionals, that proportionality will continue among the numbers that arise by those changes of them in situation and otherwise, which follow, *viz.*

Case 1. In *Alternation*, i. e. if the two extremes, or the two means severally change places between themselves.

For

For Example $2 : 7 :: 6 : 21$ for $2 \times 21 = 7 \times 6$.

And $21 : 7 :: 6 : 2$ for $21 \times 2 = 7 \times 6$.

Case 2. In *Inversion*, i.e. if at the same time the two first and the two last change places between themselves.

For Example $6 : 2 :: 21 : 7$ for $6 \times 7 = 2 \times 21$.

Case 3. In *Composition*, i.e. if the first and second terms be added together, for a new first term; and the third and fourth terms be added together, for a new third term.

For Example $2+6 : 6 :: 7+21 : 21$.

That is $8 : 6 :: 28 : 21$ for $8 \times 21 = 6 \times 28$.

Case 4. In *Division*, i.e. if the difference of the first and second terms be made the first term; and the difference of the third and fourth terms be made the third term.

For Example $6-2 : 6 :: 21-7 : 21$.

That is $4 : 6 :: 14 : 21$ for $4 \times 21 = 6 \times 14$.

Case 5. In *Conversion*, i.e. if the sums or differences, mentioned in the two last Cases, be made the second and fourth terms, instead of the first and third.

For Example $2 : 2+6 :: 7 : 7+21$

And $2 : 6-2 :: 7 : 21-7$

That is $2 : 8 :: 7 : 28$ for $2 \times 28 = 8 \times 7$.

And $2 : 4 :: 7 : 14$ for $2 \times 14 = 4 \times 7$.

Case 6. In *Mixtion*, i.e. if the said sums, and differences, are mixed among the terms.

For Example $2+6 : 6-2 :: 7+21 : 21-7$.

That is $8 : 4 :: 28 : 14$ for $8 \times 14 = 4 \times 28$.

554. The successive differences of the terms of a Geometrical progression will form another Geometrical progression, having the same ratio.

Examp. 1. In the Geometrical progression 3. 6. 12. 24. 48. &c. if each Antecedent be subtracted from its Consequent, the remainders will be 3. 6. 12. 24. &c.

Examp. 2. In the Geometrical progression 2. 6. 18. 54. 162. 486. &c. whose ratio is 3, if each Antecedent be taken from its Consequent, the remainders will be 4. 12. 36. 108. 324. &c. in which rank of remainders the ratio is also 3.

555. The successive sums of the terms of a Geometrical progression, taken two and two, three and three, four and four, &c. will form other Geometrical progressions, having the same ratio. For Example, in the progression 1. 2. 4. 8. 16. 32. 64. 128. &c. the successive sums of the terms, taken two and two, will be 3, 6, 12, 24, 48, 96, 192, &c. the succes-

five sums of the terms, taken three and three, will be 7. 14. 28. 56. 112. 224, &c. The successive sums of the terms, taken four and four, are 15. 30. 60. 120. 240, &c. In all which ranks of successive sums, each Consequent contains its Antecedent twice; that is, each Antecedent is to its Consequent, as (1) the first term of the original progression, is to (2) its ratio.

556. Hence, let a Geometrical progression consist of never so many terms;

As the sum of the Antecedents,
To the sum of the Consequents :
So is unity,
To the Ratio of the progression.

557. Therefore, by Case 5. Art. 553.

As the sum of the Antecedents,
To the difference between the sums of the Consequents
and Antecedents :
So is unity,
To the Ratio less unity.

558. But the difference between the sums of the Antecedents and Consequents, will be the difference between the last and first terms; for all the intermediate terms are included in both sums: And the sum of the Antecedents is the sum of the Progression, wanting only the last term.

559. Therefore, As the sum of the progression less the last term,
To the difference between the extreme terms :
So is unity,
To the Ratio less unity.

560. Or by Case 1. Art. 553.

As the Ratio less unity,
To unity :
So is the difference of the extreme terms,
To the sum of the progression less the last term.

561. Hence, having either the greatest or least term, and the Ratio of a Geometrical progression, given; the sum or total of any proposed number of terms thereof, may be found by the following

Rule. If the least term of the progression be given, find the greatest by Art. 548. If the greatest be given, find the least by Art. 549. Then subtract the least term from the greatest, and

and divide the remainder by the number which is left by one than the Ratio : Lastly, to the Quotient add the greatest term of the progression, and the sum will be the answer.

Example 1. What is the sum of 5 terms of a Geometrical progression, whose least term is 3, and Ratio 2?

The fifth or greatest term of this progression is 48, by Art. 548.

Then $48 - 3 = 45$ = the difference of the extremes;

And $\frac{45}{2} = 22.5$ = the Quotient of that difference, when divided by the Ratio less unity.

Lastly, $45 + 48 = 93$ = the sum of the progression 3. 6. 12. 24. 48.

Example 2. What is the sum of 5 terms of a Geometrical progression, whose greatest term is 162, and Ratio 3?

The fifth or least term of this progression is 2, by Art. 549.

Then $162 - 2 = 160$ = the difference of the extremes;

And $\frac{160}{2} = 80$ = the Quotient of that difference, when divided by the Ratio less unity.

Lastly, $80 + 162 = 242$ = the sum of the progression 162. 54. 18. 6. 2.

562. If there be two Geometrical progressions, whose Ratio's are equal: Then,

As any term of the first progression;

To any term of the second;

So is any other term of the first, whose distance from the former is known,

To a term equally distant from the former in the second.

For since the Ratio's are equal, and the distance the same, and since by Art. 548, and 549. the respective terms in each progression are produced by multiplying or dividing by the same power of the Ratio; therefore the said terms must be in proportion to the numbers, so to be multiplied or divided.

563. Hence, As the least or greatest term of the first progression,

To the least or greatest term of the second;

So is the sum of any number of terms of the first,

To the sum of the like number of terms of the second.

For the wholes will be in the same proportion as their component parts.

564. Hence, in any Geometrical progression, if the sum of any number of terms be known, the sum of a like number of terms, the first of which is situated at a given distance from the first term of the former, may be found by the following proportion.

As unity,

To that power of the Ratio, whose index is the given distance :

So is the sum of the given number of terms,

To the sum of a like number of terms, beginning at the given distance.

Example. In the Geometrical progression 2. 6. 18. 54. 162. 486. 1458, &c. if the sum 26 of the three first terms be given, to find the sum of three other terms beginning with the fifth; then the distance of the first term from the fifth being four, it will be as 1 : 81 (the fourth power of the Ratio 3); so is 26 to 2106, the sum of 162, 486 and 1458.

C H A P. XXXVI.

RULES of PRACTICE by ALIQUOT PARTS.

565. **A**N Aliquot part takes its name from the *Latin* word *aliquoties*; for (according to *Euclid*) an aliquot part is such a part of a greater number as being taken (*aliquoties* or) certain times, precisely constitutes that greater number; so 3 is an aliquot part of 12; for 3 taken 4 times, exactly makes 12, without any excess or defect: In like manner, 4 is an aliquot part of 20, because 4 taken 5 times precisely makes 20; but 7 is not an aliquot part of 20, for 7 taken twice, wants of 20; and being taken thrice, exceeds 20: This kind of part last mentioned, is by *Euclid* called *pars aliquanta*, of which there will be no use in this place.

566. When the Rule of Three Direct has 1 or an integer for the first term, it is commonly called a Rule of Practice, either from the great use and practice of it in common affairs, or else because questions of this nature may be resolved by operations more speedy and practical than those of the Rule of Three.

567. The following numbers of shillings are aliquot parts of a pound, *viz.*

$$\begin{array}{ll} 10s. = \frac{1}{2}l. & 2s. = \frac{1}{10}l. \\ 5s. = \frac{1}{4}l. & 1s. = \frac{1}{20}l. \\ 4s. = \frac{1}{3}l. & \end{array}$$

568. Any even number of shillings is either $\frac{1}{10}$ of a pound (that is 2 shillings) or else is composed of $\frac{1}{10}l.$ (to wit, 2s.) taken certain times: So 8s. is composed of $\frac{1}{10}l.$ (or 2 shillings) taken four times: In like manner, 18s. is composed of $\frac{1}{10}l.$ taken 9 times.

569. When the price of 1, or an integer of what name soever, is 2 shillings, the price of as many integers as one will of that name is discoverable at first sight; to wit, by accounting the double of the figure which stands in the first place (towards the right-hand) of the said number of integers as shillings, and the rest of the said number as pounds: So 345 yards at 2 shillings the yard will cost 34*l.*
 10s. for the double of 5 is 10, which write down a-part as shillings; then esteeming the remaining figures towards the left-hand, to wit, 34, as pounds, the answer will be *Ans. 34l. 10s.* 34*l.* 10s. This contraction is nothing else, but dividing the number of integers, whose price is required by 10.

570. The reason of this will be evident if we argue thus, 345 yards at 20s. per yard amount to 345*l.*
 But the given price 2*s.* is $\frac{1}{10}$ th of 20*s.*

Therefore 345 yards at 2*s.* per yard will amount to $\frac{1}{10}$ of 345*l.*

More Examples hereof are these:

What cost 2057 at 2*s.* per oz.

<i>l.</i>	<i>s.</i>
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Answer 205 : 14

yards.
 What cost 120 at 2*s.* per yard.

<i>l.</i>	<i>s.</i>
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Answer 12 : 0

571. When the given price of 1, or an integer, is any even number of shillings greater than two shillings, multiply the number of integers, whose price is required, by half the given number of shillings, with this caution, that the double of the figure which arises, in the first place of the product be written a-part as shillings, and the rest of the product as pounds : So if it be demanded what 218 yards, at 8 shillings the yard, amount to, the answer will be found 87 l. 4s. For

I multiply 218 by 4 (which is half 8 the	Yards,
given number of shillings) saying 4 times 8	218
is 32 ; here the double of 2 (to wit, of that	
figure which is to possess the first place in the	
product) is 4, which set a-part as shillings,	
keeping 3 in mind for the three tens : Again,	l. s.
4 times 1 is 4, which with 3 in mind makes	4
7 : Lastly, 4 times 2 makes 8, therefore the answer to the	
question is 87 l. 4s. The reason of this contraction, is evident	
from Art. 568, 570. More Examples of this Rule are these	
following :	

What cost 436 yards, at 14s. per yard ?

7

l. s.
Answer 305 : 4

What cost 320 yards, at 18s. per yard ?

9

l. s.
Answer 288 : 0

572. Any odd number of shillings is either composed of $\frac{1}{10}l.$ (or 2s.) and of $\frac{1}{10}s.$ (or 1s.) or else it is composed of $\frac{1}{10}l.$ (or 2s.) taken certain times, and of $\frac{1}{10}l.$ (or 1s.) So 3s. is composed of 2s. and 1s. Also 7s. is composed of 2s. taken three times, and of 1s. Likewise 13s. is composed of 2s. taken six times, and of 1s.,

573. When the given price of 1, or an integer is an odd number of shillings, work for the greatest even number of shillings contained in that odd number, according to Art. 571, then for the odd shilling remaining, take $\frac{1}{10}$ of the number of integers whose price is required, (by Art. 107.) These two results added together, give the answer to the question : So,

if it be demanded what 234*s.* ounces, at 13*s.* the ounce, will cost, the answere will be found 1523*l.* 12*s.* For if I multiply 234*s.* by 6, (to wit, by half the remainder, when 1 is abated from 13, the given number of shillings,) there will arise 1406*l.* 8*s.* (by Art. 571.) Then taking $\frac{1}{6}$ of 234*s.* there will arise 117*l.* 4*s.* which being added to the former product, gives 1523*l.* 12*s.* for the answer to the question.

$$\begin{array}{r}
 & 2344 \\
 \times & 6 \\
 \hline
 & 1406 : 8 \\
 & 117 : 4 \\
 \hline
 \text{Ans. } & 1523 : 12
 \end{array}$$

More Examples of this Rule are these following:

What cost 345 yards, at 17*s.* per yard?

$$\begin{array}{r}
 L \quad s. \\
 276 : 0 = 345 \times \frac{3}{5} \\
 17 : 5 = 345 \times \frac{1}{5} \\
 \hline
 \end{array}$$

$$\text{Answ. } 293 : 5$$

What cost 739 yards, at 19*s.* per yard?

$$\begin{array}{r}
 L \quad s. \\
 665 : 2 \\
 36 : 19 \\
 \hline
 \end{array}$$

$$\text{Answ. } 702 : 1$$

574. This Example might have been performed otherwise thus:

$$\begin{array}{l}
 739 \text{ yards, at } 20s. \text{ per yard, amount to } 739l. 0s. \\
 \text{And } 739 \text{ ditto, at } 1s. \text{ per yard, amount } \left. \begin{array}{l} \\ \end{array} \right\} 36l. 19s. \\
 \text{to } \frac{1}{5} \text{ thereof } \left. \begin{array}{l} \\ \end{array} \right\}
 \end{array}$$

Therefore 739 yards, at 19*s.* per yard, amount $\left. \begin{array}{l} \\ \end{array} \right\}$ 702*l.* 1*s.* to the difference of those values $\left. \begin{array}{l} \\ \end{array} \right\}$

And by parity of reasoning, when the given price is 18*s.* or 16*s.* or 15*s.* per yard, from so many pounds as there are yards, subtract severally the $\frac{1}{10}$, $\frac{1}{9}$, or $\frac{1}{8}$ thereof, and the remainders will be the answers.

575. Also Questions of this kind may be solved, by using the Aliquot parts of a pound: Thus, in the second Example, since 17 is composed of 10+5+2, it may be performed as below:

$$\begin{array}{l} 345 \text{ yards at } 20s. \text{ per yard, amount to } 345l. \\ 345 \text{ ditto at } 10s. \text{ amount to } \frac{1}{2} \text{ of } 345l. = 172 : 10 \\ 345 \text{ ditto at } 5s. \text{ amount to } \frac{1}{4} \text{ of } 345l. = 86 : 5 \\ 345 \text{ ditto at } 2s. \text{ amount to } \frac{1}{10} \text{ of } 345l. = 34 : 10 \end{array}$$

Therefore 345 ditto at 17s. amount to 293 : 5

Examp. 2. What cost 928 yards, at 5s. per yard?

$$\frac{1}{2} \text{ of } 928 = 232l. \text{ the answer.}$$

Examp. 3. What cost 928 yards, at 6s. per yard?

$$\begin{array}{l} \frac{1}{2} \text{ of } 928 = 185 : 12 = \text{the amount at } 4s. \\ \frac{1}{10} \text{ of } 928 = 92 : 16 = \text{the amount at } 2s. \end{array}$$

Therefore 278 : 8 = the amount at 6s.

Note. Since 2s. is the $\frac{1}{2}$ of 4s. the amount at 2s. might have been found, by taking the $\frac{1}{2}$ of the amount at 4s.

576. When the price of 1, or an integer, consists of pounds and shillings, first multiply the number of integers, whose price is required, by the number of pounds in the said given price, and subscribe the product as pounds; then proceed with the shillings in the said given price, according to Art. 571, &c. and having subscribed that which arises under the aforesaid product of pounds, add them all together for the answer of the question: So, if it be demanded what 328 hundred-weight amounts to at 2l. 17s. per hundred weight, the answer will appear to be 934l. 16s. as by the operation is evident.

C. wt.

328

2

l. s.

$$\begin{array}{rcl} 328 \times 2 & = & 656 : 0 \\ 328 \times \frac{1}{10} & = & 32.8 : 8 \\ \frac{1}{10} \text{ of } 328 & = & 16 : 8 \end{array}$$

Answ. 934 : 16

More Examples to illustrate this Rule, are these following:

What cost 504 C. wt. at 7l. 12s. per C. wt.

l. s.

$$\begin{array}{rcl} 504 \times 7 & = & 3528 : 0 \\ 504 \times \frac{6}{10} & = & 302.4 : 8 \end{array}$$

Answ. 3830 : 8

What

What cost 129 C. wt. at 5 l. 7 s. per C. wt.

$$\begin{array}{rcl} & l. & s. \\ 129 \times & = 645 : 0 \\ 129 \times \frac{5}{16} & = 38 : 14 \\ \frac{5}{16} \text{ of } 129 & = 6 : 9 \end{array}$$

Ans. 690 : 3

577. Any number of pence under 12 is either an Aliquot part of a shilling, or else composed of Aliquot parts thereof ; so 3 pence is an Aliquot part, to wit, $\frac{1}{4}$ of a shilling. Likewise 4 is $\frac{1}{3}$ of 12 : But farther, 5 pence is composed of 2 Aliquot parts, to wit, of 3 pence, which is $\frac{1}{4}$ of a shilling, and of 2 pence, which is $\frac{1}{8}$ of a shilling ; all which will readily appear by the following Table.

Pence.	Aliquot parts of a shilling.
1	$\frac{1}{12}$ (or $\frac{1}{3}$ of $\frac{1}{4}$)
$1\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4} + \frac{1}{8}$
6	$\frac{1}{3}$
7	$\frac{1}{4} + \frac{1}{3}$
8	$\frac{1}{3} + \frac{1}{2}$
9	$\frac{1}{2} + \frac{1}{4}$
10	$\frac{1}{2} + \frac{1}{3}$
11	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

578. When the given price of 1, or an integer, is an Aliquot part of a shilling, divide the number of integers whose value is required by the Denominator of such Aliquot part ; so will the Quotient be the number of shillings that answers the question ;

question; which number of shillings (when there is occasion) may be reduced to pounds by the brief way of dividing by 20: So if it be required to know what 2686 ounces at 4 pence the ounce amount to; the answer will be found 44*l.* 15*s.* 4*d.* For since 4*d.* is an Aliquot part, to wit, $\frac{1}{4}$ of a shilling, divide 2686 by 3, so will the Quotient be 895 $\frac{1}{3}$ *s.* or 895*s.* 4*d.* which shillings being divided by 20, give 44*l.* 15*s.* 4*d.* for the answer to the question, as you see by the following operation:

$$\begin{array}{r}
 \text{Ex.} \\
 \hline
 2686 \\
 \hline
 \begin{array}{r}
 s. \quad d. \\
 20) 89 \mid 5 : 4 = \frac{1}{3} \text{ of } 2686s. \\
 \text{Ans. } 44 : 15 : 4
 \end{array}
 \end{array}$$

More Examples of this Rule are these following:

What cost 759 yards, at 6*d.* per yard?

$$\begin{array}{r}
 s. \quad d. \\
 20) 37 \mid 9 : 6 = \frac{1}{3} \times 759s. \\
 \text{Ans. } 18 : 19 : 6
 \end{array}$$

What cost 204 yards, at 1*d.* per yard?

$$\text{Ans. } 17s. = \frac{1}{12} \text{ of } 204s.$$

579. When the given price of an integer is composed of Aliquot parts of a shilling, divide the number of integers whose price is required, by the several Denominators of the Aliquot parts contained in the given number of pence, then add the Quotients together, and the sum will be the number of shillings which answer the question. So if it be demanded what 2347 yards of linen cloth will cost at 9*d.* the yard, the answer will be found 88*l.* 0*s.* 3*d.* For since 9*d.* is composed of 6*d.* and 3*d.* to wit, of the Aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a shilling, first divide 2347 by 2, (the Denominator of the Aliquot part $\frac{1}{2}$), so there arises 1173 $\frac{1}{2}$, or 1173*s.* 6*d.* Again, dividing the said 2347 by 4, (the Denominator of the other Aliquot part) there will arise 586 $\frac{1}{4}$ or 586*s.* 9*d.* which two Quotients being added together give

$$\begin{array}{r}
 \text{Yards.} \\
 \hline
 2347 \\
 \hline
 \begin{array}{r}
 s. \quad d. \\
 20) 176 \mid 0 : 3 \\
 \text{Ans. } 88 : 0 : 3
 \end{array}
 \end{array}$$

1760s. 3d. or 88L 0s. 3d. which is the answer of the Question.

More Examples to illustrate this Rule, are these :
What cost 782 yards, at 8d. per yard?

$$\begin{array}{r} s. \quad d. \\ 260 : 8 = \frac{1}{4} \text{ of } 782s. \\ 260 : 8 = \text{ditto}, \end{array}$$

$$20) 521 : 4$$

$$l. \quad s. \quad d.$$

$$\text{Answ. } 26 : 1 : 4$$

What cost 540 oz. at 11d. per oz.

$$\begin{array}{r} 180 = \frac{1}{4} \text{ of } 540s. \\ 180 = \text{ditto}. \\ 135 = \frac{1}{4} \text{ of } 540s. \end{array}$$

$$4915$$

$$l. \quad s. \quad d.$$

$$\text{Answ. } 24 : 15 : 0$$

580. Or the Work of this Example might stand thus, viz.,
540 yards, at 12d. per yard, amount }
to $\frac{1}{12}$ of 540L } = 27L 0s,

And 540 yards, at 1d. per yard, amount }
to $\frac{1}{12}$ of the last found sum 27L } = 2L 5s,

Therefore 540 yards, at 11d. per yard, amount }
to the difference of the above values } 24L 15s

After the same manner, when the given price is 10d. 9d. or 8d. from the amount at 1s. subtract severally $\frac{1}{5}$, $\frac{1}{4}$, or $\frac{1}{3}$ of the same ; and the remainders will be the answers.

581. 'Twill be more convenient, in some cases, where the price given is not an Aliquot part of a shilling, to divide the same into two or more such parts, that the first may be an Aliquot part of a shilling, and the second, &c. Aliquot parts of the former : For Example, if the value of 540 yards at 7d. per yard be required, the same may be found as follows :

540 yards, at $12d.$ per yard amount to $27l.$
 540 ditto, at $6d.$ amount to $\frac{1}{2}$ } $= 13l. 10s.$
 that sum }
 540 ditto, at $1d.$ amount to $\frac{1}{6}$ } $= 2l. 5s.$
 of the last sum }
 Therefore 540 ditto, at $7\frac{1}{2}d.$ amount to $15l. 15s.$

582. And when the Price consists of pence and farthings, the process may be of the same kind: For Example, required the value of 540 yards, at $7\frac{1}{2}d.$ per yard.

540 yards, at $12d.$ per yard, a- }
 mount to $27l.$
 540 yards, at $6d.$ per yard, } $= 13l. 10s.$
 amount to $\frac{1}{2}$ of $27l.$
 540 yards; at $1\frac{1}{2}d.$ per yard, } $= 3l. 7s. 6d.$
 amount to $\frac{1}{4}$ of $13l. 10s.$
 540 yards, at $\frac{1}{4}d.$ per yard, } $= 11s. 3d.$
 amount to $\frac{1}{8}$ of $3l. 7s. 6d.$
 Therefore 540 yards, at $7\frac{1}{2}d.$ per yard, }
 amount to $17l. 8s. 9d.$

583. When the given price of an integer consists of shillings and pence, first multiply the number of integers, whose value is required by the said given number of shillings, and subscribe the product as shillings; then divide the said number of integers by the several Denominators which are correspondent to the Aliquot parts contained in the given number of pence, and subscribe the Quotient or Quotients under the aforesaid product of shillings, all which being added together give the number of shillings which answers the question: So if it be demanded what 347 yards of cloth will cost at the rate of $7s. 10d.$ the yard, the answer will be found $135l. 18s. 2d.$ For first 347 being multiplied by 7 (the given number of shillings,) produces 2429 shillings, then dividing 347 by 2 and 3 severally, (because 10d. is composed of $\frac{1}{2}$ and $\frac{1}{3}$ of a shilling,) the Quotients will be $173\frac{1}{2}$ and $115\frac{2}{3}$, that is $173s. 6d.$ and $115s. 8d.$ Lastly, the sum of all is $2718s. 2d.$ or $135l. 18s. 2d.$

yards.		
347		
	s.	d.
$7 \times 347 = 2429 :$		
$\frac{1}{2}$ of 347 =	173	: 6
$\frac{1}{3}$ of 347 =	115	: 8
20) 2718 : 2		
	<i>l.</i>	<i>s.</i>
		<i>d.</i>
Answ. 135 : 18 : 2		

More

More Examples of this kind, are these:

What cost 540 yards, at 17s. 9d. per yard?

17

$$17 \times 540 = \begin{cases} 3780 \\ 540 \end{cases}$$

$$\frac{1}{2} \text{ of } 540 = \underline{\underline{270}}$$

$$\frac{1}{4} \text{ of } 540 = \underline{\underline{135}}$$

$$20) \underline{\underline{95815}}$$

$$\text{Answe. } 479l. 5s. 0d.$$

What cost 313 yards, at 14s. 6d. per yard?

14

$$14 \times 313 = \begin{cases} 1252 \\ 313 \end{cases}$$

$$\frac{1}{2} \text{ of } 313 = \underline{\underline{156 : 6}}$$

$$20) \underline{\underline{453|8 : 6}}$$

$$\text{Answe. } 226l. 18s. 6d.$$

584. But if the given price be between one shilling and two shillings, the process may be the same as in Art. 578, 579, &c. excepting only, that the value of the given integers at 1s. per yard, instead of being separated from the rest by a line, must be added up with the rest. For Example, required the value of 540 yards at 1s. 7½ d. per yard.

540 yards, at 12d. per yard, amount to .27

540 ditto 6d. per yard, amount to $\frac{1}{2}$ of 27 = 13:10

540 ditto 1½ d. ditto $\frac{1}{3}$ of 13:10 = 3:7:6

540 ditto ¾ d. ditto $\frac{1}{8}$ of 3:7:6 = 11:3

Therefore 540 ditto 19½ or 1s. 7½ d. amount to 44:8:9

Compare this Work with that in Art. 582.

585. When the price of an integer consists of shillings and pence, and that such shillings and pence jointly considered do make an Aliquot part of a pound, it will be a briefer way to divide the number of integers, whose value is required, by the Denominator of such Aliquot part, so will the Quotient give the answer to the question in pounds, and known parts of a pound. Thus, if it be demanded what 767 yards cost, at the rate

rate of 6s. 8d. the yard, the answer will be found 255*l.* 13*s.* 4*d.* For since 6s. 8*d.* is an Aliquot part, to wit, $\frac{1}{3}$ of a pound, divide 767 by 3, so there arises in the Quotient 255*l.*, or 255*l.* 13*s.* 4*d.* which is the answer of the question. Note, 3}767(255 : 13 : 4 That the Aliquot parts of a pound convenient for this Rule, are these expressed in the following Table.

<i>lb.</i>	<i>d.</i>	Aliquot parts of a pound.
6 :	8	$\frac{1}{2}$
3 :	4	$\frac{1}{3}$
2 :	6	$\frac{1}{6}$
1 :	8	$\frac{1}{8}$
1 :	4	$\frac{1}{12}$
1 :	2	$\frac{1}{24}$

586. And indeed every question, wherein the price consists of shillings and pence, may be solved by the Aliquot parts of a pound, and the Aliquot parts of them: For Example, what cost 540 yards, at 17*s.* 9*d.* per yard?

540 yards, at 20*s.* per yard, amount to 540*l.*

540 ditto, at 10*s.* amount to $\frac{1}{2}$ of 540*l.* = 270

540 ditto, at 5*s.* amount to $\frac{1}{4}$ of 270*l.* = 135

540 ditto, at 2*s.* 6*d.* amount to $\frac{1}{8}$ of } = 67*s.* 10*p.*
of 135*l.* }

540 ditto, at 6*s.* 3*d.* amount to $\frac{1}{16}$ of } = 6*s.* 15*p.*
of 67*s.* 10*p.* }

Therefore 540 yards, at 17*s.* 9*d.* amount to 479 : 5

587. But if the price were 1*l.* 17*s.* 9*d.* per yard, then draw no line under the 540*l.* but add it up with the rest; and 540 yards, at 1*l.* 17*s.* 9*d.* will amount to 1039*l.* 5*s.*

588. When the price is shillings and pence, and such shillings and pence consist of the same figure.

Multiply the given quantity by the shillings, taking always the $\frac{1}{2}$ of the product for the pence, the total of which divided by 20, gives the answer in pounds.

Example. What comes 731 ells of Holland to, at 7s. 7d. or 11s. 11d. per Ell?

$$\begin{array}{r} \text{731 ells, at 7s. 7d.} \\ \hline 7 \\ \hline 5117 \\ 426 : 05 \\ \hline 2|0) 554|3 : 05 \\ \hline 277 : 03 : 05 \end{array} \qquad \begin{array}{r} \text{731 ells, at 11s. 11d.} \\ \hline 11 \\ \hline 8041 \\ 670 : 01 \\ \hline 2|0) 871|1 : 01 \\ \hline 435 : 11 : 01 \end{array}$$

589. When the given price of 1, or an integer, consists of pounds, shillings, and pence, reduce the pounds and shillings into shillings, then proceed according to Art. 583. So 5s 7d. at 3l. 17s. 5d. per C. will be found to amount to 2001l. 4s. 5d. For having reduced 3l. 17s. into 77s. multiply 517 by 77, and set down the particular products; then for the 5 pence which is composed of the Aliquot parts $\frac{1}{4}$ and $\frac{1}{2}$ of a shilling, take $\frac{1}{4}$ and $\frac{1}{2}$ of 517, and subscribe the Quotients orderly under the aforesaid products: Lastly, adding all together the sum is 40024s. 5d. or 2001l. 4s. 5d. for the answer of the question.

$$\begin{array}{c} C. \\ \hline 517 \\ \hline 77 \times 517 = \left\{ \begin{array}{l} 3619 \\ 3619 \end{array} \right. \\ \frac{1}{4} \text{ of } 517 = \quad 129 : 3 d. \\ \frac{1}{2} \text{ of } 517 = \quad 86 : 2 \\ \hline 20) 4002 \mid 4 : 5 \\ \text{Answ. } 2001 : 4 : 5 \end{array}$$

More Examples of this Rule, are these following :

What cost 108 C. at 5l. 13s. 8d. per C.

$$\begin{array}{r} 113 \times 108 = \left\{ \begin{array}{l} 324 \\ 108 \\ 108 \end{array} \right. \\ \frac{1}{3} \text{ of } 108 = \quad 36 \\ \frac{1}{3} \text{ of } 108 = \quad 36 \\ \hline 20) 122716 \\ l. \quad s. \quad d. \end{array}$$

Answ. 613 : 16 : 0

What cost 84 C. at 2l. 10s. 6d. per C.

$$\begin{array}{r} 50 \times 84 = 4200 \\ \frac{1}{2} \text{ of } 84 = \quad 42 \\ \hline 20) 424 \quad | \quad 2 \quad (212 : 2 : 0 \quad \text{Answ.} \end{array}$$

590. Questions of this kind may also be performed, by multiplying the number of integers, whose value is required, by the given number of pounds ; and then proceeding according to Art. 586. for the shillings and pence.

Examp. 1. What cost 108 C. wt. at 5l. 13s. 8d. per C. wt.

$$\begin{array}{rcl} 5 \times 108 & = & 540 = \text{the amount at } 5l. \text{ per C. wt.} \\ \frac{1}{2} \text{ of } 108 & = & 54 = \text{ditto, at } 10s. \\ \frac{1}{3} \text{ of } 54 & = & 18 = \text{ditto, at } 3s. 4d. \\ \frac{1}{16} \text{ of } 18 & = & 1 : 16 = \text{ditto, at } 0s. 4d. \end{array}$$

Therefore 613 : 16 = the amount at 5l. 13s. 8d.

Examp. 2. What cost 306 C. wt. at 2l. 12s. 4½ d. per C. wt.

$$\begin{array}{rcl} 2 \times 306 & = & 612 \quad \text{for } 2l. \\ \frac{1}{2} \times 306 & = & 153 \quad \text{for } 10s. \\ \frac{1}{3} \times 153 & = & 50 : 12 \quad \text{for } 2s. \\ \frac{1}{12} \times 50 : 12 & = & 5 : 2 \quad \text{for } 4\frac{1}{2} d. \\ \frac{1}{6} \times 5 : 2 & = & 0 : 12 : 9 \quad \text{for } 0\frac{1}{2} d. \end{array}$$

Therefore 801 : 6 : 9 for 2l. 12s. 4½ d.

591. The best way of proving the Work of Questions in Practice, is to perform the same by another method: Thus the first example in Art. 590. and the second in Art. 589. mutually prove each other.

592. When the price of an Integer is given, and the price of many Integers of the same name together with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an Integer is required, the value of those Integers may be first found by some of the preceding Rules, and then for the price of $\frac{1}{4}$ of an Integer, take $\frac{1}{2}$ of the given price of an Integer; likewise for $\frac{1}{2}$ of an Integer, take $\frac{1}{4}$ of the said given price; also for $\frac{3}{4}$ of an Integer, take the sum composed of $\frac{1}{2}$ and $\frac{1}{4}$ of the said given price: So if it be demanded what 134 C. 3 qu. (to wit, 134 hundred weight, and $\frac{3}{4}$ of an hundred weight) of sugar will cost at 4l. 16s. 3d. per C. the answer will be found 648l. 9s. 8 $\frac{1}{4}$ d. as by the subsequent operation is manifest.

134 C. wt. 3 qu.

4

$$\begin{array}{lll}
 \frac{1}{4} \times 134 & = 536 & \text{for } 4l. \\
 \frac{1}{2} \text{ of } 134 & = 67 & \text{for } 10s. \\
 \frac{1}{2} \text{ of } 67 & = 33 : 10 & \text{for } 5s. \\
 \frac{1}{2} \text{ of } 33 : 10 & = 6 : 14 & \text{for } 1s. \\
 \frac{1}{4} \text{ of } 6 : 14 & = 1 : 13 : 6 & \text{for } 3d. \\
 \text{Add } \left\{ \begin{array}{l} 2 : 8 : 1\frac{1}{2} \text{ for } \frac{1}{2} C. \text{wt.} \\ 1 : 4 : 0\frac{3}{4} \text{ for } \frac{1}{4} C. \text{wt.} \end{array} \right. & & \\
 \hline
 \end{array}$$

Answer 648 : 9 : 8 $\frac{1}{4}$

593. Whenever the parts given are similar to any known parts of a pound, questions of this kind may be then advantageously performed, by first finding what the given Integer, and the parts annexed, would amount to, at 1l. for each Integer: For instance, in the above Example, when 1 C. wt. costs 1l. 3 qu. will cost 15s.

C. wt. qu.

Then 134 : 3 at 1l. per C. wt. amount to 134 : 15.

$$134 : 3 \text{ at } 4l. = 4 \times 134 : 15 = 539 : 0$$

$$134 : 3 \text{ at } 10s. = \frac{1}{2} \text{ of } 134 : 15 = 67 : 7 : 6$$

$$134 : 3 \text{ at } 5s. = \frac{1}{4} \text{ of } 67 : 7 : 6 = 33 : 13 : 9$$

$$134 : 3 \text{ at } 1s. = \frac{1}{8} \text{ of } 33 : 13 : 9 = 6 : 14 : 9$$

$$134 : 3 \text{ at } 3d. = \frac{1}{16} \text{ of } 6 : 14 : 9 = 1 : 13 : 8\frac{1}{4}$$

Therefore 134 : 3 at 4 : 10 : 3 amount to 648 : 9 : 8 $\frac{1}{4}$

594. But the advantage of this method will be very conspicuous, in the computing the values of gold and silver plate: for in *Troy weight* the ounce (which is the Integer in those kinds of computation) is divided into 20 penny-weights, so that at 1*l.* per oz. each penny-weight will be worth 1*s.* Again the penny-weight being divided into 24 grains, every grain will be worth an half-penny, and every two grains one penny; so that the value of the given quantity, at 1*l.* per oz. may be wrote down at sight.

Examp. 1. What cost 327 oz. 17 p.w. 20 gr. of gold, at 3*l.* 17*s.* 6*d.* per oz?

oz. p.w. gr.	<i>l.</i>	<i>s.</i>	<i>d.</i>
Now 327 : 17 : 20 at 1 <i>l.</i> per oz. amount to	327 : 17 : 10		

$$\begin{array}{rcl} \text{Then for } 3l. \text{ take } 3 \times 327:17:10 = & 983:13:6 \\ 10s. \quad \frac{1}{2} \text{ of } 327:17:10 = & 163:18:\frac{1}{2} \\ 5s. \quad \frac{1}{2} \text{ of } 163:18:\frac{1}{2} = & 81:19:\frac{5}{2} \\ 2s. 6d. \quad \frac{1}{2} \text{ of } 81:19:\frac{5}{2} = & 40:19:\frac{3}{4} \end{array}$$

Answer 1270 : 11 : 7 *$\frac{1}{4}$*

595. There are many persons who are absurd enough to propose, as a trial of the Arithmetical Skill of others, the multiplying of a number of pounds, shillings, and pence, by a number of pounds, shillings, and pence; the reader is therefore desired to take notice, that if any such person should propose to him the multiplying of 327*l.* 17*s.* 10*d.* by 3*l.* 17*s.* 6*d.* that he may proceed as in the above Example, and give the answer 1270*l.* 11*s.* 7 *$\frac{1}{4}$* *d.* for the product required.

Examp. 2. What cost 327 oz. 17 p.w. 21 gr. of silver, at 5*s.* per oz?

oz. p.w. gr.	<i>l.</i>	<i>s.</i>	<i>d.</i>
Now 327 : 17 : 21 at 1 <i>l.</i> per oz. amount to	327 : 17 : 10 <i>$\frac{1}{2}$</i>		
Therefore for 5 <i>s.</i> take $\frac{1}{4}$ of it =	$8\frac{1}{2}:19:5\frac{1}{2}+\frac{1}{8}$		

C. qu. lb.

596. What cost 218 : 3 : 24 at 5l. 15s. 7 $\frac{1}{4}$ d. per C.

$115 \times 218 =$	{	1090
$\frac{1}{2}$ of 218		218
$\frac{1}{4}$ of 109		218
$\frac{1}{8}$ of 27s. 3d.		109 d. far.
The Quotients arising for by Art. 592.—	{	27 : 3 : 0
$\frac{1}{2} C.$		4 : 6 : 2
$\frac{1}{4} C.$		57 : 9 : 3 $\frac{1}{2}$
14lb.		28 : 10 : 3 $\frac{3}{4}$
7lb.		14 : 5 : 1 $\frac{1}{4}$ +
3lb.		7 : 2 : 2 $\frac{1}{4}$ +
		3 : 1 : 0 +

$$20) 2532 \frac{1}{2} : 3 : 2 +$$

l. *s.* *d.*

$$\text{Answe. } 1266 : 2 : 3\frac{1}{2} +$$

The Example last-mentioned being (of those questions which ordinarily happen in trade) one of the hardest to be resolved by the Rule of Practice; therefore in order to explain the foregoing operation, you may observe; that the price of 218 C. 3 qu. is found after the manner of former Examples; then for 14lb. part of the 24lb. in the question, take $\frac{1}{2}$ of the price $\frac{1}{4}$ C. Likewise for 7lb. take half the price of 14lb. and so there yet remains 3lb. whose price is found by taking $\frac{3}{7}$ of the price of 7lb. viz. the price of 7lb. being very near 7s. 2 $\frac{1}{4}$ d. or 86 $\frac{1}{2}$ d. Multiply 86 $\frac{1}{2}$ by 3, and divide the Quotient by 7; so there arises 37d. or 3s. 1d. very near. Lastly, all being added together, the sum is found to be very near 2532s. 3 $\frac{1}{2}$ d. or 1266l. 2s. 3 $\frac{1}{2}$ d.

Note. That a quarter of a farthing (or $\frac{1}{16}$ of a penny) is the smallest money expressed in the Example; and where any thing arises less than a quarter of a farthing, it is omitted, but it is supposed to follow this note +, for which surpluses some respect ought to be had in adding all together: Now although in resolving questions after this practical manner there will be some error, yet the loss for the most part will be less than a farthing, which is inconsiderable.

597. Altho' questions of the last-mentioned kind cannot be performed by the method in Art. 593. because the division of a quarter of an hundred weight into 28 pounds, has nothing similar to it in the subdivisions of a pound Sterling; yet if the Decimal, equivalent to the given parts of an C. wt. (taken from

the Tablet explained in Art. 350. viz. ,75 for the 3 quarters and ,2142857 for the 24 lb. making together ,9642857) be annexed to 218 the number of hundred weights, the number (218,9642857) so arising will express, in pounds Sterling and decimal parts of a pound Sterling, the value of 218 C. 3 qu. 24 lb. at 1 l. Sterling, per C. wt. and the operation may be as follows :

The given Quantity.

At 1 l. per hundred weight amounts to 218,9642857

For 5l. take $5 \times 218,9642857 = 1094,8214085$

For 14s. take $\frac{1}{3}$ thereof = $7 \times 21,8964286 = 153,2750002$

For 1s. take $\frac{1}{10}$ thereof = $\frac{1}{10}$ of 21,8964286 = 10,9482143

For 6d. take $\frac{1}{2}$ of 10,9482143 = 5,4741071.

For 1 $\frac{1}{2}$ d. take $\frac{1}{4}$ of 5,4741071 = 1,3685268

For $\frac{1}{4}$ d. take $\frac{1}{8}$ of 1,3685268 = 0,2280878

Then will their sum be the answer = 1266,1153447

Which (when the decimal part is reduced to shillings and pence) is 1266l. 2s. 3 $\frac{3}{4}$ d. nearly.

598. When the price of 1 lb. weight is known, and the price or value of 1 C. (to wit 112 lb.) is required, the answer may sometimes be given more speedily, than by any of the former Rules, by this Rule which follows, viz. find the number of farthings contained in the given price of 1 lb. weight, then take twice that number of shillings, and once that number of groats, and having added them together, the sum will give the value of 1 C. to wit, 112 lb. weight: So if it be demanded what 1 C. or 112 lb. weight of cheese will cost at the rate of 3 $\frac{1}{4}$ pence the pound weight, the answer will be 1l. 10s. 4d.

For according to the said Rule, the number of farthings contained in 3 $\frac{1}{4}$ d. (the price of 1 pound weight) l. s. d. is 13, therefore the double of 13 shillings is — 1 : 6 : 0
13 groats make 0 : 4 : 4

Therefore the sum (which is the price of { 1 : 10 : 4
of 1 C. or 112 lb. weight) is 1 : 10 : 4

The reason of this Rule is as follows, if 1 lb. weight cost 13 farthings, then 112 lb. must necessarily cost 112 times 13 farthings, or (which is the same) 13 times 112 farthings; but 13 times 112 farthings, are equal to twice thirteen shillings together with once thirteen groats; because 112 farthings are composed of twice 48 farthings (or two shillings) and of 16 farthings (or one groat;) wherefore the truth of the said Rule is evident.

Another

Another Example. When sugar is at $5\frac{1}{2} d.$ the pound weight, what is the value of 1 C. (or 112 lb. weight?) *Answer 2l. 11s. 4d.* For in $5\frac{1}{2} d.$ are contained 22 farthings, 1. s. d. therefore the double of 22 shillings is _____ 2 : 4 : 9
22 groats make _____ 0 : 7 : 4

Which added together, give the price of 1 C. }
or 112 l. to wit _____ 2 : 11 : 4

599. Here follow the quantities of particular Goods in Wholesale-trade, found in the Book of Rates, with concise and ready ways of casting up the same, deduced from the same principle with that in Art. 598.

600. Goods sold by sixscore to the hundred, are Lamb-skins, Bärlings, Balks great, middle, and small; Clap-boards, Pipe-boards, Bow-staves, Cap-ravens, Deal-boards, Spars of all sorts, Cod, Cole, Ling, and all sorts of Stock-fish; with many sorts of linen, *viz.* Hamburg, Silesia, Irish, Muscovia, Westphalia, Hanover, &c.

If the price of one of any of those sorts of goods be given to find the price of an hundred.

Rule. Divide the number of farthings contained in the price of one by 8, and the Quotient will be the price of 120 in pounds. *Note,* Reckon half a crown for every one that remains, after Division.

Example. What cost 120 ells of Canvas at $5\frac{1}{2} d.$ per Ell?
 $5\frac{1}{2} d.$ is 22 farthings, and $\frac{22}{8} = 2\frac{1}{4}$; therefore 120 will cost 2l. and 6 half-crowns, or 2l. 15s.

601. Goods sold by the thousand, are Paste-boards for Books, Tennis-balls, Lemons, Oranges, Teazels, Flanders-paving and pan-tiles, Lantern-horns, Barrel-hoops, and Boards; Quills, Lamperns, Squirrel-skins, Ox-bones, Yards of Lift, &c.

If the price of one of any of those sorts of goods be given to find the price of a thousand.

Rule. To as many pounds as there are farthings in the price of 1, add the same number of ten-pences, so shall the sum be the price of a thousand.

Example. What cost a thousand of Tiles at 3 d. each?

3 d. is 12 farthings: Then to	12 l.
Add 12 ten-pences = 10 twelve-pences =	<u>0 l. 10 s.</u>
Therefore a thousand will cost	12 l. 10 s.

602. Goods sold by the great Gros, containing 12 small Gros, or 144 dozen, *viz.*

Buttons of Metal, Glafs, Thread, Silk or Hair; Beads of Bone, Box, Glafs or Wood; Cap-hooks, Chess-men; To-

bacco-pipes, Combs of Light or Box-wood, Thread or Silk-points, Playing-cards, &c.

If the price of 1 dozen of any of these sorts of goods be given to find the price of a great gross.

Rule. Multiply the farthings contained in the price of 1 dozen by 3, and the product will be the price of a great gross in shillings.

Example. What cost a great gross of Buttons, at $8\frac{1}{2}d.$ per dozen?

$$8\frac{1}{2}d. \text{ is } 34 \text{ farthings; and } 34 \times 3 = 102s. = 5l. 2s.$$

Therefore a great gross will cost 5l. 2s.

603. Goods sold by the small gross of 12 dozen, viz. Tobacco and Pepper-boxes, Nest and Tinder-boxes, Buckles of some sorts, Wash-balls, Ink-horns, Bodkins; several sorts of Knives, Comb-caſes, &c.

Goods of this quantity may be cast up by the last preceding Rule, ſeeing there are as many particulars in a ſmall gross, as dozens in a great one.

604. Goods ſold by the fiftyscore to the hundred, are Annotto, Capers, Safflowers, Thrums, Ginger, Cloves, Indico, Croſs-bow-thread, Pack-thread, Kids or Goats-hair, Quick-silver, Cotton-wool, English Hard-wax, Brafs and Latten-work, as Chafing-dishes, And-irons, Laver-cocks, &c.

If the price of 1 of any of these sorts of goods be given, to find the price of an hundred.

Rule. To twice as many shillings as there are farthings in the price of one, add once as many pence, ſo ſhall the ſum be the price of an hundred.

Example. What cost 100 balls of Pack-thread at $2\frac{1}{2}d.$ per ball?

$$2\frac{1}{2}d. \text{ is } 10 \text{ farthings; then to } 10 \times 2 = 20s.$$

add $10 \times 1 = 0s. 10d.$

And an hundred will cost 20s. 10d.

605. Questions to exercise the Rules of Practice:

	<i>l.</i>	<i>s.</i>	<i>d.</i>
75C. 2q. 19lb. at $4\frac{1}{2}d.$ per lb.	Answer 158	18	$1\frac{1}{2}$
7108lb. at 35s. 6d. per C.	Answer 1126	10	1
67108oz. at $16\frac{1}{4}d.$ per lb.	Answer 283	12	$8\frac{1}{2}$
71lb. 10oz. 15p.w. at 5s. 9d. per oz.	Answer 248	0	$9\frac{1}{2}$
319 yards, at 5s. 6d. per ell.	Answer 70	3	$7\frac{1}{2}$
419 $\frac{1}{2}$ yards, at 4s. 10 $\frac{1}{4}$ d.	Answer 101	13	$9\frac{1}{2}$
47 quar. at 4s. 9d. per bush.	Answer 89	6	0
715 $\frac{1}{4}$ yards at 4s. 10 $\frac{1}{4}$ d.	Answer 173	4	$5\frac{1}{2}$

	l.	s.	d.
59 oz. 11 p.w. 18 gr. at 5s. 7d. per oz.	Answer	16	12 : 8
50 $\frac{3}{4}$ lb. of silk at 20d. per oz.	Answer	67	13 : 4
66 gross at 3s. 9d. per dozen.	Answer	148	10 : 0
56 load of hay at 18d. per truss.	Answer	151	4 : 0

C H A P. XXXVII.
Of TARE and TRET, &c.

606. **T**HESE allowances are called beyond sea, *The courtesies of London*, because not practised in any other place; concerning which observe, That,

607. Gross is the weight of the commodity, and that which contains it without any deduction. And Tare is an allowance for that which contains the commodity, whether it be Bag, Barrel, Sack, Frail, &c. and is either,

1. At so much *per Bag, Barrel, Frail, &c.*
2. At so much *per cent.* or,
3. By so much of the gross weight, called Invoice Tare.

608. When the Tare is so much *per bag*, as Almonds; *per barrel*, as Oil in candy barrels; *per frail*, as Raisins, &c.

Rule. Multiply the number of the said bags, barrels, frails, hogsheads, &c. by the allowance of Tare, which product subtract from the gross, and (when no Tret is mentioned) the remainder is neat.

Example. What is the neat weight of 38 hogsheads of Tobacco, weighing gross 102 : 2 : 17, Tare *per hogshead* 70l.

C.	q.	l.	38
102	: 2	: 17	
112			2660
<hr/>			
1297			
<hr/>			
11497	Gross		
2660	Tare		
<hr/>			
8837	Neat		

609. When the Tare is at so much *per cent.* and that,
 An Aliquot part, or parts of the hundred weight;
 As Figs, Almonds, Argol, &c.
 Caroteels, and butts of Currants, &c. 14 }
 Oil in uncertain casks, &c. 16 } per Cent.
 18 }

Rule. Divide the whole gross by the said part or parts that the Tare is of an hundred, gives the Tare of the whole; for such a part of the gross must be the Tare of the whole, as the given Tare is of an hundred.

Example. What is the neat weight of 15 barrels of Argol, gross $C. 51 : 2 : 18$. Tare 14 *per cent.* which being the 8th of an hundred, take the 8th of the gross for Tare,

$$\begin{array}{r} C. \quad q. \quad lb. \\ 8) 51 : 2 : 18 \qquad \text{Gross} \\ \underline{6 : 1 : 23 : 4} \qquad \text{Tare} \\ 45 : 0 : 22 : 12 \qquad \text{Neat.} \end{array}$$

Example 2. How many neat hundreds in 18 quarter-rolls of Currants, each gross 8 : 3 : 10. Tare 21 *l. per cent.*

$$\begin{array}{r} C. \quad q. \quad lb. \\ 8 : 3 : 10 \quad \text{each} \\ \underline{3} \\ 26 : 2 : 02 \\ \underline{6} \\ lb. \\ 14 \frac{1}{2}) \quad \underline{159 : 0 : 12 \text{ oz. gross}} \\ 7 \frac{1}{2}) \quad \underline{19 : 3 : 15 : 8} \\ \underline{9 : 3 : 21 : 12} \\ 29 : 3 : 09 : 04 \quad \text{Tare} \\ \underline{129 : 1 : 02 : 12} \quad \text{neat.} \end{array}$$

610. But if the Tare is not readily found to be an Aliquot part or parts of a hundred.

Rule. Multiply the pounds gross by the allowance of Tare, dividing the product by 112, the quotient is the Tare of the whole.

Example. What is the neat produce of 12 barrels of pot-ashes, each gross 203 *l.* Tare 10 *l. per cent.*

$$\begin{array}{r}
 203 \\
 120 = 12 \times 10 \\
 \hline
 112) 24360 \quad (217 \quad 2436 \quad \text{gross} \\
 \hline
 196 \quad \quad \quad 217 : \frac{1}{2} \text{ Tare} \\
 \hline
 840 \\
 \hline
 56 \quad \quad \quad 2218 : \frac{1}{2} \text{ neat.}
 \end{array}$$

611. If the allowance for Tare is not by the hundred, but according to the gross weight, as Sumach in bags, bales or fangots of *Aleppo*, *Cyprus*, and *Smyrna* silk, &c. such Tare is called Invoice Tare; of which the Book of Rates, in the Table of Allowances for Tare, says, *viz.*

Cyprus and Smyrna silk.

Bales	about or above 300 lb.	allowed	lb.
from 300 to 200	for Tare.	16	16
from 200 downwards		14	14

Tobacco from *Virginia*.

Hogsheads	5 C. and upwards	allowed	lb.
from 4 to 5,	for Tare.	100	100
from 3 to 4,		90	90
under 3 hundred		80	80

Example. What is the neat weight of four hogsheads of *Virginia* tobacco, weighing gross and number'd? *viz.*

N. ^o .	G.	q.	lb.	lb.
27	2	: 2 : 19		70
9	5	: 0 : 15	Tare found	100
4	3	: 2 : 11	as above.	80
19	4	: 2 : 19		90

$$\begin{array}{r}
 16 : 0 : 08 \quad \text{gross } 112) 340 (3 \\
 3 : 0 : 04 \quad \text{Tare} \quad \quad \quad 4
 \end{array}$$

13 : 0 : 04 neat, and so of Silk, &c.

Also

Also Sugar from India.

In casks and canisters,
In chests, and casks, St. Thom. } Tare { $\frac{1}{6}$
{ $\frac{1}{3}$

612. Tret, is an allowance of 4 lb. per 104 , on some sort of goods, (to the buyer by the merchant) for the Dust and Waste in them; which when allowed, after you have subtracted the Tare from the gross, the remainder is suttle, or suttile.

Rule. This suttle take out, and divide by 26, because the allowance of 4 lb. per 104 , is $\frac{1}{26}$ part, and the quote is the Tret, which Tret subtract from the suttle, and the neat remains.

Example. How many pound neat in 12*C.* 3*q.* 12*lb.* gross, Tare 2*lb. per cent.* Tret 4 lb. per 104 ?

$$\begin{array}{rcl}
 26) 1415 (54 & C. q. lb. \\
 \hline
 & 115 & 12 : 3 12 \\
 & \hline
 & 11 & 112 \\
 & \hline
 & 8) 1440 gross & \\
 & 7) 180 & \\
 & \hline
 & 25 + \text{Tare} & \\
 & \hline
 & 1415 suttle & \\
 & 54 \text{ Tret} & \\
 & \hline
 & 1361 neat, &
 \end{array}$$

Otherwise, to find and deduct Tret.

Rule. Add 2 cyphers to the suttle pounds, which number divide by 104, and the quote is the neat pounds.

Example. What is the neat weight of 4 Bar, of Spanish Tobacco? containing, viz.,

Nº C. q. lb.

4	1 : 0 : 04	Tare 14 per cent.
7	0 : 3 : 25	
9	1 : 0 : 05	
3	0 : 2 : 26	

3 : 3 : 04
112

8) 424 gross
53 Tare

104) 37100 (356 + neat.

613. Cloff, is another allowance of 2 lb. weight, to citizens of London, on every draught above three hundred weight, on some goods, as Galls, Madder, Sumack, Argol, &c.

614. Neat, is the pure weight of the commodity, when all given allowances are deducted.

615. As for other allowances not so common, as Break, Damage, &c. the first being generally at so much per Cask, Bale, Bag, &c. the other so much in the whole, can have no difficulty.

Example. What is the neat weight of 7 hogheads gross, 28 C. 19 lb. Tare 16 per cent. Break 8 per hoghead, Damage 98½?

C.	q.	lb.	
7)	28 : 1 : 19		gross
<hr/>			
4 : 0 : 06 : 11			Tare
<hr/>			
24 : 1 : 12 : 5			Break
<hr/>			
23 : 3 : 12 : 5			Damage
<hr/>			
22 : 3 : 25 : 13			neat.

616. In Oil the Tare is of two sorts.

- (1.) On Candy barrels _____ at 29 lb. per barrel.
- (2.) On uncertain casks _____ at 18 lb. per cent.

The neat pounds of which are found by the foregoing Rules.
But to reduce those pounds into gallons;

Rule. Multiply those pounds neat by 2, the product of which divide by 3, and that quote by 5, gives neat gallons, $\frac{7}{5}$ lb. making a gallon.

Example. Oil, gross 127 C. 3 q. 7 lb. Tare 18 lb. per cent.
How many neat gallons?

$$\begin{array}{r}
 \text{C.} \quad \text{q.} \quad \text{lb.} \\
 127 : 3 : 7 \\
 \hline
 112 \\
 \\
 \hline
 1615 \\
 \\
 \hline
 7) 14315 \text{ gross} \\
 8) 2045 \\
 255 \\
 \hline
 2300 \text{ Tare} \\
 \\
 \hline
 12015 \text{ neat pounds} \\
 2 \\
 \hline
 3) 24030 \\
 5) 8010 \\
 \hline
 1602 \text{ gallons.}
 \end{array}$$

C H A P. XXXVIII.
*Concerning EXCHANGES of COINS, WEIGHTS, and
 MEASURES.*

617. THE rate or proportion between Coins, Weights, &c. of different kinds being known, either from some good author, or rather by experience; it will not be difficult, to such as understand the before-going Rules, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind: But since in some cases the common manner of working may be much contracted, it will be convenient to shew the most compendious ways to perform this business.

618. In exchanging of things of different kinds, (whether they be Coins or Weights, &c.) when two things of different kinds are compared together, the question may be resolved by one single Rule of Three, as will be evident by the subsequent Examples, *viz.*

Quesⁿ. 1. How many Riders at 21s. 2½d. Sterling the piece, ought to be received for 251l. 6s. 4½d. of Sterling money? Answer 237 Riders. For the first and third terms in the Rule of Three, which arise from this question, being converted into half-pence, the proportion will be this:

$$509 : 1 :: 120633 : 237.$$

Quesⁿ. 2. If 100 ells of *Antwerp* make 75 yards of *London*, how many yards of *London* measure will 27 ells of *Antwerp* make? Answer 20½ yards.

$$100 : 75 :: 27 : 20\frac{1}{2}.$$

619. In *London*, and throughout *England*, accounts are kept in pounds, shillings, and pence Sterling, and are cast up as *Flemish* and *French* money are, by 12 and 20, and the Exchange made with all places by pence, *i.e.* giving so many pence Sterling for the pieces on which the Exchange is made, except

1. *Portugal*, and with it in shillings and pence, on their Milrea.
2. *Antwerp*, *Hamburg*, &c. Countries bordering upon *Flanders* and *Holland*, and with them by the pound of 20s. Sterling.
3. *Ireland*, and the *American* plantations, by the hundred pound.

620. In Italy,

*Genoa, Leghorn, Venice, &c.**Genoa and Leghorn exchange with London by the dollar, or piece of eight; Venice by the ducat.**Genoa and Leghorn keep their accounts in livres, sols, and deniers; 5 livres is a piece of eight at Genoa, and 6 at Leghorn.*

621. In Spain,

*Madrid, Cadiz, Bilboa, &c.**Exchange also by the piece of eight.*

622. In Portugal;

*Lisbon, and Oporto, &c.**Exchange on the Milrea.*

623. In France,

*Paris, Lyons, and Rouen.**Exchange by the Crown.**But keep their accounts in Livres, Sols, and Deniers;*

12 Deniers	make	1 Sol.
12 Sols		1 Livre.
3 Livres		1 Crown.

624. In the Netherlands,

*Antwerp, Brussels, Amsterdam, Rotterdam, Hamburgh, &c.**Exchange with London by shillings and pence Flemish. Accounts are in these places kept in Flemish pounds, shillings and pence; by some in guilders, stivers, and pennicks. The Flemish pounds, shillings, and pence, are divided as our moneys, viz. one pound into 20s. and one shilling into 12d. But,*

16 Pennicks,	one	Stiver.
6 Stivers,		Flemish Shilling.
20 Stivers,		Guilder.
6 Guilders,		Flemish Pound.
30 Stivers,		Common Dollar.
50 Stivers,		Specie Dollar.
63 Stivers,		Ducatoon.

625. Italian, Spanish, and French Exchange.

*Case 1. The quantity of Sterling money being given.**Rule. Take the aliquot parts for the pence of the Exchange out of a pound, dividing the given crowns by those parts, the total of which is the answer, as in Art. 586.**Example. How much Sterling must I pay here to receive in France 479 crowns, Exchange at 52 d. Sterling per crown?*

$4s. \frac{1}{2} 479$ crowns.

$$\begin{array}{r} 4 \quad \frac{1}{2} \quad 95 : 16 \\ \quad 7 : 19 : 8 \\ \hline 103 : 15 : 8 \end{array}$$

Example (2.) How much Sterling for 7439 French crowns Exchange, at 55 pence per crown?

s.	d.	7439.
2	6	$\frac{1}{4}$
1	8	$\frac{1}{2}$
5		$\frac{1}{4}$
$\frac{1}{2}$		$\frac{1}{8}$
		$929 : 17 : 06$
		$619 : 18 : 04$
		$154 : 19 : 07$
		$19 : 07 : 05 : \frac{1}{8}$
		$1724 : 02 : 10 : \frac{1}{8}$

Case 2. The quantity of Foreign money being given.

Rule. Use the universal Rule of Proportion, Art. 407. as in Example 22. annexed thereto.

Example. How many French Crowns must I give for 103L 15s. 8d. Sterling: Exchange at 52d. Sterling per French Crown?

Fr. Cr.	l.
Conditional terms	$1 \frac{52}{240}$
Corresponding	A $\frac{6227}{60}$. For 103L 15s. 8d. = 6227 groats.

$$\text{Therefore } A = \frac{1 \times 6227 \times 240}{60 \times 52} = \frac{6227 \times 4}{52} = \frac{6227}{13} = 479 \text{ crowns.}$$

626. Of Flemish and German Exchange.

627. First. To reduce Sterling into Flemish pounds.

Rule. Take the aliquot parts for what the rate of the Exchange is above a pound, dividing the given Sterling by the said parts, the total of which, with the given Sterling, is the Flemish money, as in Art. 587.

Example 1. One in Antwerp delivering money by Exchange for London at 35s. 6d. Flemish per pound Sterling; how much must he pay there to receive here 597L Sterling?

$$\begin{array}{r} 10s. \frac{1}{2} & 597 \\ 5 \frac{1}{2} & 298 : 10 \\ 6 \frac{1}{2} & 149 : 05 \\ & 14 : 18 : 6 \end{array}$$

$$1059 : 13 : 6$$

Answer 1. 1059 : 13 : 6 Flemish.

Example. 2. A Merchant at Rotterdam has a bill drawn on him for the value of 673l. 16s. 8d. Sterling, Exchange at 33s. 4d. Flemish per pound Sterling; how much must he pay there?

$$\begin{array}{r} 10s. \frac{1}{2} & 673 : 16 : 8 \\ 3 \frac{1}{4} & 336 : 18 : 4 \\ & 112 : 06 : 1 \frac{1}{2} \\ \hline & 1123 : 01 : 1 \frac{1}{2} \end{array}$$

Answer 1. 1123 : 01 : 1 $\frac{1}{2}$ Flemish.

628. *Secondly,* To reduce Sterling into Guilders.

Rule. Reduce the Sterling to Flemish, by the foregoing Rule, which Flemish money multiply by 6.

Example. If 397l. 15s. Sterling, Exchange at 34s. 8d. Flemish per pound Sterling, were remitted to Amsterdam; how many guilders, stivers, &c. may be received at the said rate of Exchange?

$$\begin{array}{r} 10s. \frac{1}{2} & 397 : 15 \\ 4 \frac{1}{2} & 198 : 17 : 06 \\ 8 \frac{1}{2} & 79 : 11 : 00 \\ & 13 : 05 : 02 \end{array}$$

$$689 : 08 : 08$$

6

Gul. Stiv.

Answer 4136 : 12

4136 : 12 : 00

629. *Thirdly,* To reduce Flemish pounds into Sterling, use the universal Rule of Proportion, as in Case 2. Art. 625.

Example. A Merchant at Brussels delivers 579l. 10s. Flemish, Exchange at 35s. 6d. per l. Sterling; how much Sterling must be received?

579l.

$$579 l. 10s. = \frac{1159}{2} l. \text{ and } 35s. 6d. = \frac{71}{40} l.$$

	Sterl.	Flem.
Conditional terms	I	$\frac{71}{40}$
Corresponding	A	$\frac{1159}{2}$

$$\text{Therefore } A = \frac{1 \times 1159 \times 40}{2 \times 71} = \frac{1159 \times 20}{71} = 326 l. 9s. 6\frac{3}{4}d.$$

630. But those are ready in the use of Decimal Fractions, may proceed as follows:

$$579 l. 10s. = 579.5; \text{ and } 35s. 6d. = 1.775$$

$$\text{Then } A = \frac{579.5}{1.775} = 326.4789. \text{ See the operation.}$$

$$\begin{array}{r}
 1,775) 579,5000(326,4789 \\
 \cdots \quad 5325 \\
 \hline
 \cdot 4700 \\
 3550 \\
 \hline
 11500 \\
 10650 \\
 \hline
 \cdots 8500 \\
 7100 \\
 \hline
 1400 \\
 1243 \\
 \hline
 \cdot 157 \\
 142 \\
 \hline
 15
 \end{array}$$

631. *Fourthly*, To reduce Guilders, Stivers, and Pennicks into Sterling.

Rule. Divide the Guilders, &c. by 6, which reduces them to Flemish pounds, and proceed as in Art. 629, and 630.

632. Of gain or loss by Exchange.

Note. That when Exchange is made with *Italy, France, Spain, &c.* with which, as aforesaid, *London* exchanges for so many pence for their pieces, the gain thereby is so much the more, by how much the course of Exchange runs low; because it is evident, that I can receive more ducats for 500*l.* when the Exchange is at 53*d.* than at 56 per piece.

But when I remit for *Flanders* or *Holland*, with which *London* exchanges on the pound Sterling, the higher the Exchange is above Par, the more we have the advantage of gain; for according to reason, I may receive more *Flemish* money for 1000*l.* Sterling, when the Exchange is at 36*s. 8d.* than at 35*s. 4d.*

**633. Decimal TABLES for Flemish Exchanges,
at the most usual Rates of Exchange.**

Rate of Exchan.		The Value of pounds Ster- ling in pounds <i>Flemish.</i>	The Value of pounds <i>Flemish</i> in pounds Ster- ling.
	s. d.		
32			
	1	.6625
	1	.60416	.6233766
2	1	.60836217616
3	1	.61256201550
4	1	.6166185567
5	1	.620836169665
6	1	.6256153846
7	1	.62916	.6138107
8	1	.636122449
9	1	.63756106870
10	1	.64166091370
8. 11	1	.645836075949
33			
	1	.656060606
	1	.65416	.6045343
2	1	.65836030150
3	1	.66256015037
4	1	.66
5	1	.670835985037
6	1	.6755970149
7	1	.679165955335
8	1	.6835940594
9	1	.68755925925
10	1	.69165911330
11	1	.695835896806
34			
	1	.75882352
	1	.704165867970
2	1	.70835853658
3	1	.71255839416
4	1	.7165825242

Rate of Exchan.	s. d.	The Value of pounds Ster- ling in Guild- ers.	The Value of Guilders in pounds Ster- ling.
32—	9 .610416666	
1	9 .62510389610	
2	9 .6510362694	
3	9 .67510335916	
4	9 .710309278	
5	9 .72510282775	
6	9 .7510256410	
7	9 .77510230178	
8	9 .810204081	
9	9 .82510178116	
10	9 .8510152283	
11	9 .87510126581	
33—	9 .910101010	
1	9 .92510075567	
2	9 .9510050250	
3	9 .97510025061	
4	10.01	
5	10.02509975061	
6	10.0509950248	
7	10.07509925558	
8	10.109900990	
9	10.12509876541	
10	10.1509852216	
11	10.17509828011	
34—	10.209803921	
1	10.22509779951	
2	10.2509756097	
3	10.27509732360	
4	10.309708736	

Rate

Rate of Exchan.	The Value of pounds Ster- ling in pounds <i>Flemish.</i>		The Value of pounds <i>Fle- mish</i> in pounds Sterling.
	s.	d.	
34	5	I .72083	.5811138
	6	I .7255797101
	7	I .72918	.5783132
	8	I .73	.5769230
	9	I .73755755395
	10	I .7418	.5741628
	11	I .74583	.5727923
35	—	I .755714285
	1	I .75418	.5700712
	2	I .7583	.5687209
	3	I .76255673758
	4	I .78	.5660377
	5	I .77083	.5647058
	6	I .7755633802
	7	I .77918	.5620609
	8	I .783	.5607476
	9	I .78755594405
	10	I .7918	.5581395
	11	I .79583	.5568445
36	—	I .85555555
	1	I .80418	.5542725
	2	I .8083	.5529954
	3	I .81255517241
	4	I .818	.5504587
	5	I .82083	.5491990
	6	I .8255479452
	7	I .82918	.5466970
	8	I .83	.5454545
	9	I .83755442176
	10	I .8418	.5429864
	11	I .84583	.5417607

Rate of Exchan.	s. d.	The Value of pounds Sterling in Guilders.	The Value of Guilders in pounds Sterling.
34	5	10.32509685231
	8	10.3509661835
	1	10.37509638553
	8	10.409615383
	9	10.42509592325
	10	10.4509569368
	11	10.47509546538
<u>35</u>		10.509523808
	1	10.52509501186
	2	10.5509478681
	3	10.57509456263
	4	10.609433961
	5	10.62509411763
	6	10.6509389671
	7	10.67509367681
	8	10.709345793
	9	10.72509324008
	10	10.7509302325
	11	10.77509280741
<u>36</u>		10.809259259
	1	10.82509237871
	2	10.8509216590
	3	10.87509195402
	4	10.909174311
	5	10.92509153316
	6	10.9509132420
	7	10.97509111616
	8	11.009090909
	9	11.02509070293
	10	11.0509049773
	11	11.07509029345

Rate

Chap. XXXVIII. for Flemish Exchanges.

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Rate of Exchang.	s. d.	The Value of pounds Sterl. in pounds <i>Flemish.</i>	The Value of pounds <i>Flem.</i> in pounds Sterling.
37—			
1	1 .855405405	
2	1 .85416	.5393258	
3	1 .8583	.5381166	
4	1 .86255369127	
5	1 .866	.5357143	
6	1 .87083	.5345211	
7	1 .8755333333	
8	1 .87916	.5321508	
9	1 .883	.5309734	
10	1 .88755298013	
11	1 .8916	.5286343	
38—			
1	1 .89583	.5274725	
2	1 .9526357	
3	1 .90416	.5251641	
4	1 .9083	.5240174	
	1 .91255228758	
	1 .916	.5217391	

T 4

Rate

Rate of Exchang.	The Value of pounds Sterling in Guilders.	The Value of Guilders in pounds Sterling.
s. d.		
37—	II. I09009009
1	II. 125 ..	.08988763
2	II. 1508968610
3	II. 175 ..	.08948545
4	II. 208928571
5	II. 225 ..	.08908685
6	II. 2508888888
7	II. 275 ..	.08869180
8	II. 308849556
9	II. 325 ..	.08830021
10	II. 3508810571
11	II. 375 ..	.08791208
38—	II. 408771928
1	II. 425 ..	.08752735
2	II. 4508733624
3	II. 475 ..	.08714596
4	II. 508695651

The EXPLICATION.

634. The first Column shews the Rate of the Exchange from 32*s.* to 38*s.* 4*d.* and is extended no farther; for a regulation being made of the coin, it is not probable it will rise higher, or fall lower than those limitations.

635. The second Column on the left-hand, shews the value of pounds Sterling, expressed in pounds *Flemish*, whereby at the first view, you discern what is the amount of 1, 10, 100, 1000, &c. to ten millions of Sterling, in pounds *Flemish*, at any of the inserted rates of Exchange. For example, if the Exchange is at 35*s.* 2*d.* one pound Sterling, being equivalent to one pound fifteen shillings and two-pence at one view; therefore it appears that

<i>L.</i>	<i>L.</i>	<i>s.</i>	<i>d.</i>	
10 <i>L.</i> Sterling makes	17	: 11	: 8	
100	175	: 16	: 8	{ <i>Flem.</i>
1000	1758	: 6	: 8	&c.

All which Flemish monies are found in the line against the said given rate of Exchange : But if the Sterling money, for which the value in pounds Flemish is required, consists of odd pounds, or hath shillings, &c. annexed ; then multiply the Flemish money, at the given Exchange, by the sum in Sterling, and the product answers the question. Thus $67l. 10s.$ Sterling, multiplied by $1,758$ the Flemish money at $35 : 2$ makes $118l. 13s. 9d.$ Flemish.

636. The third Column on the left-hand page shews the value or amount of pounds Flemish in Sterling, after the same manner as the second shews the value or amount of pounds Sterling in Flemish money :

For instance,

At the aforesaid Exchange of $35s. 2d.$ I see that one pound Flemish being equivalent to $.5687209$, or $l. 11s. 4\frac{1}{2}d.$ Sterling. Therefore 10 Flem. makes 5.687209 , or $5 : 13 : 9 : \{$ ling
 $100l. \quad \quad \quad 56.87209$, or $56 : 17 : 6 : \{$ very
and $1000 \quad \quad \quad 568.7209$, or $568 : 14 : 5 : \{$ near.

But if your Flemish money, the value of which is required in Sterling, consists of odd pounds, or hath shillings, &c. annexed, multiply the same by the Sterling, as before directed, and the product of such Multiplication answers the question ; so if it be required to find how much Sterling for $137l. 10s.$ Flemish, Exchange being at $35s. 2d.$ then $137,5 \times .5687209 = 78,19912$, or $78l. 4s.$ Sterling nearly.

637. The fifth Column, or the second on the left-hand page, shews the value or amount of pounds Sterling in Guilders, or what number of Guilders I ought to receive, or pay, for any sum of Sterling, at any of the inserted rates of Exchange :

For Example,

At the aforesaid Exchange of $35s. 2d.$ it appears that 10 Guilders, 11 Stivers, are equivalent to one pound Sterling ; as likewise that

Guild.	Stiv.		l.
105	10	are to	10
1055		to	$100 \}$
10550		to	$1000 \}$

Sterling.

If the Sterling-money is in odd pounds, or hath shillings, &c. annexed, you are to multiply the Guilders corresponding to the rate of Exchange by your Sterling sum, as above directed ; so $132l. 10s.$ Sterling, Exchange at $35s. 2d.$ will be found to make 1397 Guilders, 17 Stivers, 8 Pennicks.

638. The last Column shews the value of Guilders in pounds Sterling, or how much Sterling I ought to receive, or pay, for any number of Guilders, at any of the specify'd rates of Exchange.

So at the aforesaid rate of Exchange, viz. 35s. 2d. one Guilder being worth .09478681, or 1s. 10 $\frac{1}{4}$ d. Sterling.

Therefore,

		l. s. d.
10 make	.9478681, or	18 : 11 : $\frac{1}{4}$ } Sterling
100	9.478681, or	9 : 09 : 6 : $\frac{1}{4}$ } very
1000	94.78681, or	94 : 15 : 8 : $\frac{1}{4}$ } near.

If your Guilders be in odd numbers, or have Stivers annexed, multiply them also by the value in Sterling corresponding to the rate of Exchange, and the product will answer your demand.

Guild. Stiv. Penn.

So 1397 : 17 : 8, Exchange at 35s. 2d. being multiplied by .09478681, amounts to 132l. 10s. Sterling, as above-faid.

Note, The reason of the Tables being calculated so high, is, that you may be more exact in long sums, tho' you'll seldom have occasion to make use of more than five or six figures: And the dots placed to the right-hand are inserted for the more easy and safe observation of large even sums. But if the casting up of odd sums should be thought as troublesome to be effected by these Tables, as by the common method of doing them, (tho' in all cases performed by one single Multiplication) yet even sums most commonly happening in drawing and remitting by Exchange, the usefulness of these Tables will be sufficiently manifest.

639. Many questions proposed in Reduction, by persons in the common method of teaching, belong to this place: Some of the most useful follow, and by attending to the manner of solution of them, the reader may form Rules for other cases.

640. To reduce guineas into pounds.

Rule. To the given number of guineas, add $\frac{1}{20}$ thereof.

Example. How many pounds in 927 guineas?

$$\frac{1}{20} \text{ of } 927 = 46 : 7s.$$

Answer 973l. 7s.

641.

641. To reduce pounds into guineas.

Rule. From the given pounds subtract $\frac{1}{21}$ thereof.

Example. How many guineas in 973*l.* 7*s.*

$$\frac{1}{21} \text{ of } 973l. 7s. = 139 : 1$$

$$\text{Subtract } \frac{1}{3} \text{ of } 139l. 1s. = 46 : 7 = \frac{1}{21} \text{ of } 973l. 7s. \text{ by Art. 113.}$$

Answer 927*l.* 6*s.*

642. To reduce moidores at 1*l.* 7*s.* into pounds.

Rule. To the given number of moidores add $\frac{1}{4}$ and $\frac{1}{5}$ thereof.

Example. How many pounds in 927 moidores?

$$\frac{1}{4} \text{ of } 927 = 231 : 15$$

$$\frac{1}{5} \text{ of } 927 = 92 : 14$$

Answer 1251*l.* 9*s.*

643. To reduce moidores into guineas.

Rule. To the given number of moidores add $\frac{2}{7}$ thereof.

Example. How many guineas in 924 moidores?

$$\frac{1}{7} \text{ of } 924 = 132$$

132

Answer 1188 guineas.

644. To reduce guineas into moidores.

From the given number of guineas take $\frac{2}{7}$ thereof.

Example. How many moidores in 1188 guineas?

$$\frac{2}{7} \text{ of } 1188 = 132$$

$$\frac{2}{7} \text{ of ditto} = 264$$

Answer 924 moidores.

645. To reduce Portugal pieces of 1*l.* 16*s.* each to pounds.

From twice the given number of pieces take $\frac{1}{10}$ thereof.

Example. How many pounds in 925 pieces, each 1*l.* 16*s.*

$$2 \times 925 = 1850$$

$$\frac{1}{10} \text{ of } 1850 = 185$$

Answer 1665*l.*

646.

646. To reduce *Portugal* pieces of 1*l.* 16*s.* to guineas.

From twice the given number of pieces take $\frac{1}{7}$ thereof.

Example. How many guineas in 924 pieces, at 1*l.* 16*s.*

$$\begin{array}{r} 2 \times 924 = 1848 \\ \frac{1}{7} \text{ of } 1848 = 264 \end{array}$$

Answer 1584

647. When more than two different Coins, Weights, Measures, &c. are compared together, *viz.* when one kind of Coin is compared with a second of another kind; that second with a third; the third with a fourth; the fourth with a fifth, &c. two different cases are ordinarily raised from such comparison, *viz.*

It may be required to know, 1. How many pieces of the first Coin are equal in value to a given number of pieces of the last Coin. Or,

2. How many pieces of the last coin are equal in value to a given number of pieces of the first kind of Coin?

An Example of the first Case.

If 35 ells of *Vienna* make 24 ells of *Lyons*; 3 ells of *Lyons* 5 ells of *Antwerp*; and 100 ells of *Antwerp* 125 ells at *Frankfort*; how many ells of *Vienna* are equal to 50 ells at *Frankfort*? Answer 35 ells of *Vienna*.

For the more easy understanding of the resolution of this question, and others of like nature: Let *a* represent an ell at *Vienna*, *b* an ell at *Lyons*, *c* an ell at *Antwerp*, and *d* an ell at *Frankfort*; then may the given terms in the question be stated in the following order.

$$\text{Suppositions } \left\{ \begin{array}{l} 35a = 24b \\ 3b = 5c \\ 100c = 125d \end{array} \right.$$

$$\text{The Question } 50d = ?a$$

648. This order of placing the said given numbers (or terms) being observed, it appears, that if $35a$ be accounted to stand in the first place; $24b$ in the second; $3b$ in the third; $5c$ in the fourth; $100c$ in the fifth, &c. then all the terms which stand in odd places, to wit, in the first, third, fifth, and seventh places, will necessarily fall under the first row or column on the left-hand, and all the terms that stand in even places, to wit, in the second, fourth, and sixth places, will fall under the latter column.

649. These things premised, all questions which come under Case I. before-mentioned, may be resolved by this Rule, viz.

Rule I. Multiply all the given terms that stand in odd places (to wit, in the first column) according to the Rule of continual Multiplication, and reserve the last product for a Dividend: Again, multiply continually all the terms which stand in even places: so shall the product be a Divisor, and the Quotient arising from the said Dividend and Divisor will be the answer of the question.

650. But instead of performing the Multiplications and Division, make use of the method in Art. 266. applying it to the numbers, as they stand on different sides of the marks of equality, in the same manner as it is there applied to the Numerator and Denominator of a Fraction. See the above Example so managed.

$$\begin{array}{rcl} 7 & 35a & = 74b \quad \$ \quad \frac{1}{1} \\ 1 & 3b & = \frac{8c}{1} \\ 1 & 4 \times 30c & = 173d \quad \frac{8}{1} \\ 5 & 30d & = ?a \end{array}$$

Therefore $7 \times 5 = 35$ the Answer.

651. The reason of this Rule will be manifest by solving the question propounded by three single Rules of Three, thus:

$$\text{I. } 24b : 35a :: 3b : \frac{35 \times 3}{24}a$$

$$\text{II. } 5c : \frac{35 \times 3}{24}a :: 100c : \frac{35 \times 3 \times 100}{24 \times 5}a$$

$$\text{III. } 125d : \frac{35 \times 3 \times 100}{24 \times 5}a :: 50d : \frac{35 \times 3 \times 100 \times 50}{24 \times 5 \times 125}a$$

Which number at last found, to wit,

$$\frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}$$

with the before-mentioned order of placing the terms proposed in the question, gives the very Rule before expressed in words.

652. An Example of the latter of the two cases before-mentioned.

If 10lb. of Avoirdupoise-weight at London be equal to 9lb. of Amsterdam; 45lb. at Amsterdam, to 49lb. at Bruges; and 98lb. at Bruges, equal to 116lb. at Dantzick; how many lb. of Dantzick are equal to 112lb. of Avoirdupoise-weight at London? Answer 129.92lb. of Dantzick.

That

That the operation may be the more clear, let a represent one pound of Avoirdupoise-weight; b one lb. of *Amsterdam*; c one lb. of *Bruges*; and d one lb. of *Dantzick*; then let the question be stated after the order in the first case, *viz.*

$$\begin{array}{l} \text{Suppositions} \\ \left\{ \begin{array}{l} 10a = 9b \\ 45b = 49c \\ 98c = 116d \\ \hline \end{array} \right. \\ \text{The Question} \quad ?d = 112a \end{array}$$

653. These things premised, all questions which fall under *Cafe 2.* before-mentioned, may be resolved by this Rule, *viz.*

Rule 2. Multiply all the given terms which stand in even places (to wit in the latter column) by the Rule of continual Multiplication, and reserve the last product for a Dividend: Again, multiply continually the rest of the terms that stand in odd places (to wit in the first column) for a Divisor; so shall the Quotient arising be the answer of the question.

654. If the method in Art. 266. be applied to the above Example, it may stand as below:

$$\begin{array}{l} 10a = gb \\ 5gb = 49c \\ 2gb = 116d \\ \hline ?d = 112a \end{array}$$

Therefore $\frac{116 \times 112}{10 \times 5 \times 2} = \frac{116 \times 112}{100} = 116 \times 1.12 = 129.92 =$
the answer.

655. A little practice will render the use of the letters and marks of equality unnecessary, and will enable the reader to write the result down, with only a line between the figures as in the following Example.

A merchant having occasion to remit 385*l.* Sterling to *Rome*, may do the same two ways, as follows, *viz.* He may remit by the way of *Paris*, 56 shillings Sterling being equal to 12 French crowns, and 55 French crowns being worth 52 dollars at *Genoa*; and 2 *Genoa* dollars amounting to 3 Roman crowns; or he may remit by the way of *Amsterdam*, 20 shillings Sterling, and 33 shillings *Flemish*; 15 shillings *Flemish*, and 2 rixdollars at *Frankfort*; 11 rixdollars at *Frankfort*, and 12 Venetian ducats; and 11 Venetian ducats and 12 Roman crowns being equal: Now the question is, which way he can remit to the best advantage? Answer, by the way of *Paris*.

For by the underwritten operation it appears, that if he remits by the way of *Paris* he will receive 2340 Roman crowns.

$$38l. = 7700 \text{ shillings.}$$

$\begin{array}{r} 14 \\ 11 \\ 1 \\ 2 \end{array}$	$\begin{array}{r} 56 \\ 85 \\ 7 \\ 1 \end{array}$	$\begin{array}{r} 17 \\ 87 \\ 3 \\ 9 \end{array}$
		6 13 3 10

$$\text{And } 6 \times 13 \times 3 \times 10 = 2340.$$

But if he remits by way of *Amsterdam*, he will receive but 2016 Roman crowns.

$\begin{array}{r} 1 \\ 3 \\ 11 \\ 11 \\ 2 \end{array}$	$\begin{array}{r} 17 \\ 17 \\ 12 \\ 1 \end{array}$	$\begin{array}{r} 33 \\ 2 \\ 4 \\ 12 \\ 7 \end{array}$
		3 2 4 12 7

$$\text{And } 2 \times 3 \times 4 \times 12 \times 7 = 2016.$$

656. Questions concerning Exchanges of Coins.

1. A bill of 200*l.* is remitted to *Paris* by a merchant in *London*: What is the value in *French crowns*, at 4*s.* 6*d.* each? Answer 888 *Crowns* $\frac{18}{42}$.
2. A merchant at *London* received 100*l.* *Sterling*, for the value paid by his correspondent, at the rate of 3*s.* 6*d.* *Sterling per crown*: How many crowns were paid at *Paris*? Answer 571 *crowns* $\frac{18}{42}$.
3. Admit a bill drawn in *Lyons*, and payable in *London* for 1510 crowns: How much *English money* comes it to, the exchange at 55*d.* $\frac{1}{2}$? Answer 346*l.* 16*s.* 6*d.*
4. There are 2000 ducats, at 4*s.* 4*d.* each, remitted to *London*, to be paid in pounds *Sterling*: What is the amount? Answer 433*l.* 6*s.* 8*d.*
5. A bill of 220*l.* 16*s.* 8*d.* is drawn from *London*: What is the value at *Florence*, in ducatoons, at 53*d.* each? Answer 1000 ducatoons.
6. If 247*l.* 18*s.* 4*d.* *Sterling* be remitted to *Frankfort*: What is the value in *Florins*, at 59*d.* Answer 1000 *Florins*.
7. A factor has sold goods, at *Cadiz*, for 1468 pieces of eight, at 4*s.* 6*d.* per piece: How much *Sterling* is the sum? Answer 333*l.* 7*s.* 2*d.*
8. Being desirous to remit to my correspondent at *London*, the sum of 2000*l.* 12*s.* 6*d.* *Flemish*, to dispose of according to my order (Exchange at 34*s.* 6*d.* *Flemish per pound Sterling*):

Sterling :) How much money Sterling shall I be creditor
for? Answer 1159*l.* 15*s.* 7*1*/*4**d.*

C H A P. XXXIX.

The Application of the Doctrine of Proportion to Questions relating to Loss and Gain, Barter, Fac- torship, &c.

- 657.** Questions relating to *Loss* and *Gain*.
 Q. I. A parcel of goods being bought for 60*l.* and sold for 75*l.* what was the rate of gain *per cent.* Answer 25*l.*

For 60 : 75 :: 100 : 125.

2. Suppose I have goods to the value of 415*l.* 12*s.* 6*d.* that come to a bad market, and know they impair by lying: What will they come to, if I am obliged to sell them at the loss of 12 in the 100? Answer 365*l.* 15*s.*
 For 100 - 12 = 88 and 100 : 88 :: 415,625 : 365,75.
 2. Bought 18*C.* wt. of cheese at 28*s.* per *C.* wt. which I

$$\text{For } 1 : \frac{28}{20} :: 18 : \frac{18 \times 28}{20} = 25 l. 4 s.$$

$$\text{And } 1 : \frac{7}{480} :: 18 \times 112 : \frac{7 \times 18 \times 112}{480} = 291.8s.$$

4. Having sold 10 yards of cloth for 4l. 16s. and thereby gained at the rate of 10 per cent. What was the prime cost of 1 yard? Answer 8s. 8 $\frac{3}{4}$ d.

For 110 : 100 :: 4,8 : 4,36

And 10 : 4,36 :: 1 : 0,436.

5. Having sold 1 yard of cloth for 11s. 6d. I gained, at the rate of 15*l.* per cent. but, had I sold it for 12*s.* what would be the rate of gain per cent. Answer 20*l.*

For : 115 : 100 :: $\frac{23}{48}$: $\frac{20}{48}$ = prime cost of 1 yard.

And ($\frac{20}{15} \equiv$) $\frac{1}{3} : \frac{1}{2} :: 100 : 120.$

6. A merchant has bought linen cloth at 11*s.* per ell; which proving worse than he expected, he is willing to sell at such a price, that he may lose precisely after the rate of 1*7*/₁₀l. for every 20*l.* that he laid out; the question is to know at what price he ought to sell the ell, that the proportion in the said loss may be observed? Answ. 10*s.* 1*d.* per ell.

$$\text{For } 20 - \frac{1}{7} = 18\frac{1}{7}$$

And $20 : 18\frac{1}{7} :: 11 : 10\frac{1}{7}s.$ shillings.

7. If 100*lb.* weight of any commodity cost 30*s.* at what price must 1*lb.* weight of that commodity be sold, to gain after the rate of 10*lb.* for every 100 laid out? Answer 3*7*/₁₀ d. per lb. weight.

$$\text{For } 100 : 110 :: 30 : 33$$

And $100 : 33 :: 1 : 10\frac{1}{3}s.$ (or 3*2*/₃ d.)

8. A merchant sells a parcel of jewels, which cost him 250*l.* ready money, for 559*l.* payable at the end of 6 months; the question is (his security being supposed to be good) what his gain was worth in ready-money, upon rebate of interest at the rate of 6*l.* for 100*l.* for a year? Answ. 300*l.*

$$\text{For } 559 - 250 = 309$$

And $100 : 100 :: 309 : 300.$

9. Bought 3 oxen for 24*l.* 10*s.* which I sell again for 2*s.* per stone; what ought the three oxen to weigh together, the hides and offal being the only clear gain? Answer 245 stone.

$$\begin{array}{rcl} s. & \text{stone.} & s. & \text{stone.} \\ \text{For } 2 : 1 :: 490 : 245 \end{array}$$

10. Suppose I buy 28 pieces of stuff at 4*l.* per piece, and sell ten of the pieces at 6*l.* and eight at 5*l.* at what rate must I sell the rest, to gain 10 per cent. by the whole? Answ. 2*l.* 6*s.* 5*d.*

For 28 pieces at 4*l.* are worth 112*l.*

10 ditto	6 <i>l.</i> amount to	60
8 ditto	5 <i>l.</i> ditto	40

Therefore 18 pieces were sold for 100*l.*

But 28 pieces must sell for 123*l.* 4*s.*

$$\text{For } 100 : 112 :: 123,4s.$$

Therefore 10 pieces must sell for 23*l.* 4*s.*

And 1 piece ditto 2*l.* 6*s.* 5*d.*

658. Questions relating to Barter.

1. *B* delivered 3 hogsheads of brandy at 6s. 8d. per gallon, to *C*, for 126 yards of cloth, what was the cloth per yard? Answ. 10s.

For 1 gall. : $\frac{3}{4}$ l. :: 3 \times 63 gall. : 63 l.
And 126 yards : 63 l. :: 1 yard : $\frac{1}{2}$ l.

2. How much sugar at 8d. per lb. weight, may be bought for 20 C. of tobacco, at 3l. per C. Answ. 1800 lb. weight of sugar.

For 1 : 3 :: 20 : 60
And $\frac{1}{30}$: 1 :: 60 : 1800

3. *C* has candles at 6s. per dozen ready money; but in barter he will have 6s. 6d. per doz. *D* has cotton at 9d. per lb. ready money; what price must the cotton be at in barter; and how much cotton must be bartered for 100 doz. of candles? Answ. the cotton $9\frac{1}{2}$ d. per lb. in barter, and 7 C.wt. 16 lb. of cotton must be given for 100 doz. of candles.

For 72 d. : 78 d. :: 9 d. : $9\frac{1}{2}$ d.
Also 1 doz. : 6s. :: 100 doz. : 30 l.
And $\frac{3}{5}$ l. : 1 lb. :: 30 l. : 800 lb. = 7 C. 0 qu. 16 lb.

4. Two merchants barter, *A* has 20 C. wt. of Cheese at 21s. 6d. the C. wt. *B* has 8 pieces of Irish cloth, at 3l. 14s. per piece; Whether of them must receive money, and how much? Answ. *A* must pay to *B* 8l. 2s.

For 8 pieces at 3l. 14s. are worth 29l. 12s.
And 20 C. wt. at 1l. 1s. 6d. 21l. 10s.

Difference 8l. 2s.

5. *A* and *B* barter, *A* has 41 C. of hops at 30s. per C. wt. for which *B* gives him 20l. in money, and the rest in prunes at 5d. per pound; how many prunes did *B* give *A*, beside the 20l.? Answ. 17 C. wt. 3 q. 4 lb.

For 41 C. wt. at 1l. 10s. are worth 61l. 10s.

Deduct 20l.

Remains 41l. 10s.

And $\frac{4}{5}$ l. : 1 lb. :: $\frac{4}{5}$ l. : 1992 lb. = 17 C. wt. 3 qu. 4 lb.

6. *A* has 100 pieces of silks, which are worth but 3l. per piece in ready money; yet he barters them with *B* at 4l. per piece, and at that rate takes their value of *B* in wool, at 7l. 10s. per C. which is worth but 6l. per C. in ready money; the question is to know what quantity of wool

wool pays for the silks, and which of the two *A* or *B*, is the gainer, and how much? Answ. $53\frac{1}{3}C.$ of wool pays for the silks, and *A* gains $20\text{l}.$ by the barter.

$$\text{For } 7\frac{1}{2} : 1 :: 400 : 53\frac{1}{3}$$

$$\text{And } 1 : 6 :: 53\frac{1}{3} : 320$$

So it is evident, that the true worth of the wool which *B* delivered was $320\text{l}.$ for which he received only of *A* the worth of $300\text{l}.$

7. *A* has linen cloth, worth $20d.$ an ell, ready money; but, in barter he will have $2s.$ *B* has broad cloth worth $14s.$ $6d.$ per yard ready money: At what price ought the broad cloth to be rated in barter? Answ. $17s.$ $4d.$ $3\frac{1}{3}f.$

$$\text{For } 20 : 24 :: 174 : 208\frac{1}{3}$$

659. Questions relating to Factorship.

1. A merchant delivers to his factor $100\text{l}.$ allowing him to join to it $30\text{l}.$ and values his service worth $40\text{l}.$ what share of the gain ought the factor to have? Answ. $\frac{1}{7}$ parts.

$$\text{For } 30 + 40 = 70 \text{ the factor's stock.}$$

$$\text{And } 170 : 1 :: 70 : \frac{1}{7}.$$

2. A merchant's real stock being $100\text{l}.$ and the factor's service valued at $20\text{l}.$ who received $\frac{1}{2}$ of the gain; what was the factor's real stock? Answ. $80\text{l}.$

$$\text{For } 100 - 20 = 80.$$

3. A merchant delivered to his factor $600\text{l}.$ upon condition that if the factor add to it $250\text{l}.$ of his own money, and bestow his pains in managing the whole stock, he should then have $\frac{2}{3}$ parts of the total gain. The question is to know what stock the factor's service was estimated at? Answer $150\text{l}.$

The factor's part of the gain being $\frac{2}{3}$, the merchant must necessarily have the remainder, which is $\frac{1}{3}.$

$$\text{Then } \frac{2}{3} : \frac{1}{3} :: 600 : 400$$

$$\text{And } 400 - 250 = 150.$$

4. The merchant's real stock being $100\text{l}.$ and the factor being allowed $\frac{1}{4}$ of the gain for his service, what real stock must he join to have $\frac{1}{3}$ of the gain? Answ. $16\frac{2}{3}\text{l}.$

$$\text{For } \frac{1}{4} : 1 :: 100 : 133\frac{1}{3} \text{ the whole stock.}$$

Of which the factor's service is $33\frac{1}{3}$

$$\text{But } 100\text{l. is } \frac{2}{3} \text{ of } 150.$$

$$\text{Therefore } 50 - 33\frac{1}{3} = 16\frac{2}{3} \text{ the factor must advance.}$$

5. A merchant delivers to his factor 320*l.* and permits him to add to it 64*l.* of his own money, to be employed in traffick; and by agreement between them, the factor's service is estimated equivalent to a certain stock; which is such, that if the total gain be divided proportionably according to those three stocks, the factor is to receive $\frac{1}{3}$ of the total gain, in consideration of the said imaginary stock (being the value of his service); the question is to know the full part of the gain belonging to each, and what stock the factor's service was valued at? Answer the merchant $\frac{5}{7}$ of the gain, and the factor $\frac{1}{3}$, whose service is valued at 96*l.* stock.

$$\begin{array}{rcl} \text{For} & 320 + 64 = 384 \\ \text{And} & \frac{1}{3} : \frac{1}{3} :: 384 : 96 \\ \text{Again, } & \left\{ \begin{array}{r} 320 \\ 64 \\ 96 \\ \hline 480 \end{array} \right. & \left. \begin{array}{r} 320 : \\ 160 : \\ \frac{1}{3} \end{array} \right. \end{array}$$

660. It will often happen, that the solution of questions in Fellowship may be facilitated by the use of Fractions Vulgar or Decimal: For since by Art. 159. and 161, the first and second term are the same, in every operation of the Rule of Three, that is necessary to solve the question; it will follow, that if the second term be divided by the first, the Quotient, so obtained, may be used as a common Multiplier to each of the respective third terms: Thus, in the second Example of Art. 159.

$$\begin{aligned} \frac{5}{7} : \frac{1}{2} &= \frac{1}{2} ; \text{ and } \frac{1}{2} \times 48 = 5 \times 4 = 20 \\ \frac{1}{2} \times 36 &= 5 \times 3 = 15 \\ \frac{1}{2} \times 24 &= 5 \times 2 = 10 \end{aligned}$$

Again, in the following Example, viz. A chapman breaking, owes unto four men the following sums of money, viz.

To	$\left\{ \begin{array}{l} A \ 21 : 9 : 6 \\ B \ 72 : 19 : 3 \\ C \ 114 : 13 : 9 \\ D \ 264 : 17 : 6 \\ \hline \text{Sum} \quad 474 : 00 : 0 \end{array} \right.$	His whole estate is found to be but 148 <i>l.</i> 2 <i>s.</i> 6 <i>d.</i> what must each have of the same, and what will it be per pound?
----	--	---

Now 148*l.* 2*s.* 6*d.* = 148,125; and $\frac{148,125}{474} = ,3125$.

Then

Then by the method in Art. 396.

$$\begin{array}{l}
 (21l. 9s. 6d.) = 21,475 \times .3125 = 6,7109 = 6l. 14s. 2\frac{1}{2}d. \\
 (72l. 19s. 3d.) = 72,9625 \times .3125 = 22,8008 = 22l. 16s. 0d. \\
 (114l. 13s. 9d.) = 114,6875 \times .3125 = 35,8399 = 35l. 16s. 9\frac{1}{2}d. \\
 (264l. 17s. 6d.) = 264,875 \times .3125 = 82,7733 = 82l. 15s. 5\frac{1}{2}d. \\
 \\
 \hline
 148,1249 = 148l. 2s. 6d.
 \end{array}$$

661. Questions relating to Fellowship.

1. Three merchants trading to *Virginia*, lost goods to the value of 800*l.* Now if *A*'s stock was 1200*l.* *B*'s 4800*l.* and *C*'s 2000*l.* what sum did each man lose? Answer *A* lost 120*l.* *B* 480*l.* *C* 200*l.*
2. Four men traded with a stock of 800*l.* and they gained in two years time twice as much, and 40*l.* over: *A*'s stock was 140*l.* *B*'s 260*l.* *C*'s 300*l.* I demand *D*'s stock and what each man gained by trading? Answ. *D*'s stock was 100*l.* and *A* gain'd 287*l.* *B* 533*l.* *C* 615*l.* and *D* 205*l.*
3. *A*, *B*, and *C* put in money together. *A* put in 20*l.* *B* and *C* put in 85*l.* they gained 63*l.* of which *B* took up 21*l.* what did *A* and *C* gain, and *B* and *C* put in?
Answer, *A* gained 12*l.* and *C* 30*l.*
B put in 35*l.* and *C* 50*l.*
For 105 : 63 :: 20 : 12. and 63 - 12 - 21 = 30.
Also 12 : 20 :: 21 : 35. and 85 - 35 = 50.
4. *A*, *B*, and *C* put in money together; *A* put in 20*l.* *B* 30*l.* *C* a sum unknown, they gained 36*l.* *C* took up 16*l.* what did *A* and *B* gain, and *C* put in?
Answer *A* gained 8*l.* and *B* 12*l.*
C put in 40*l.*
For 36 - 16 = 20 the gain of *A* and *B*.
Then (20 + 30 =) 50 : 20 :: { 20 : 8*l.* *A*'s gain.
{ 30 : 12*l.* *B*'s ditto.
Lastly 8 : 20 :: 16 : 40 *C*'s stock.
5. *A*, *B*, and *C*, put in money together; *A* put in 20*l.* for 3 months, *B* put in 30*l.* for 5 months, and *C* put in 40*l.* for 7 months; they gained 60*l.* what must each have of the gain?

Answer $\left\{ \begin{array}{l} A, 7\frac{17}{45} \\ B, 18\frac{15}{45} \\ C, 34\frac{4}{45} \end{array} \right.$

$$\begin{aligned} \text{For } 20 \times 3 &= 60 \\ 30 \times 5 &= 150 \\ 40 \times 7 &= \frac{280}{490} \end{aligned}$$

$$\text{Then } 490 : 60 :: \left\{ \begin{array}{l} 60 \\ 150 \\ 280 \end{array} \right\} : \left\{ \begin{array}{l} 7\frac{17}{45} \\ 18\frac{15}{45} \\ 34\frac{4}{45} \end{array} \right\}$$

6. *A* and *B* are in company ; *A* put in the first of January 50*l.* but *B* could not put any money in till the first of May ; what must *B* then put in, to have an equal share with *A*, at the year's end ? Answer 75*l.*

$$\begin{aligned} \text{For } 50 \times 12 &= 600 \\ \text{And } \frac{600}{4} &= 75. \end{aligned}$$

C H A P. XL.

Concerning the INTEREST of MONEY, and the Construction of TABLES to that purpose.

662. IN resolving questions concerning Interest of Money, four things are to be well observed ; to wit, first, the principal or money lent for gain or interest ; secondly, the time for which the said principal is lent ; thirdly, the rate or proportion which the principal bears to the sum of the principal and interest ; and fourthly, the interest itself : So if 100*l.* be lent, upon condition that 106*l.* shall be repaid at the end of a year, the said 100*l.* is called principal ; the time for which the said principal is lent, is one year ; the proportion which the principal bears to the sum of the principal and interest is such, as 100 has to 106*l.* Lastly, the interest itself is 6*l.*

663. Interest is either Simple, or Compound.

664. Simple Interest is that which arises, or is computed from the principal only : So if 100*l.* be lent for two years, the simple interest thereof, after the rate of 6*l.* for 100*l.* for one

one year, will be 12*l.* *viz.* 6*l.* due at the first year's end, and 6*l.* due at the second year's end.

665. Compound Interest is that which arises from the principal, and also from the interest thereof, and therefore it is called Interest upon Interest: So if 100*l.* be lent and forborn three years, and the compound interest thereof is to be computed, after the rate of 6*l.* for 100*l.* for one year, there will arise, besides the simple interest of the principal for three years, the interest of 6*l.* (due at the first year's end) for two years, and the interest of 6*l.* (due at the second year's end) for one year following.

666. Rebate, or discount of money is, when a sum of money due at any time to come, is satisfied by the payment of so much present money, which if it was put forth at a certain rate of interest for the said time, would become equal to the sum first due: So if 100*l.* be due at the end of two years, and is to be satisfied by the payment of present money upon Rebate, after the rate of 6*l.* *per cent., per ann.* simple interest, there ought to be so much ready money paid, which after the said rate of interest would in two years be augmented to 100*l.* In like manner, if the Rebate or Discount were to be made after any rate of compound interest, so much ready money ought to be paid, as at such rate of compound interest, for the time agreed on, would become equal to the sum first due.

667. In the taking of interest, or use-money, for the loan or forbearance of money lent, respect must be had to the rate limited by act of parliament, which hath restrained all persons from taking more than 5*l.* for the interest or use of 100*l.* lent for a year; but what part of 5*l.* may be taken for the interest of 100*l.* lent for half a year, a quarter of a year, a month, or any other part of a year, is not expressed in the Act: In this case, therefore, we must observe custom and daily practice; so we shall find that 2*l.* 10*s.* is usually taken for half a year's interest of 100*l.* and 25*s.* for a quarter of a year, &c. by which practice, this following analogy (which is the ground or reason of the common Rules for computing simple interest) seems to be assumed for a safe exposition of the Statute, *viz.* that such proportion as the whole year (supposed to consist of 365 days) has to any propounded space of time more or less than a year, such proportion any interest (not exceeding the rate limited by the Act) for any principal lent for a year, ought to have to the interest of the same principal for the time agreed upon: This analogy being granted, the manner of computing simple interest, for any principal lent and

forborn any time proposed, will be such as is expressed in the two next sections.

668. The interest or gain of 100*l.* principal money forborn for a year being known, the interest of any other principal money for the same time may be found out by one single operation of the Rule of Three; for as 100*l.* principal is in proportion to the interest thereof, so is any other principal to its interest: So if it be demanded, what 270*l.* will gain in a year, at the rate of 5*l.* for 100*l.* for one year, the answer will be found to be 13*l.* 10*s.* For,

$$\begin{array}{r} l. \quad l. \quad l. \quad l. \quad l. \quad s. \quad d. \\ 100 : 5 :: 270 : 13,5 \text{ (or } 13 : 10 : 0) \end{array}$$

669. A second Example. What is the interest of 246*l.* 18*s.* 10*d.* for a year, at the rate of 3*l.* for 100*l.* for a year? Answ. 7*l.* 8*s.* 1*d.*

This question, and all others wherein the gain of (or allowance for) 100 integers is concerned, may be answered as follows:

First, Multiply 246*l.* 18*s.* 10*d.* by 3, after the manner delivered in Art. 87. and write down the product which is 740*l.* 16*s.* 6*d.* Then divide the said product by 100, in this manner, viz. divide 740 pounds by 100, which is performed by cutting off, towards the right-hand, the two last places of

$$\begin{array}{r} l. \quad s. \quad d. \\ 246 : 18 : 10 \\ \hline 3 \\ \hline \begin{array}{r} 1.7 | 40 : 16 : 06 \\ \hline 20 \\ \hline 1.8 | 16 \\ \hline 12 \\ \hline 2.1 | 98 \end{array} \end{array}$$

740, so the Quotient gives 7 pounds, and there will be a remainder of 40 pounds, which 40 pounds reduce into shillings, so there will arise 800*s.* to which adding the 16*s.* which stand in the place of shillings, the sum will be 816*s.* these are also to be divided by 100 (by cutting off two places as before, so the Quotient will give 8*s.* and there will remain 16*s.* which being reduced to pence, and unto them 6 pence being added (to wit, the 6 pence which stands in the place of pence) there will arise 198 pence; these also are to be divided by 100 (by cutting

cutting off two places to the right-hand as before) so the Quotient gives 1 penny, and there will remain 98 pence; so the exact Quotient or answer of the question is found to be 7*l.* 8*s.* 1*d.*

More Examples of this Rule are these following:

$$\begin{array}{r} l. \quad l. \quad l. \quad s. \quad d. \\ 100 : 9 :: 793 : 12 : 7 \\ \hline & & & & 6 \end{array}$$

$$\begin{array}{r} l. 47 | 61 . 15 . 6 \\ \hline & 20 \end{array}$$

$$\begin{array}{r} s. 12 | 35 \\ \hline & 12 \end{array}$$

$$\begin{array}{r} d. 4 | 26 \\ \hline \end{array}$$

$$\begin{array}{r} l. \quad l. \quad l. \quad s. \quad d. \\ 100 : 8 :: 43 : 14 : 3 \\ \hline & & & & 8 \end{array}$$

$$\begin{array}{r} l. 3 | 49 . 14 . 0 \\ \hline & 20 \end{array}$$

$$\begin{array}{r} s. 9 | 94 \\ \hline & 12 \end{array}$$

$$\begin{array}{r} d. 11 | 28 \\ \hline \end{array}$$

670. After the same manner may this following question and such like be resolved, viz. when 100 ells of linen-cloth costs 30*l.* 18*s.* 9*d.* what is the price of 1 ell? Answ. 6*s.* 2*d.* 3*farth.*

Ells. l. s. d. Ell.

$$\begin{array}{r} 100 : 30 . 18 . 9 :: 5 \\ \hline & 20 \end{array}$$

$$\begin{array}{r} Shill. 6 | 18 \\ \hline & 12 \end{array}$$

$$\begin{array}{r} Pence 2 | 25 \\ \hline & 4 \end{array}$$

$$\begin{array}{r} Farth. 1 | 00 \\ \hline \end{array}$$

671. When the given gain of (or allowance for) 100 integers consists of some number of pounds, together with some aliquot part or parts of a pound, the operation will be little different from the last-mentioned Examples, as may appear by the resolution of the subsequent question, *viz.* what must be allowed for 2156*l.* 13*s.* 4*d.* at the rate of 6*l.* 15*s.* for 100*l.* Answ. 145*l.* 11*s.* 6*d.* thus found. First multiply the said 2156*l.* 13*s.* 4*d.* by 6 (the number of pounds in the given allowance 6*l.* 15*s.*) after the manner of the last Examples, and subscribe the product which is 12940*l.* under the line; then since 15*s.* are equal to $\frac{1}{4}$ *l.* together with $\frac{1}{4}$ *l.* take $\frac{1}{2}$ of the said 2156*l.* 13*s.* 4*d.* which is 1078*l.* 6*s.* 8*d.* likewise $\frac{1}{4}$ of the said 2156*l.* 13*s.* 4*d.* to wit, 539*l.* 3*s.* 4*d.* and having subscribed these Quotients under the product first found, add them all together, and find 14557*l.* 10*s.* 0*d.* the total product, with which proceed as in the former Examples; and so at length the Answer is found to be 145*l.* 11*s.* 6*d.* View diligently the operation,

$$\begin{array}{r}
 l. \quad l. \quad l. \quad s. \quad d \\
 \hline
 100 : 6\frac{1}{4} :: 2156 . 13 . 4 \\
 \hline
 & 12940 . 0 . 0 \\
 & 1078 . 6 . 8 \\
 & 539 . 3 . 4 \\
 \hline
 & 145 | 57 . 10 . 0 \\
 & \quad 20 \\
 \hline
 & 11 | 50 \\
 & \quad 12 \\
 \hline
 & 6 | 00
 \end{array}$$

672. By this Rule, Brokage, Commission, Provision, Storage, Insurance, &c. are work'd, having no respect to time.

Example 1. Unto what comes the Commission of 356*l.* 17*s.* 11*\frac{1}{2}**d.* at 2*\frac{1}{2}* per cent.

$$\begin{array}{r}
 \frac{1}{2}) \quad 359 : 17 : 11 : \frac{1}{2} \\
 \hline
 719 : 15 : 11 : 0 \\
 179 : 18 : 11 : \frac{3}{4} \\
 \hline
 6 \mid 99 : 14 : 10 : \frac{3}{4} \\
 \hline
 19 \mid 94 \\
 \hline
 11 \mid 38 \\
 \hline
 1 \mid 55
 \end{array}$$

Example 2. At $1\frac{1}{4}$ per cent. what comes the Brokage of 198*l.*
11*s.* 7*d.* to?

$$\begin{array}{r}
 \frac{1}{4}) \quad 198 : 11 : 7 \\
 \hline
 49 : 12 : 10 : \frac{3}{4} \\
 \hline
 2 \mid 48 : 4 : 5 : \frac{3}{4} \\
 \hline
 9 \mid 64 \\
 \hline
 12 \text{ Answ. } 2l. 9s. 7\frac{1}{2}d. \\
 \hline
 7 \mid 73 \\
 \hline
 2 \mid 95
 \end{array}$$

673. But questions of this nature are more conciley and readily worked by this

Rule. Divide the given sum by the aliquot part, that the rate of the Commission or Brokage, &c. is of 100*l.* As for instance, in the last Examples.

Example 1. The Commission being $2\frac{1}{2}$ per cent. which is the 40th part of 100*l.* Therefore

$$\begin{array}{r}
 4 \mid 0) \quad 25 \mid 9l. 17s. 11d. \frac{1}{2} \\
 \hline
 \text{Answ.} \quad 6 : 09 : 11 : \frac{1}{4}
 \end{array}$$

Example

Example 2. The Brokerage being $1\frac{1}{2}$ per cent. which is the 80th part of 100*l.* Therefore

$$8 | 0 \ 19 \ 8 \text{ l. } 11 \text{ s. } 7 \text{ d.}$$

$$\text{Answ } 2 : 09 : 07 : \frac{1}{2}$$

674. When Time is concerned.

Rule. Multiply the principal by the interest of 100*l.* for the time required, and the product (striking off two figures on the right-hand) is the answer?

Example 1. What is the interest of 479*l.* 18*s.* for two years, at $4\frac{1}{2}$ per cent. per annum.

The interest of 100*l.* for two years at $4\frac{1}{2}$ per cent. is 9*l.* Therefore

$$\begin{array}{r} 479 : 18 \\ \hline 9 \\ \hline 43 | 19 : 02 \\ \hline 20 \\ \hline 3 | 82 \\ \hline 12 \\ \hline 9 | 84 \\ \hline 4 \end{array}$$

$$3 | 36 \text{ Answ. } 43\text{l. } 03\text{s. } 09\frac{3}{4}\text{d.}$$

Example. What is the interest of 571*l.* 15*s.* for 8 months, at 6 per cent. per annum?

The interest of 100*l.* for 8 months, at 6 per cent. is 4*l.* Therefore

$$\begin{array}{r} 571\text{l. } 15\text{s.} \\ \hline 4 \\ \hline 22 | 87 : 00 \\ \hline 20 \\ \hline 17 | 40 \\ \hline 12 \\ \hline 4 | 80 \\ \hline 4 \end{array}$$

$$3 | 20 \text{ Answ. } 22\text{l. } 17\text{s. } 04\frac{3}{4}\text{d.}$$

Note.

Note. If it be for 3 months, at 5 per cent. per annum, then you need only add $\frac{1}{4}$ of the principal to itself, working as before directed, &c.

675. Since the interest of any principal,

$$\text{at } \left\{ \begin{array}{l} 5 \\ 6 \\ 4 \\ 3 \end{array} \right\} \text{ per cent. for } \left\{ \begin{array}{l} 12 \\ 10 \\ 15 \\ 20 \end{array} \right\} \text{ Months.}$$

is just so many shillings as it has pounds. Therefore,

- (1.) What is 379*l.* 15*s.* for 12 months, at 5 per cent.?
 (2.) What is 461*l.* 12*s.* 6*d.* for 10 months, at 6 per cent.?
 $2|0 \quad 37|9 : 15$ $2|0 \quad 41|6 : 12 : 6d.$

$$\text{Answ. } 18 : 19 : 9$$

$$\text{Anfw. } 20 : 16 : 7\frac{1}{2}$$

- (3.) What is 427*l.* 13*s.* 9*d.* for 20 months, at 3 per cent.?
 (4.) What is 927*l.* 11*s.* 3*d.* for 15 months, at 4 per cent.?
 $2|0 \quad 42|7 : 13 : 9$ $2|0 \quad 92|7 : 11 : 3$

$$\text{Anfw. } 21 : 7 : 8\frac{1}{2}$$

$$\text{Anfw. } 46 : 7 : 6\frac{1}{2}$$

676. Hence, to find the interest of a given principal, for any other time,

Rule. Take parts, for the time required, out of the time affixed.

Thus if the interest of 379*l.* 15*s.* for 12 months, at 5 per cent. is so many shillings, then for 6 months, it is the $\frac{1}{2}$ of so many shillings, or so many six-pences; and for 4 months the $\frac{1}{3}$ of it, or so many groats, as the principal has pounds.

Example 1. What is the interest of 27*l.* 11*s.* for 4 months, at 5 per cent.?

$$2|0) \quad 27|9 : 11$$

$$3) \quad 13 : 19 : 6\frac{1}{2}$$

$$\text{Anfw. } 4 : 13 : 2$$

Example 2. What is the interest of 197*l.* 11*s.* for 2 months, at 6 per cent.?

$$2|0) \quad 19|7 : 11$$

$$5) \quad 9 : 17 : 6\frac{1}{2}$$

$$\text{Anfw. } 1 : 19 : 6$$

Example

Example 3. What is 491*l.* 17*s.* 10*d.* for 5 months, at 4 per cent.?

$$\begin{array}{r} 2|0) \ 49|1 : 17 : 10 \\ \hline \end{array}$$

$$\begin{array}{r} 3) \ 24 : 11 : 10\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \text{Answ. } 8 : 3 : 11\frac{1}{2} \\ \hline \end{array}$$

Example 4. What is 279*l.* 11*s.* 8*d.* for 7*½* months, at 4 per cent.?

$$\begin{array}{r} 2|0) \ 279 : 11 : 8 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \ 13 : 19 : 7 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Answ. } 6 : 19 : 9\frac{1}{2} \\ \hline \end{array}$$

For what odd money is adjoined to the principal, value every 20*s.* as 12*d.* and lesser sums in proportion.

677. Interest for days.

To find the exact number of days between any two given times, without any Book or Table.

This is done by carrying in memory that old distich, *viz.*

*Thirty days has September, April, June, and November,
February has twenty-eight alone; all the rest have thirty-one.*

And according to this reckoning, the year contains 365 days; therefore, for instance,

To find the number of days between the 7th of June, 1701, and the 15th of September, 1703; say thus,

From the 7th of June 1701, to the 7th of June 1703, is 365

2

730

or two years.

From the 7th of June 1703, to the end of the same month is

Then in { July 31
August 31

From which to the 15th of September, is 15

So that from the 7th of June 1701, to the 15th } of September 1703, is } days 830

Note. That 1 day must be added to every 4 years contained in the time. For every fourth year, being Leap-year, February has then 29 days.

After this manner may any number of days, even to $\frac{1}{2}$ or $\frac{1}{4}$ of a day, be exactly and readily discovered. Now,

678. If the interest of 100*l.* principal for one whole year, or 365 days be known, the simple interest of any other principal, for any number of days more or less than 365, may be found out by the following Rule, *viz.*

Multiply these three numbers, according to the Rule of continual Multiplication, to wit, the given interest of 100*l.* for a year, the principal, whose interest is required, and the number of days prescribed, reserving the last product for a Dividend: Also multiply 365 by 100, and reserve this product for a Divisor: Lastly, finish Division, so shall the Quotient be the interest or gain sought.

Note here, That the two principals, to wit 100*l.* and the other propounded, are supposed to be of one and the same denomination: Also the interest required will be of the same denomination with the given interest of 100*l.*

For an Example of this Rule, let it be required to find out the interest of 400*l.* for a week or 7 days, at the rate of 6*l.* for 100*l.* for a year, or 365 days: First multiplying these three numbers 6, 400, and 7 continually (*viz.* multiplying 6 by 400, and the product thence arising by 7) the last product will be 16800 for a Dividend; also multiplying 365 by 100, the product is 36500 for a Divisor: Lastly, dividing 16800 by 36500 (after cyphers at pleasure are added to 16800) the Quotient (according to Art. 401.) will be discovered to be this Decimal .4602, which is equal to 9*s.* 2*d.* 1*f.*

679. The reason of the above-mentioned Rule for the computing of interest for days, will be manifest by this following way of solving the same question by two single Rules of Three, *viz.*

$$\text{I. } 100 : 6 :: 400 : \frac{6 \times 400}{100}$$

$$\text{II. } 365 : \frac{6 \times 400}{100} :: 7 : \frac{6 \times 400 \times 7}{365 \times 100}.$$

680. Or the same will appear from the universal Rule of proportion, Chap. 30. Art. 406.

	Princip.	Inter.	Days.
Conditional Terms	100	6	365
Corresponding	400	A	7

Now it is evident that the principal and time produce the interest.

$$\text{Therefore } A = \frac{400 \times 7 \times 6}{365 \times 100} \text{ as above.}$$

681. Hence it appears, that the above directed operation may be often contracted by the method in Art. 266.

682. But the most concise way of computing the interest is by using Decimal Fractions ; for Example, if it be required to find the interest of 2156*l.* 13*s.* 3*d.* for 6*½* years, at 4*½**l.* per cent. ?

$$2156\text{l. } 13\text{s. } 3\text{d.} = 2156,6625 \text{ and } 4\frac{1}{2}\text{l.} = 4,5$$

5,4 Multiplier inverted.

$$\begin{array}{r} 862665 \\ 107833 \end{array}$$

9704,98 this product divided by

$$100 \text{ will be } 97,0498 \quad 6\frac{1}{2} = 6,75$$

57,6 Multiplier inverted.

$$\begin{array}{r} 5822988 \\ 679349 \\ 48525 \end{array}$$

Answ. $655,0862 = 655\text{l. } 1\text{s. } 8\frac{1}{2}\text{d.}$

683. And to prevent the trouble of dividing by 365 in every computation of interest for days, the following Tables have been calculated ; the first term of each of which, is the interest of 1*l.* for one day at the given rate ; that is, ,000136986, the first number of the Table of 5 per cent. is found from

$\left(\frac{1 \times 1 \times 5}{365 \times 100} \right)$ the general Rule given in Art. 678, 679, and

680. And the other numbers are made by Addition ; the second number being the double of the first ; the third the sum of the first and second, &c.

*TABLES of Simple Interest serving at any rate
for any time.*

	2 per cent.	3 per cent.	$3\frac{1}{2}$ per cent.
1	,0000547945	,0000821918	,0000958904
2	,0001095890	,0001643836	,0001917808
3	,0001643836	,0002465753	,0002876712
4	,0002191781	,0003287671	,0003835616
5	,0002739726	,0004109589	,0004794521
6	,0003287671	,0004931507	,0005753425
7	,0003835616	,0005753425	,0006712329
8	,0004383561	,0006575342	,0007671233
9	,0004931506	,0007397260	,0008630137
10	,0005479452	,0008219178	,0009589041
20	,0010958904	,0016438356	,0019178082
30	,0016438356	,0024657534	,0028767123
40	,0021917808	,0032876712	,0038356164
50	,0027397260	,0041095890	,0047945205
60	,0032876712	,0049315068	,0057534247
70	,0038356164	,0057534246	,0067123288
80	,0043835616	,0065753425	,0076712329
90	,0049315069	,0073972603	,0086301370
100	,0054794521	,0082191781	,0095890411
200	,0109589042	,0164383562	,0191780822
300	,0164383563	,0246575343	,0287671233
	Months.		
3	,005	,0075	,00875
6	,01	,015	,0175
9	,015	.0225	,02625
	Years.		
1	,02	,03	,035
2	,04	,06	,070
3	,06	,09	,105
4	,08	,12	,140
5	,10	,15	,175
6	,12	,18	,210
7	,14	,21	,245
8	,16	,24	,280
9	,18	,27	,315
10	,20	,30	,350

	4 per cent.	4½ per cent.	5 per cent.
1	,0001095890	,0001232877	,0001369863
2	,0002191780	,0002465753	,0002739726
3	,0003287671	,0003698630	,0004109589
4	,0004383561	,0004931506	,0005479452
5	,0005479452	,0006164383	,0006849315
6	,0006575342	,0007397259	,0008219178
7	,000671232	,0008630136	,0009589041
8	,0008767123	,0009863012	,0010958904
9	,0009863013	,0011095899	,0012328767
10	,0010958904	,0012328767	,0013698630
20	,0021917808	,0024657534	,0027397260
30	,0032876712	,0036986301	,0041095890
40	,0043835616	,0049315068	,0054794520
50	,0054794520	,0061643835	,0068493150
60	,0065753424	,0073972602	,0082191780
70	,0076712328	,0086301369	,0095890410
80	,0087671232	,0098630136	,0109589040
90	,0098630137	,0110958904	,0123287671
100	,0109589041	,0123287671	,0136986301
200	,0219178082	,0246575342	,0273972602
300	,0328767123	,0369863013	,0410958903
	Months.	.	.
3	,01	,01125	,0125
6	,02	,0225	,025
9	,03	,03375	,0375
	Years.		
1	,04	,045	,05
2	,08	,090	,10
3	,12	,135	,15
4	,16	,180	,20
5	,20	,225	,25
6	,24	,270	,30
7	,28	,315	,35
8	,32	,360	,40
9	,36	,405	,45
10	,40	,450	,50

6 per

	6 per cent.	7 per cent.	8 per cent.
1	,0001643836	,0001917808	,0002191780
2	,0003287671	,0003835616	,0004383561
3	,0004931506	,0005753424	,0006575342
4	,0006575342	,0007671232	,0008767122
5	,0008219178	,0009589041	,0010958904
6	,0009863013	,0011506849	,0013150684
7	,0011506849	,0013424657	,0015342464
8	,0013150684	,0015342465	,0017534244
9	,0014794520	,0017260275	,0019726026
10	,0016438356	,0019178082	,0021917808
20	,0032876712	,0038356164	,0043835616
30	,0049315068	,0057534246	,0065753424
40	,0065753424	,0076712328	,0087671232
50	,0082191781	,0095890411	,0109589041
60	,0098630136	,0115068492	,0131506848
70	,0115068492	,0134246574	,0153424656
80	,0131506848	,0153424656	,0175342464
90	,0147945206	,0172602738	,0197260273
100	,0164383562	,0191780822	,0219178082
200	,0328767124	,0383561644	,0438356166
300	,0493150686	,0575342466	,0657534249
Months.			
3	,015	,0175	,02
6	,03	,035	,04
9	,045	,0525	,06
Years.			
1	,06	,07	,08
2	,12	,14	,16
3	,18	,21	,24
4	,24	,28	,32
5	,30	,35	,40
6	,36	,42	,48
7	,42	,49	,56
8	,48	,56	,64
9	,54	,63	,72
10	,60	,70	,80

Explication.

684. The first column of every page being time, against it stands the interest of one pound for that time, at the rate *per cent.* specified on the top of the column, and at the same view you have the interest of 10*l.* 100*l.* 1000*l.* and all in one line. Let the interest of 500*l.* for 73 Days at 5 *per cent.* be required.

Then, against 70 days stands	,0095890410
Against 3 days stands	,0004109589

Therefore the interest of 1*l.* } is ,0099999999
for 73 days ————— } or ,01

Now since the interest at 5 *per cent.* for 73 days,

	<i>l.</i>	<i>l. s. d.</i>
Of 1 is	.01 or	2 : $\frac{1}{4}$
That of 10 is	.10 or	02 : 0 : 0
And of 100 is	1.00 or	1 : 00 : 0 : 0

Also of 1.11 or 1 : 02 : 2 : $\frac{1}{4}$

So that one pound after that manner being found to be the interest of 100*l.* at 5 *per cent.* *per annum* for 73 days, 5 times that money, to wit, 5*l.* must therefore be the interest of 500*l.* Again, if the interest of 111*l.* at the same rate and for the same time were required; take the total of the above sums for an answer, being 1.11, or 1*l.* 2*s.* 2*d.* So that let the given principal be what number of pounds, shillings and pence it will, it may be either taken in parts, as 111*l.* above, or by multiplying the interest of one pound by the given principal; so that no sum, at any of the inserted rates or time, can require more than one Multiplication.

685. Although this is calculated only for ten years, beyond which time simple interest seldom happens, yet the interest from thence for any longer time may as easily and readily be found, *viz.*

Example. Unto what amounts the interest of 79*l.* 15*s.* for 16 years and 73 days, at 5 *per cent.* *per annum*?

The interest of 1 <i>l.</i> for —	10 years is ,50
	6 years is ,30
	73 days is ,01

Therefore the interest of 1*l.* } is ,81 which multiplied by
for 16 years 73 days ————— } the principal produces the answer.

$$\begin{array}{r} 7975 \\ - 381 \\ \hline \end{array}$$

$$\begin{array}{r} 7975 \\ - 63800 \\ \hline \end{array}$$

64,5975 Answer 64*l.* 11*s.* 1*d.*

686. Likewise may the interest of any sum at any other rate per cent. whatsoever be readily found, according to the directions in Art. 675.

Example 2. What comes the interest of 432*l.* 10*s.* for 1 year $\frac{1}{2}$, at $5\frac{1}{2}$ per cent. per annum?

First take the interest of one pound for the given time at 5 per cent. per annum, which is } .06250
To which adding one tenth } .00625
gives the interest of one pound at $5\frac{1}{2}$ per cent. } ,06875
Multiply'd by the given principal } 432,5

$$\begin{array}{r} 34375 \\ + 3750 \\ \hline 20625 \\ + 27500 \\ \hline 29,734375 \end{array}$$

Ans. 29*l.* 14*s.* 08*d.*

29,734375

687. When an annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount to, simple interest being computed for every particular yearly payment, from the time it became due, until the end of the term of years, the Work will be as in this following Example, viz. If an annuity, or yearly rent of 134*l.* 10*s.* 6*d.* be all forborn till the end of 4 years, what will it then amount to, simple interest being allowed at the rate of 6 per cent. per annum, for each year's rent, from the time on which it was due, until the end of the said term of 4 years? Ans. 6*l.* 9*s.* 5*d.*

It is evident by the question, that at the rate of interest proposed, there must be computed the interest of 134*l.* 10*s.* 6*d.* (due at the third year's end) for one year (to wit, the fourth year;) also the interest of the like sum due at the second year's end, for 2 years (to wit, the 3d and 4th years;) likewise the interest of the same sum due at the first year's end for three

years (to wit, the second, third, and fourth years:) All which interest being added to the sum of the four years rent, the total sum shews what the said annuity will amount to at the end of the said term of 4 years.

Explication.

		l.	s.	d.
The interest of 134 <i>l.</i> 10 <i>s.</i> 6 <i>d.</i> at 6 per cent. per annum.	for	1 is . . .	8 : 1 :	5,16
		2 is . . .	16 : 2 :	10,32
		3 is . . .	24 : 4 :	3,48
The sum of the 4 years rent (to wit, 4 times 134 <i>l.</i> 10 <i>s.</i> 6 <i>d.</i>)		is . . .	538 : 2 : 0	
All which added together, give the answer of the question, to wit,			586 : 10 : 6,96	

688. When it is required to find out how much ready money will satisfy a debt due at the end of any space of time to come, by rebating or discounting at a given rate of simple interest, it may be effected by this Rule, *viz.* first, find out the interest of 100*l.* at the settled rate of interest, for the time which the ready money is to be paid before-hand; then adding the interest so found to 100*l.* make always the sum of that addition the first term in an operation of the Rule of Three; 100*l.* the second term; and the debt proposed to be satisfied the third term; lastly, the fourth proportional found out by the said operation, will be the ready money which ought to be paid in satisfaction of the debt propounded.

Example 1. If a debt of 100*l.* be payable at the end of a year to come, how much ready money will discharge that debt, by rebating or discounting at the rate of 6 per cent. per annum? Answ. 94*l.* 6*s.* 9*d.* 2*far.* very near; for by the Rule of Three,

$$106 : 100 :: 100 : 94,3396 +$$

That is to say, if 106*l.* (which is composed of 100*l.* principal, and 6*l.* interest) proceeds from 100*l.* principal forbore for a year, what principal forbore for a year does 100*l.* (composed of principal and interest) proceed from? Answ. 94,3396*l.* +, (or 94*l.* 6*s.* 9*d.* very near) principal money; therefore 94*l.* 6*s.* 9*d.* in ready money, is of equal value with 100*l.* due at the end of a year to come; for if the said 94*l.* 6*s.* 9*d.* be put forth at interest for a year, at the rate of 6 per cent. per annum, it will gain 5*l.* 13*s.* 2*d.* very near, which together with the said 94*l.* 6*s.* 9*d.* makes the 100*l.* first proposed to be discharged by rebate.

Example

Example 2. If 150*l.* 10*s.* be payable at the end of 37 days to come, how much present money will discharge the said debt, by rebating after the rate of 6 per cent. per annum? Answ. 148*l.* 14*s.* 3*½ d.* +, as by the following operation is manifest.

$$\begin{array}{ccccccc} \text{days.} & l. & \text{days.} & l. \\ \text{I.} & 365 : 6 & :: & 73 : 1,2 \\ & l & l & l & l \\ \text{II.} & 101,2 : 100 & :: & 150,5 : 148,7154 + \end{array}$$

That is to say, first, I seek the interest of 100*l.* for 73 days, at the rate of interest proposed, by the above process, or by the Table? Answ. 1, $\frac{2}{15}$ *l.* or 1, $\frac{1}{2}$ *l.* Then adding the said 1, $\frac{1}{2}$ to 100, say, if 101, $\frac{1}{2}$ *l.* principal and interest, payable at the end of 73 days to come, be equivalent to 100*l.* ready money, what ready money is 150*l.* 10*s.* (or 150, $\frac{1}{2}$) payable at the end of 73 days to come equivalent unto? So by multiplying and dividing (according to the Rules of Decimal Multiplication and Division explained in Chap. 28. and 29.) the Quotient or answer of the question will be found 148,7154 +; that is, 148*l.* 14*s.* 3*½ d.* +; for the Decimal ,7154 being valued according to Art. 365. will by inspection only be discovered to be 14*s.* 3*½ d.* which Rule I shall here once for all, advise the learner to be well acquainted with.

The Proof.

Seek what the ready money found as aforesaid will gain, in so much time as it is paid beforehand, at the rate of interest propounded: Then having added this gain to the said ready money, if the sum be equal to the debt first proposed to be satisfied by rebate, the ready money was rightly found out. So the last Example will be thus proved:

$$\begin{array}{ccccccc} l. & l. & l. & l. \\ 100 : 1,2 & :: & 148,7154 & : & (1,7845 \end{array}$$

Which fourth proportional 1,7845 being added to 148,7154, the sum will be 150,4999 +, which does not want a farthing of 150*l.* 10*s.* the debt first proposed.

689. When it is required to find the present worth of an annuity, by rebating or discounting at a given rate of simple interest, the operation will be as in the following Example, viz. How much present money is equivalent to an annuity or rent of 100*l.* per annum, to continue 5 years, rebate being made at the rate of 6*l.* for 100*l.* for one year at simple interest? Answ. 425*l.* 18*s.* 9*½ d.* very near.

It is manifest that there must be computed the present worth of 100*l.* due at the first year's end, also the present worth of 100*l.* due at the second year's end, and in like manner for the third, fourth, and fifth years; all which particular present worths being added together, the aggregate or sum will be the total present worth of the annuity.

The operation by Decimals is as follows:

	<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>	
1.	106 : 100 :: 100 : 94,33962				+
2.	112 : 100 :: 100 : 89,28571				+
3.	118 : 100 :: 100 : 84,74576				+
4.	124 : 100 :: 100 : 80,64516				+
5.	130 : 100 :: 100 : 76,92307				+

Answ. 425,93932 +

C H A P. XLI.

Of COMPOUND INTEREST.

690. **C**OMPOUND INTEREST is derived from the following supposition, *viz.* that the interest of money ought to be punctually paid when due (which is commonly at the end of a year from the time of lending the principal) and that, if it be not so done, then the said interest should be considered as, and added to the, principal from that time; and interest be computed on their sum for the next year or other period of time.

Suppose, for instance, that 100*l.* were lent for one year, at 5 per cent. then, if the 5*l.* interest be not paid at the year's end, there will arise a principal of 105*l.* on which interest must be computed for the second year; which by Art. 668, will amount to 5*l.* 5*s.* now, if this be not paid at the end of the second year, then, to the second year's principal 105*l.* add its interest 5*l.* 5*s.* and the sum 110*l.* 5*s.* will be the principal, on which interest must be computed for the third year; and so on for succeeding years.

691. But since the interest of any sum of money for one year is found by the following proportion. See Art. 668.

As

As $100l.$: to its interest :: the given principal : to its interest,
Therefore by composition, see Art. 553. Case 5.

As $100l.$: to $100l.$ + its interest :: the given principal : to
the principal + its interest.

692. Whence the amounts of any given principal (suppose $300l.$) for any number of years (suppose 4) at any rate (suppose 5%) per cent. may be found as follows, viz.

$$100 : 105 :: \left\{ \begin{array}{l} 300 : 315 \\ 315 : 330,75 \\ 330,75 : 347,2875 \\ 347,2875 : 364,651875 \end{array} \right\} \text{The amount of } 300l. \text{ and its interest at } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{years.}$$

693. But the principal, and the several amounts, are a rank of numbers in Geometrical Progression, increasing in proportion as 100 to 105, or as 1 to 1.05.

As will appear from their production, or may be proved from the equality of the products of any extreme and mean terms.

694. Since by Art. 692.

As $100l.$: its amount :: any principal : its amount.
Therefore $100l.$: its amount :: $1l.$: its amount.

And $1l.$: its amount :: any principal : its amount.

695. Now the amounts of $1l.$ for any number of years at any rate per cent. (suppose 5%) will be the powers of (1.05) the amount of $1l.$ for one year by Art. 693. and 547.

696. Therefore, if the powers of the amount of $1l.$ for one year be first calculated, the amount of any principal may be found by Multiplication only.

Thus, if the powers of 1.05 be computed, they, with their indexes, will be as follow :

$$\begin{matrix} 0 & 1 & 2 & 3 & 4 & \text{&c. indexes.} \\ 1 & 1.05 & 1.1025 & 1.157625 & 1.21550625 & \text{&c. powers.} \end{matrix}$$

And the amount of $300l.$ in 4 years at 5% per cent. will be the product of (1.21550625) the 4th power of 1.05, multiplied by the principal $300l.$ See the operation,

$1,21550625$

$\overline{300}$

$364,65187500$

697. In order to facilitate the computations relating to compound interest, the powers of the amounts of $1l.$ for one year at the several rates therein mentioned, are inserted in the following Table.

TABLE

TABLE I.

Which shews what one pound will amount to, being forborn to the end of any number of years under 6¹, compound interest being computed yearly, at any of these rates, viz. 2, 3, 3*1*, 4, 4*1*, 5, 6, 7, and 8 per cent. per annum.

Years.	at 2 per C _{t.}	3 per C _{t.}	3 <i>1</i> per C _{t.}	4 per C _{t.}	4 <i>1</i> per C _{t.}	5 per C _{t.}	6 per C _{t.}	7 per C _{t.}	8 per C _{t.}
1	1,00000	1,03000	1,03500	1,04000	1,04500	1,05000	1,06000	1,07000	1,08000
2	1,04040	1,06090	1,07123	1,08160	1,09203	1,10250	1,12360	1,14490	1,16640
3	1,06121	1,09773	1,10872	1,12486	1,14117	1,15763	1,19102	1,22504	1,25971
4	1,08243	1,12551	1,14752	1,16986	1,19252	1,21551	1,26248	1,31080	1,36049
5	1,10408	1,15927	1,18769	1,21665	1,24618	1,27628	1,33823	1,40255	1,46933
6	1,12616	1,19405	1,22926	1,26532	1,30226	1,34010	1,41852	1,50073	1,58687
7	1,14869	1,22987	1,27228	1,31593	1,36086	1,40710	1,50363	1,60578	1,71382
8	1,17166	1,26677	1,32677	1,36881	1,36857	1,42210	1,47746	1,59385	1,71819
9	1,19509	1,30477	1,36290	1,42331	1,48610	1,55133	1,68948	1,83846	1,99901
10	1,21899	1,34392	1,41060	1,48024	1,55297	1,62890	1,79085	1,96715	2,15893
11	1,24337	1,38423	1,45997	1,53945	1,62285	1,71034	1,89830	2,10485	2,33164
12	1,26824	1,42576	1,51107	1,60103	1,69588	1,79586	2,01220	2,25219	2,51817
13	1,29361	1,46853	1,56396	1,66507	1,77220	1,88565	2,13293	2,40985	2,71962
14	1,31948	1,51259	1,61870	1,73168	1,85195	1,97993	2,26090	2,57853	2,93719
15	1,34587	1,55797	1,67535	1,80994	1,93528	2,07893	2,39656	2,75993	3,17217

TABLE E

TABLE I.

Which shews what one pound will amount to, being born to the end of any number of years under 61, compound interest being computed yearly, at any of these rates, viz. 2, 3, 3*½*, 4, 4*½*, 5, 6, 7, and 8 per cent. per annum.

Years. at 2 per Cent.	3 per Cent.	3 <i>½</i> per Cent.	4 per Cent.	4 <i>½</i> per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
16	1,37279	1,60471	1,73399	1,87298	2,02237	2,18288	2,54035	2,95216
17	1,40024	1,65285	1,79468	1,94790	2,11338	2,29202	2,69277	3,15882
18	1,42825	1,70243	1,85749	2,02582	2,20848	2,40662	2,85434	3,37993
19	1,45681	1,75351	1,92250	2,10685	2,30786	2,52695	3,02560	3,61653
20	1,48595	1,80611	1,98979	2,19112	2,41171	2,65330	3,20714	3,86968
21	1,51567	1,86030	2,05943	2,27877	2,52024	2,78596	3,39956	4,14056
22	1,54598	1,91610	2,13151	2,36992	2,63365	2,92526	3,60354	4,43040
23	1,57690	1,97359	2,20611	2,46472	2,75217	3,07152	3,81975	4,74053
24	1,60844	2,03279	2,28333	2,56330	2,8601	3,22510	4,04894	5,07237
25	1,64061	2,09378	2,36325	2,66584	3,00543	3,38636	4,29187	5,42743
26	1,67342	2,15659	2,44596	2,77247	3,14068	3,55567	4,54938	5,80735
27	1,70689	2,22129	2,53157	2,88337	3,28201	3,73346	4,82235	6,21387
28	1,74102	2,28793	2,62017	2,99870	3,42970	3,92013	5,11169	6,64884
29	1,77585	2,35657	2,71188	3,11865	3,58404	4,11614	5,41839	7,11426
30	1,81136	2,42726	2,80679	3,24340	3,74532	4,32194	5,74349	7,61226

TABLE

TABLE I.

Which shews what one pound will amount to, being born to the end of any number of years under 61, compound interest being computed yearly, at any of these rates, viz. 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 per cent. per annum.

Years.	at 2 per Cts.	3 per Cts.	3½ per Cts.	4 per Cts.	4½ per Cts.	5 per Cts.	6 per Cts.	7 per Cts.	8 per Cts.
31	1,84759	2,50008	2,90503	3,37313	3,91386	4,5384	6,08810	8,14511	10,86767
32	1,88454	2,57508	3,00671	3,50806	4,08998	4,76494	6,45339	8,71527	11,73703
33	1,92223	2,65234	3,11194	3,64838	4,27403	5,00319	6,84059	9,32534	12,6605
34	1,96068	2,73191	3,22086	3,79432	4,46636	5,25335	7,25103	9,97811	13,69013
35	1,99989	2,81386	3,33359	3,94609	4,66735	5,51602	7,68609	10,67658	14,78534
36	2,03989	2,89828	3,45027	4,10293	4,87738	5,79182	8,14725	11,42394	15,96817
37	2,08069	2,98523	3,57103	4,26809	5,09686	6,08141	8,63609	12,22362	17,24563
38	2,12230	3,07478	3,69601	4,43881	5,32622	6,38548	9,15425	13,07927	18,62528
39	2,16475	3,16703	3,82537	4,61637	5,56590	6,70475	9,70351	13,99482	20,11530
40	2,2084	3,26204	3,95926	4,80102	5,81637	7,03999	rc,28572	14,97446	21,72452
41	2,25220	3,35990	4,09783	4,99306	6,07810	7,39199	rc,90286	16,02267	23,46248
42	2,29724	3,46070	4,24126	5,19278	6,35162	7,76159	rc,55703	17,14426	25,33948
43	2,34319	3,56452	4,38970	5,40050	6,63744	8,14967	rc,25046	18,34436	27,36664
44	2,39005	3,67145	4,54334	5,61652	6,93612	8,55715	rc,98548	19,62846	29,55597
45	2,43785	3,78160	4,70236	5,84118	7,24825	8,98501	rc,76461	21,00245	31,92045

TABLE

TABLE I.

Which shews what one pound will amount to, being forbore to the end of any number of years under 6 $\frac{1}{2}$, compound interest being computed yearly, at any of these rates, viz. 2, 3, 3 $\frac{1}{2}$, 4,
4 $\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum.

Years.	at 2 per C.	3 per C.	3 $\frac{1}{2}$ per C.	4 per Cent.	4 $\frac{1}{2}$ per Ct.	5 per Ct.	6 per Cent.	7 per Cent.	8 per Cent.
46	2,48661	3,89504	4,86694	6,07482	7,57442	9,43426	14,59049	22,47262	34,47409
47	2,53634	4,01190	5,03728	6,31782	7,91527	9,90597	15,46592	24,04570	37,23201
48	2,58707	4,13225	5,21359	6,57053	8,27146	10,40127	16,39387	25,72891	40,21057
49	2,63881	4,25622	5,39607	6,83335	8,64367	10,92133	17,37750	27,52993	43,42742
.50	2,69159	4,38391	5,58493	7,10668	9,03264	11,46740	18,42015	29,45703	46,90161
51	2,74542	4,51542	5,78040	7,39095	9,43911	12,04077	19,52536	31,51902	50,65374
52	2,80033	4,65089	5,98271	7,68659	9,86387	12,64281	20,69689	33,72535	54,70604
53	2,85634	4,79041	6,19211	7,99405	10,30774	13,27495	21,93870	36,08612	59,08252
54	2,91346	4,93413	6,40883	8,31381	10,77159	13,93870	23,25502	38,61215	63,80913
55	2,97173	5,08215	6,63314	8,64637	11,25631	14,63563	24,65032	41,31500	68,91386
56	3,03117	5,23461	6,86530	8,99222	11,76284	15,36741	26,12934	44,20705	74,42697
57	3,09179	5,39165	7,10559	9,35191	12,29217	16,13578	27,69710	47,30155	80,38112
58	3,15362	5,55340	7,35428	9,72599	12,84532	16,94257	29,35893	50,61265	86,81161
59	3,21670	5,72000	7,61168	10,11503	13,42336	17,78970	31,12046	54,15554	93,75654
60	3,28103	5,89160	7,87809	10,51963	14,02741	18,67919	32,98769	57,94643	101,25706

The Construction of the preceding Table.

698. The numbers 1, 2, 3, 4, &c. to 60, in the first column on the left-hand signify years; the numbers 2, 3, 3 $\frac{1}{2}$, 4 $\frac{1}{2}$, 5, 6, 7, and 8, placed at the head of the rest of the columns, denote rates of interest for 100*l.* lent for a year, and the numbers set in the several columns under those rates of interest, are the powers of the numbers which stand on the first line, which are found by continually multiplying by those numbers. Thus in the column under 5.

$$\begin{array}{c} \text{The number } \left\{ \begin{array}{l} 1,05 \\ 1,1025 \\ 1,15763 \\ 1,21551 \end{array} \right\} \text{ Standing against } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{ is the amount} \\ \text{of } 1\text{l. in } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{ Years at } \left\{ \begin{array}{l} 5 \\ 10 \\ 15 \\ 20 \end{array} \right\} \text{ per Cent.} \end{array} = 1,05 \times 1 \\ = 1,05 \times 1,05 \\ = 1,05 \times 1,1025 \\ = 1,05 \times 1,15763$$

The calculation of which numbers will be very easy, if performed according to the following method :

$$\begin{array}{r} \text{To } 1,05 \\ \text{Add } 525 \\ \hline \end{array} = 1,05 \times .05 = \frac{1}{20} \times 1,05$$

$$\begin{array}{r} \text{The sum } = 1,1025 \\ \text{Add } 55125 \\ \hline \end{array} = 1,05 \times 1,05 \quad \quad \quad = 1,1025 \times .05 = \frac{1}{20} \times 1,1025$$

$$\begin{array}{r} \text{The sum } = 1,157625 \\ \text{Add } 57881 \\ \hline \end{array} = 1,1025 \times 1,05 \quad \quad \quad = 1,157,625 \times .05 = \frac{1}{20} \times 1,157,625$$

$$\text{The sum } = 1,215506 = 1,157,625 \times 1,05$$

699. PROBLEM I.

The Principal, Rate, and Time being given, to find the Amount.

Example. What will 136*l.* 15*s.* 6*d.* be augmented to, being forborn 20 years, interest upon interest being computed at the rate of 6 per cent. per annum? Answ. 438*l.* 13*s.* 1 $\frac{1}{2}$ *d.* very near, which is thus found out.

First, looking into the 7th column of the above Table, to wit, that column which has the figure 6 placed at the head of it, right against 20 years is the number 3,20714, which shews that 1*l.* being continued 20 years at 6 per cent. per annum, compound interest, and all forborn until the end of the said term, will be augmented to 3,20714*l.* (that is, 3*l.* 4*s.* 1*d.* 2*f.* and somewhat more;) therefore after the 15*s.* 6*d.* in the question is reduced to the Decimal ,775, multiply the said tabular number 3,20714, by 136,775 (the principal pro-

pounded in the question), so the product will be 438,6565,
&c. that is, 438*l.* 13*s.* 1½*d.* for the answer of the question.
View the operation here following:

$$\begin{array}{r}
 136,7750 \\
 41702,3 \\
 \hline
 4103250 \\
 273550 \\
 9574 \\
 137 \\
 54 \\
 \hline
 438,6565
 \end{array}$$

700. PROBLEM 2.

The Principal, Rate, and Amount being given, to find the Time.

Example. A person having lent 136*l.* 15*s.* 6*d.* at 6*l. per cent.* compound interest, received 438*l.* 13*s.* 1½*d.* in full payment; how many years was the money out of his hands?

Divide (438,6565) the amount, by (136,775) the principal and find (3,207, &c.) the Quotient, or the nearest number to it, in the Table, in the column under (6) the rate: So shall (20) the number standing in the left-hand column, even therewith, be the time required; exact, if the Quotient be exactly found in the Table, or nearly so, if otherwise.

701. PROBLEM 3.

The Principal, Time, and Amount being given, to find the Rate.

Example. A person having lent 136*l.* 15*s.* 6*d.* received in full payment thereof, at the end of 20 years, 438*l.* 13*s.* 1½*d.* at what rate *per cent.* was compound interest computed?

Divide (438,6565) the amount, by (136,775) the principal, and find (3,207, &c.) the Quotient, or the nearest number thereto, in the Table, on a line with the given time (20); so shall (6) the number at the head of the column be the rate required; exact, if the Quotient be exactly found in the Table, or nearly so, if otherwise.

After the same manner, the numbers belonging to any of the other rates of interest mentioned in the first Table, are to be used.

702. When an annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount to, compound interest being computed for each particular payment, from the time it became due until the end of the term of years, the Work will be as in the following Example, *viz.* suppose an annuity of 300*l.* payable at yearly payments be forborn, and all unpaid until the end of four years; the question is, what will then be due, compound interest being computed at the rate of 5 per cent. per annum, for every yearly payment from the time it becomes due, to the end of the said term of four years? Answ. 1293*l.* 0*s.* 9*d.*

It is evident by the question, that there must be computed what 300*l.* due at the third year's end will be augmented to in one year, (to wit, the fourth year) at 5 per cent. Also what 300*l.* due at the second year's end will be increased to in two years, (to wit, the third and fourth years;) likewise what 300*l.* due at the first year's end will be augmented to, in the three following years, (to wit, the second, third, and fourth years,) all which sums being added to 300*l.* (the payment due at the end of the fourth year, which is incapable of any improvement,) the aggregate or sum will be the total money in arrear at the end of the fourth year, to wit, 1293,0375*l.* as may appear by the following operation, *viz.*

The last payment of the annuity due at the end of the fourth year is	300
Again the 300 <i>l.</i> due at the third year's end, will in one year, after the rate of 5 per cent. be augmented to	315
Also 300 <i>l.</i> due at the second year's end will in two years, at the rate of 5 per cent. per an- num, compound interest, be increased to, (as appears by Art. 692.)	330,75
In like manner 300 <i>l.</i> due at the first year's end, will in three years be augmented to	347,2875
The sum due at four years end is	1293,0375

703. The invention of the numbers before-mentioned being well examined, it will appear, that if an annuity or rent payable at yearly payments be improved to the utmost at interest upon interest, and all forborn or respite to the end of certain years, the total then due will be the sum of a Geometrical Progression consisting of as many terms as there are yearly payments, the first of which proportionals is the annuity or rent:

And

And the second proportional proceeds from the first in the same rate as 105 proceeds from 100, if the rate of interest be 5 per cent. (or as 108 proceeds from 100, if the rate of interest be 8 per cent. &c.) and so likewise the third from the second, the fourth from the third, &c. after the manner of the operation in Art. 692.)

704. The Construction of the following Table II.

Upon the aforesaid grounds, the following Table II. is calculated, to shew what one pound Annuity, payable at yearly payments, and forborn any number of years under 61 will amount to, by computing interest upon interest, at any of the rates expressed at the head of the said Table.

And the same Table may be easily composed, by the addition of the numbers in the preceding Table I. in this manner:

To 1	standing under 5 in the latter Table,
Add 1,05	the first number in the former Table,
Makes 2,05	the second number in the latter Table.
Add 1,1025	the second number in the former Table,
Makes 3,1525	the third number in the latter Table.
Add 1,15763	the third number in the former Table,
Makes 4,31013	the fourth number in the latter Table,
&c.	

The numbers in the preceding Table I. ought to be continued to more places than are there expressed; to prevent the errors, which may happen by the adding of defective Decimal Fractions.

TABLE II.

Which shows what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound interest being computed at any of these rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
2	2,02000	2,03000	2,03500	2,04000	2,04500	2,05000	2,05600	2,07000	2,08000
3	3,06040	3,09090	3,10623	3,12160	3,13703	3,15250	3,18360	3,21490	3,24640
4	4,12161	4,18363	4,21494	4,24646	4,27819	4,31013	4,37462	4,43994	4,50611
5	5,20404	5,30914	5,36247	5,41632	5,47071	5,52563	5,63709	5,75074	5,86660
6	6,30812	6,46841	6,55015	6,63298	6,71689	6,80191	6,97532	7,15329	7,33593
7	7,43428	7,66246	7,77941	7,89829	8,01915	8,14201	8,39384	8,65402	8,92280
8	8,58297	8,89234	9,09169	9,321423	9,38001	9,54911	9,89747	10,25980	10,63663
9	9,75463	10,15911	10,36850	10,58280	10,80211	11,02656	11,49132	11,97799	12,48756
10	10,94972	11,46388	11,73139	12,00611	12,28821	12,57789	13,18080	13,81645	14,48656
11	12,16872	12,80780	13,14199	13,48635	13,84118	14,20679	14,97164	15,78360	16,64549
12	13,41209	14,19203	14,60196	15,02581	15,46403	15,91713	16,86994	17,88845	18,97113
13	14,68033	15,61779	16,11303	16,62684	17,15991	17,71298	18,88214	20,14064	21,49530
14	15,97394	17,08632	17,567699	18,29191	18,93211	19,59863	21,01507	22,55049	24,21492
15	17,29342	18,59891	19,29568	20,02359	20,78405	21,57856	23,27597	25,12902	27,15211

TABLE

Chap. XLII.

Interest.

TABLE II.

Which shews what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound interest being computed at any of these rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
16	18,63929	20,15688	20,97103	21,82453	22,71934	23,65749	25,67253	27,88805	30,32429
17	20,01207	21,576159	22,70502	23,69751	24,74171	25,84037	28,21288	30,94022	33,75023
18	21,41231	23,41444	24,49969	25,64541	26,85508	28,13239	30,90565	33,99903	37,45024
19	22,84056	25,11687	26,35718	27,67123	29,06356	30,53900	33,75999	37,37897	41,44626
20	24,29737	26,87037	28,27968	29,77808	31,357142	33,05595	36,78559	40,99549	45,76196
21	25,78332	28,67649	30,26947	31,96920	33,78314	35,71925	39,99273	44,86518	50,42292
22	27,29898	30,53678	32,32890	34,24797	36,30338	38,0521	43,39229	49,00574	55,45676
23	28,84495	32,45288	34,46041	36,61789	38,93703	41,43048	46,99583	53,43614	60,89330
24	30,42185	34,42647	36,66653	39,08260	41,68920	44,50200	50,85558	58,76667	66,76476
25	32,93030	36,45925	38,94986	41,64591	44,56521	47,72710	54,86451	63,24904	73,10594
26	33,67091	38,55304	41,31310	44,31175	47,57065	51,11345	59,15638	68,67647	79,95442
27	35,34432	40,70963	43,5905	47,08421	50,71132	54,66913	63,70577	74,48382	87,35077
28	37,05121	43,93092	46,29063	49,96758	53,99333	58,40258	68,52811	80,69769	95,33883
29	38,79224	45,28885	48,91080	52,96629	57,42303	62,32271	73,63980	87,34653	103,96594
30	40,56808	47,57542	51,62268	56,08494	61,90707	66,43885	79,05819	94,46079	113,8321

TABLE

TABLE

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TABLE II.

Which shows what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound interest being computed at any of these rates, to wit, 2, 3, $\frac{1}{2}$, 4, $\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per cent.	$\frac{1}{2}$ per cent.	3 per cent.	4 per cent.	4 $\frac{1}{2}$ per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
35	43,37944	50,000268	54,42947	59,32834	64,75239	70,76079	84,80168	102,97394	123,34587
32	44,22703	52,50276	57,33450	62,70147	68,66625	75,29883	90,88978	110,21815	134,21354
33	46,71157	55,07784	60,34121	66,20953	72,75623	80,06377	97,34317	118,93343	145,95062
34	48,03380	57,73018	63,45315	69,85791	77,03026	85,06696	104,18376	128,25877	158,62667
35	49,99449	60,46208	66,67401	73,65223	81,49662	90,32031	111,43478	138,23688	172,31680
36	51,99437	63,2794	70,00760	77,59831	86,16397	95,83632	119,12087	148,91346	187,10215
37	54,03426	66,17422	73,45787	81,70225	91,04334	101,62814	127,26812	160,33740	203,07032
38	56,11494	69,15945	77,02890	85,97034	96,13821	107,70955	135,90421	172,56192	220,31595
39	58,23724	72,23423	80,72491	90,40915	101,46442	114,09502	145,95846	185,64029	238,94122
40	60,40198	75,40126	84,5028	95,02552	107,03032	120,79977	154,76197	199,63511	259,05652
41	62,61002	78,66330	88,50954	99,82654	112,84669	127,83976	165,94768	214,60957	280,78104
42	64,83222	82,02320	92,60737	104,81960	118,92479	135,23175	175,95055	230,63224	304,24352
43	67,15947	85,48389	96,84863	110,01238	125,27640	142,99334	187,50758	247,77650	329,58301
44	69,50266	89,04841	101,23833	115,41288	131,91384	151,14301	199,75803	266,12085	356,94965
45	71,89271	92,71986	105,78167	121,02939	138,84997	159,70016	212,74351	285,74931	386,59562

TABLE

TABLE II.

Which shews what one pound annuity payable yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound interest being computed at any of these rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. *per annum*

Years	2 per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
46	74,3306	96,5015	110,4840	126,8706	146,0982	168,6852	226,5081	306,7518	418,4261
47	76,8172	100,3965	115,3510	132,9454	153,6726	178,1194	241,0986	329,2244	452,5002
48	79,3535	104,4084	120,3883	139,2632	161,5879	188,0254	256,5645	353,2701	490,1322
49	81,9406	108,5406	125,6018	145,8337	169,8594	198,4267	273,9584	378,9990	530,3427
50	84,5794	112,7969	130,9979	152,6671	178,5030	209,3480	290,1359	406,5289	573,7702
51	87,2740	117,1808	136,5828	159,7738	187,5357	220,8154	308,1561	435,9860	620,6718
52	90,0164	121,6962	142,3632	167,1647	196,9748	232,8562	328,2814	467,5050	671,3255
53	92,8167	126,3471	148,3460	174,8513	206,8386	245,4990	349,9783	501,2303	726,0316
54	95,6731	131,1375	154,5381	182,8454	217,1464	258,7739	370,9170	537,3164	785,1114
55	98,5865	136,0716	160,9469	191,1592	227,9180	272,7116	394,1720	575,9286	848,9232
56	101,5583	141,1538	167,5800	199,8055	239,1743	287,3482	418,8223	617,2436	917,8371
57	104,5894	146,3884	174,4453	208,7978	250,9371	302,7157	444,9517	661,4506	992,2640
58	107,6812	151,7800	181,5509	218,1497	263,2293	318,8514	472,6488	708,7522	1072,6451
59	110,8348	157,1334	188,9052	227,8757	276,0746	335,7940	502,0077	759,3648	1159,4568
60	114,0515	163,0534	196,5169	237,9987	289,4980	355,35837	537,1282	813,5204	1253,2133

705. The Use of the preceding Table II.

PROBLEM 4. The Annuity, Time and Rate being given, to find the Amount.

Example. If an annuity of $20\text{l}.$ payable by yearly payments for fifteen years, be all forborn or unpaid until the end of the said term, what will it then amount to, upon a computation of interest upon interest, at the rate of 6 per cent. per annum?

In the column which has (6) the rate at the head of it, and on a line with (15) the time, you will find (23,27597) the amount of an annuity of $1\text{l}.$ for the given time at the given rate; multiply that amount (23,27597) by the annuity (20) and the product (465,51940 or $465\text{l}. 10\text{s}. 4\frac{1}{2}\text{d}.$) will be the answer. View the following operation,

$$\begin{array}{r} 23,27597 \\ \times 20 \\ \hline 465,51940 \end{array}$$

706. PROBLEM 5. The Annuity, Amount and Time being given, to find the Rate.

Example. A person who had an annuity of $20\text{l}.$ suffered it to be in arrear for 15 years, and at the end of that time received $465\text{l}. 10\text{s}. 4\frac{1}{2}\text{d}.$ in full payment; at what rate was compound interest allowed him?

Divide (465,519, &c.) the amount, by (20) the annuity, and find (23,275, &c.) the Quotient or the nearest number to it, on a line with (15) the time; so shall (6) the number at the head of the column be the rate required; exact, if the Quotient be exactly found in the Table, or nearly so, if otherwise.

707. PROBLEM 6. The Annuity, Amount and Rate being given, to find the Time.

Example. In what time will an annuity of $20\text{l}.$ amount to $465\text{l}. 10\text{s}. 4\frac{1}{2}\text{d}.$ allowing compound interest at 6. per cent.?

Divide (465,519, &c.) the amount, by (20) the annuity, and find (23,275, &c.) the Quotient, or the number nearest to it, in the column under (6) the rate; so shall (15) the number standing in the left-hand column, even therewith, be the time required; exact, if the Quotient be exactly found in the Table, or nearly so, if otherwise.

708. PROBLEM 7. *The Amount, Time and Rate being given, to find the Annuity.*

Example. A person received 465*l.* 10*s.* 4*d.* in full payment of an annuity, which had been 15 years unpaid, and was allowed 6*l.* per cent. compound interest: What was the annuity?

In the column which has (6) the rate at the head of it, and on a line with (15) the time, you will find (23,27597) the amount of an annuity of 1*l.* for the given time at the given rate; by which divide (465,519, &c.) the given amount, and the Quotient (20) will be the annuity required.

709. When a sum of money is due at a time to come, and it is required to know what it is worth in ready-money, rebate being made at a given rate of compound interest, the Work will be the reverse of Art. 692. viz.: there must be found a Geometrical Progression whose first term is the money proposed to be rebated, the second must decrease or lessen from the first, the third from the second, &c. in such manner or rate as 100 decreases from 105, for as 100 from 108, if the rate of interest be 8 per cent.) then if the progression be continued to a number of terms, more by one than the number of years in question, the last term will be the required answer: So if 364,651875*l.* be due at the end of four years to come, it will be found to be worth in ready-money 300*l.* rebate being made at compound interest at 5 per cent. as by the four following proportions is manifest. See Art. 692.

$$105 : 100 :: \begin{cases} 364,651875 : 347,2875 \\ 347,2875 : 330,75 \\ 330,75 : 315 \\ 315 : 300 \end{cases}$$

710. Upon this ground the following Table III. is calculated, to shew what one pound due at the end of any number of years to come, is worth in present money, rebate being made, at the rates of compound interest, mentioned in the said Table; by the help of which, and of Multiplication, Questions of Rebates for any sum proposed may be performed without considerable error.

TABLE III.

Which shews what one pound, payable at the end of any number of years to come under £1, is worth in ready money; discount or rebate being yearly computed at any of these rates, to wit, 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 per cent. per annum compound interest.

Years.	2 per C _{t.}	3 per C _{t.}	3½ per C _{t.}	4 per C _{t.}	4½ per C _{t.}	5 per C _{t.}	6 per C _{t.}	7 per C _{t.}	8 per C _{t.}
1	,980392	,970874	,966184	,961538	,956938	,952381	,934579	,925926	
2	,961170	,942596	,933511	,924556	,915730	,907029	,889996	,873439	,857339
3	,942322	,915142	,901943	,888996	,876297	,863838	,839619	,816298	,793832
4	,923845	,888487	,871442	,854804	,838561	,822702	,792094	,762895	,735030
5	,905732	,862609	,841973	,821927	,802451	,783526	,747258	,712986	,680583
6	,887971	,837484	,813501	,790315	,767896	,746215	,704961	,666342	,630170
7	,870560	,813092	,785991	,759918	,734828	,710681	,665057	,622750	,583490
8	,853490	,789409	,759412	,730690	,703185	,676839	,627412	,582009	,550269
9	,836755	,766417	,733731	,702587	,672904	,644609	,591898	,543934	,500249
10	,820348	,744094	,708919	,675564	,643928	,613913	,558395	,508349	,463193
11	,804263	,722421	,684946	,649581	,616199	,584679	,526788	,475093	,428883
12	,788493	,701380	,661173	,624597	,589664	,556837	,496699	,444012	,397114
13	,773033	,680951	,639404	,600574	,564272	,530321	,468839	,414965	,367698
14	,757875	,661118	,617782	,577475	,539973	,505068	,442301	,387817	,340461
15	,743015	,641862	,596891	,555265	,516720	,481017	,417265	,362446	,315242

TABLE

TABLE III.

Which shews what one pound payable at the end of any term of years to come under 61, is worth in ready money; discount or rebate being yearly computed at any of these rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum compound interest.

Years.	2 per Crt.	3 per Crt.	3½ per Crt.	4 per Crt.	4½ per Crt.	5 per Crt.	6 per Crt.	7 per Crt.	8 per Crt.
6	,728446	,623367	,576706	,533908	,494469	,458112	,393646	,338755	,291890
17	,714163	,605016	,557204	,513373	,473176	,436297	,371364	,316574	,270269
18	,700159	,587395	,538361	,493628	,452800	,415521	,350344	,295864	,250249
19	,686431	,570286	,520156	,474642	,433302	,395734	,330513	,276508	,231712
20	,672971	,5553676	,502566	,456387	,414643	,376889	,311805	,258419	,214548
21	,659776	,537549	,485571	,438834	,396787	,358942	,294155	,241513	,198656
22	,646839	,521893	,469151	,421955	,379701	,341850	,277505	,225713	,183941
23	,634156	,506692	,453286	,405726	,363350	,325571	,261197	,210947	,170315
24	,621721	,491934	,437957	,390121	,347703	,310068	,248979	,197147	,157699
25	,609531	,477606	,423147	,375117	,332731	,295303	,232999	,184249	,146018
26	,597579	,463695	,408838	,360689	—	—	—	—	—
27	,585862	,450189	,395012	,346817	,304691	,267848	,207368	,160930	,135202
28	,574375	,437077	,381654	,333447	,291571	,255094	,195630	,150402	,125187
29	,563112	,424346	,368748	,320651	,279015	,242946	,184557	,140563	,107328
30	,552071	,411987	,356278	,308319	,267000	,231377	,174116	,131367	,099377

TABLE

TABLE III.

Which shew what one pound, payable at the end of any term of years to come under 6*s*, is worth in ready money; discount or rebate being yearly computed at any of these rates, to wit, 2, 3, 3*s*, 4, 4*s*, 5, 6, 7, and 8 per cent. *per annum*, compound interest.

Years.	2 per Ct.	3 per Ct.	3 <i>s</i> per Ct.	4 <i>s</i> per Ct.	4 per Ct.	4 <i>s</i> per Ct.	5 per Ct.	6 per Ct.	7 per Ct.	8 per Ct.
31	,541246	,399987	,344230	,296460	,255502	,220359	,164255	,122773	,092016	
32	,530633	,388337	,332590	,285058	,244501	,209866	,154957	,114741	,085200	
33	,520229	,377026	,321343	,274094	,233971	,199873	,146186	,107235	,078889	
34	,510028	,366045	,310470	,265552	,223896	,190355	,137912	,100219	,073045	
35	,500028	,355383	,299977	,253415	,214254	,181290	,130105	,093663	,067635	
36	,490223	,345022	,289833	,243669	,205028	,172657	,122741	,087535	,062625	
37	,480611	,334983	,280032	,234297	,196199	,164436	,115793	,081809	,057986	
38	,471187	,325226	,270562	,225285	,187750	,156605	,109239	,076457	,053690	
39	,461948	,315754	,261413	,216621	,179665	,149148	,103056	,071455	,049713	
40	,452890	,306557	,252572	,208829	,171929	,142046	,097222	,066780	,046031	
41	,444010	,297628	,244031	,200278	,164525	,135282	,091719	,062412	,042521	
42	,435304	,288959	,235779	,192575	,157440	,128840	,086527	,058329	,039464	
43	,426769	,280543	,227806	,185168	,150661	,122704	,081630	,054513	,036541	
44	,418401	,272372	,220102	,178046	,144173	,116861	,077009	,050946	,033834	
45	,410197	,264439	,212629	,171198	,137964	,111297	,072650	,047613	,031328	

TABLE

TABLE III.

Which shews what one pound, payable at the end of any term of years to come under 61, is worth in ready money; discount or rebate being yearly computed at any of these rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum compound interest.

Years.	2 per Ct.	3 per Ct.	$3\frac{1}{2}$ per Ct.	4 per Ct.	$4\frac{1}{2}$ per Ct.	5 per Ct.	6 per Ct.	7 per Ct.	8 per Ct.
46	,402154	,256737	,205468	,164614	,132023	,105997	,068538	,044498	,029007
47	,394268	,249259	,198520	,158283	,126338	,100949	,064658	,041587	,026859
48	,386538	,241999	,191806	,152195	,122898	,96142	,060998	,038867	,024869
49	,378958	,234959	,185302	,146341	,115692	,911564	,057546	,036324	,023027
50	,371528	,228107	,179053	,140713	,110710	,087204	,054288	,033947	,021321
51	,364243	,221463	,172998	,135301	,105942	,083051	,051215	,031727	,019742
52	,357101	,215013	,167148	,130097	,101380	,079096	,048316	,029651	,018280
53	,350099	,208750	,161496	,125093	,97041	,075330	,045582	,027711	,016925
54	,343234	,202670	,156035	,120282	,92837	,071743	,043001	,025899	,015672
55	,336504	,196767	,150758	,115656	,088839	,068326	,040567	,024204	,014511
56	,329906	,191036	,145660	,111208	,085013	,065073	,038271	,022621	,013436
57	,323437	,185472	,140734	,106930	,081353	,061974	,036105	,021141	,012441
58	,317095	,180070	,135975	,102817	,077849	,059023	,034061	,019758	,011519
59	,310888	,174825	,131377	,098863	,074497	,056212	,032133	,018465	,010666
60	,304782	,169733	,126034	,095060	,071289	,053536	,030314	,017257	,009876

711. *The Construction of the preceding Table.*

The numbers 1, 2, 3, 4, &c. to 60, (in the first column on the left-hand) signify years; the numbers 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8, placed at the head of the rest of the columns, express rates of interest for 100*l.* lent for a year, and the numbers set in the several columns under those rates of interest are a rank of the negative powers of the same numbers, of which the numbers in Table I. were the affirmative powers; and therefore from Art. 410. it is evident that the numbers in this Table are the Reciprocals of those in Table I.

712. Let it be proposed to calculate the numbers in the column of 5 per cent. Divide unity by 18,67919 the last number in the column of 5 per cent. in Table I. and the Quotient (0,053536) will be the last number of that column in this Table; or the present worth of 1*l.* due at the end of 60 years; The remaining numbers may be found by multiplying continually by 1,05 as in Table I.

The num-
ber

{ ,053536
,056212
,059023
,061974 }

standing
against

{ 60
59
58 }

is the pre-
sent worth
of 1*l.* due
at the end
of

{ 60
59
58 }

years at 5
per cent.

{ } = 18,67919 = 1,05

{ } = ,053536 X 1,05

{ } = ,056212 X 1,05

{ } = ,059023 X 1,05

The

The following is the process necessary to compute the three last numbers.

$$\begin{aligned}
 \text{To } ,053536 &= \frac{1}{18,679.19} \\
 \text{Add } 2676 &= ,053536 \times 0.05 = \frac{1}{20} \times ,053536 \\
 \text{Sum } ,056212 &= ,053536 \times 1.05 \\
 \text{Add } 2811 &= ,056212 \times 0.05 = \frac{1}{20} \times ,056212 \\
 \text{Sum } ,059023 &= ,056212 \times 1.05 \\
 \text{Add } 2951 &= ,059023 \times 0.05 = \frac{1}{20} \times ,059023 \\
 ,061974 &= ,059023 \times 1.05.
 \end{aligned}$$

713. The Use of the preceding Table III.

PROBLEM 8. *The Amount, Time, and Rate being given, to find the present Worth.*

Example. How much present money will discharge a debt of 356*l.* payable at the end of 7 years to come, and allowing 7*l. per cent.* compound interest?

In the column under (7) the rate, and on a line with (7) the time, you will find (,622750) the present worth of 1*l.* due at the end of the given time at the given rate; multiply that present worth (,622750) by the given amount (356) and the product (221,699 or 221*l.* 14*s.*) will be the present worth required.

714. The method of finding the present worth of an annuity, may be explained by the following Example.

Let it be required to find the present worth of an annuity of 347,287*5l.* to continue 3 years, allowing compound interest at 5*l. per cent.*

Now the present worth of 347,287*5l.* due at the end of 1 year will, by Prob. 8. be _____ } 330,75

And the present worth of 347,287*5l.* due at the end of 2 years, will be _____ } 315,

Also the present worth of 347,287*5l.* due at the end of 3 years, will be _____ } 300,

Therefore the present worth of an annuity of 347,287*5l.* to continue three years will be their sum _____ } 945,75

715. Hence it will be very easy to perceive that the following Table IV. of the present worths of annuities of 1*l.* may be composed of the successive sums of the numbers in Table III. in a manner similar to that of composing Table II. from Table I. viz. the first number in every of the columns (except the column of years) in the following Table IV. is the same with the first number in the like columns respectively in the preceding Table III. the second number in each of the said columns of the fourth Table, is the sum of the first and second numbers in the respective columns of the third Table; the third number in the said columns of the fourth Table, is the sum of the first, second and third numbers in the respective columns of the third Table: Or, yet more easily thus, the third number in the fourth Table, is composed of the third in the third Table, and of the second in the fourth; the fourth number in the fourth Table is composed of the fourth in the third, and of the third in the fourth; the like is to be understood of the rest. But you are to observe, that according to this method of composing the fourth Table by Addition, the numbers of the third Table must be continued to more places than are there expressed, to prevent errors arising by the addition of defective Decimal Fractions.

TABLE

TABLE IV.

Which shews the present worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound interest being computed at any of these rates, viz. 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	0.98039	.97087	.96618	.96154	.95694	.95238	.94340	.93458	.92593
2	1.94156	1.91347	1.89969	1.88610	1.87267	1.85941	1.83339	1.80802	1.78327
3	2.88388	2.82861	2.80164	2.77309	2.74896	2.72325	2.67301	2.62432	2.57710
4	3.80773	3.71710	3.67308	3.62990	3.58753	3.54595	3.46511	3.38721	3.31213
5	4.71346	4.57971	4.51505	4.45182	4.38998	4.32948	4.21236	4.10020	3.99271
6	5.60143	5.41719	5.32855	5.24214	5.15787	5.07569	4.91732	4.76654	4.62288
7	6.47199	6.23028	6.11454	6.00206	5.89270	5.78637	5.58238	5.38929	5.20637
8	7.32548	7.01969	6.87396	6.73275	6.59589	6.46321	6.20979	5.97130	5.74664
9	8.16224	7.78611	7.60769	7.43533	7.26879	7.10782	6.80169	6.51523	6.24689
10	8.98259	8.53020	8.31661	8.11090	7.91272	7.72174	7.36009	7.02358	6.71008
11	9.78685	9.35262	9.00155	8.76048	8.52892	8.30641	7.88688	7.49867	7.13896
12	10.57534	9.95400	9.66333	9.38507	9.11858	8.86325	8.38384	7.94269	7.53608
13	11.34837	10.63496	10.30274	9.99856	9.68285	9.39357	8.85268	8.35765	7.90378
14	12.10625	11.29607	10.92052	10.56314	10.22283	9.89864	9.29498	8.74547	8.24424
15	12.84926	11.93794	11.51741	11.11839	10.73955	10.37966	9.71245	9.10791	8.55948

TABLE

TABLE IV.

Which shews the present worth of one pound annuity, to continue any term of years under 6*i*, and payable by yearly payments, compound interest being computed at any of these rates, viz. 2, 3, 3*i*, 4*i*, 4*ii*, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	3 <i>i</i> per Cent.	4 per Cent.	4 <i>i</i> per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
16	13,57771	12,56110	12,09412	11,65230	11,23402	10,83777	10,10590	9,44665	8,85137
17	14,29187	13,16612	12,65132	12,16567	11,70719	11,27497	10,47726	9,76322	9,13164
18	14,99203	13,75351	13,18968	12,65930	12,15999	11,68958	10,82760	10,05909	9,37189
19	15,67846	14,32380	13,70984	13,13394	12,59329	12,08532	11,15812	10,33560	9,60360
20	16,35143	14,87748	14,21240	13,59033	13,00794	12,46221	11,46992	10,59401	9,81815
21	17,01121	15,41502	14,69797	14,02916	13,40472	12,82115	11,76408	10,83553	10,01680
22	17,65805	15,93692	15,16713	14,55112	13,78443	13,16300	12,04158	11,06124	10,20074
23	18,29220	16,44361	15,62041	14,85684	14,14778	13,48857	12,30338	11,27219	10,37106
24	18,91393	16,93554	16,05837	15,24696	14,49548	13,79864	12,55036	11,46933	10,52876
25	19,52346	17,41315	16,48152	15,62208	14,82821	14,09395	12,78336	11,65358	10,67478
26	20,12104	17,87684	16,89035	15,98277	15,14661	14,37519	13,00317	11,82578	10,80998
27	20,70690	18,32703	17,28537	16,32959	15,45130	14,64303	13,21053	11,98671	10,93517
28	21,28127	18,76411	17,66702	16,66306	15,74287	14,89813	13,40616	12,13711	11,05108
29	21,84439	19,18846	18,03577	16,98372	16,02189	15,14107	13,59072	12,27767	11,15841
30	22,39646	19,60044	18,39205	17,29203	16,28889	15,37245	13,76483	12,40904	11,25778

8

TABLE

TABLE IV.

Which shows the present worth of £1 the pound annuity, to continue any term of years under 61, and payable by yearly payments, compound interest being computed at any of these rates, viz. 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
31	22,93770	20,00043	18,73628	17,58849	16,54439	15,19281	13,92909	12,53181	11,34980
32	23,46834	20,38877	19,06887	17,87355	16,78889	15,80268	14,08404	12,64656	11,43500
33	23,98856	20,76579	19,39021	18,14765	17,02286	16,00255	14,23023	12,75379	11,51389
34	24,49859	21,13184	19,70068	18,41120	17,24676	16,19290	14,36814	12,85404	11,58693
35	24,99862	21,48722	20,00066	18,66461	17,46101	16,37419	14,49825	12,94767	11,65457
36	25,48884	21,83225	20,29049	18,90828	17,66604	16,54685	14,62099	13,03521	11,71719
37	25,96943	22,16724	20,57053	19,14258	17,86224	16,71129	14,73678	13,11702	11,71518
38	26,44064	22,49246	20,84109	19,36786	18,04999	16,86789	14,84602	13,19347	11,84887
39	26,90259	22,80822	21,10250	19,58449	18,22966	17,01704	14,94908	13,26493	11,87858
40	27,35548	23,11477	21,35507	19,79277	18,40198	17,15909	15,04630	13,33171	11,92401
41	27,79949	23,41240	21,59910	19,99305	18,56611	17,29437	15,13802	13,39412	11,96724
42	28,23479	23,70136	21,83488	20,18563	18,72355	17,44232	15,22454	13,45245	12,00670
43	28,66156	23,98190	22,06269	20,37080	18,87421	17,54591	15,30617	13,50696	12,04324
44	29,07996	24,25427	22,28279	20,54884	19,01838	17,66277	15,38318	13,55791	12,07707
45	29,49016	24,51871	22,49545	20,72004	19,15635	17,277407	15,45583	13,60552	12,10840

N

TABLE

TABLE IV.

Which shows the present worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound interest being computed at any of these rates, viz. 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 per cent. per annum.

Years.	2 per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
46	29,89231	24,77545	22,70092	20,88465	19,28837	17,88007	15,52437	13,65002	12,13741
47	30,28658	25,02471	22,89944	21,04294	19,41471	17,98102	15,58903	13,69161	12,16427
48	30,67312	25,26671	23,009124	21,19513	19,53561	18,07716	15,65003	13,73047	12,18914
49	31,05208	25,50166	23,27656	21,34147	19,65130	18,16871	15,70757	13,76680	12,21216
50	31,42361	25,72976	23,45562	21,48219	19,76201	18,25593	15,76186	13,80075	12,23349
51	31,78785	25,95123	23,62862	21,61749	19,86795	18,33898	15,81308	13,89247	12,25323
52	32,14495	26,16624	23,79577	21,74758	19,96933	18,41807	15,86139	13,86212	12,27151
53	32,49505	26,37499	23,95726	21,87268	20,06635	18,49340	15,90697	13,88984	12,28843
54	32,83828	26,57766	24,11330	21,99296	20,15918	18,56515	15,94998	13,91574	12,30410
55	33,17479	26,77443	24,26405	22,10861	20,24802	18,63347	15,99054	13,93994	12,31861
56	33,50469	26,96546	24,46405	22,21082	20,33303	18,69855	16,02881	13,96256	12,33205
57	33,82813	27,15094	24,50945	22,32675	20,41439	18,76052	16,06492	13,98370	12,34449
58	34,14523	27,33101	24,68642	22,42957	20,49224	18,81954	16,09898	14,00346	12,35601
59	34,45610	27,50583	24,81780	22,52843	20,56673	18,87575	16,13111	14,02192	12,36668
60	34,76089	27,67756	24,94473	22,62349	20,63802	18,92929	16,16143	14,03918	12,37655

716. *The Use of the preceding Table IV.*

Prob. 9. *The Annuity, Time and Rate being given, to find its present Worth.*

Example. What is the present worth of an annuity of 56*l.* per annum, to continue 21 years, allowing compound interest at 6*l.* per cent. per annum?

In the column under (6) the rate, and on a line with (21) the time, you will find (11,76408) the present worth of an annuity of 1*l.* to continue the given time, at the given rate: Multiply that present worth (11,76408) by the annuity (56) and the product (658,78848; or 658*l.* 15*s.* 9*d.*) will be the present worth required.

717. Prob. 10. *The Annuity, present Worth, and Time being given, to find the Rate.*

Example. A person sold an annuity of 56*l.* to continue 21 years, for 658*l.* 15*s.* 9*d.* at what rate was compound interest computed?

Divide (658,78848) the present worth, by (56) the annuity, and find the quotient (11,764, &c.) or the nearest number to it, on a line with (21) the time, so shall (6) the number at the head of the column, be the rate required; exact, if the quotient be exactly found in the Table; or nearly so, if otherwise.

718. Prob. 11. *The Annuity, present Worth, and Rate being given, to find the Time.*

Example. A person bought an annuity of 56*l.* at 6*l.* per cent. compound interest for 658*l.* 15*s.* 9*d.* how long was the annuity to continue?

Divide (658,78848) the present worth, by (56) the annuity, and find the quotient (11,764, &c.) or the nearest number to it, in the Table in the column under (6) the rate; so shall (21) the number standing in the left-hand column, even therewith, be the time required; exact, if the quotient be exactly found in the Table; or nearly so, if otherwise.

719. Prob. 12. *The present Worth, Time and Rate being given, to find the Annuity.*

Example. What annuity or yearly income, to continue 21 years, can be purchased for 658*l.* 15*s.* 9*d.* allowing compound interest at 6*l.* per cent.

In the column under (6) the rate, and on the line with (21) the time, you will find (11,76408) the present worth of an annuity of 1*l.* to continue during the given time, at the given rate; by this divide (658,78848) the given purchase-money, and the quotient (56) will be the annuity required.

720. But farther, the numbers of the said fourth Table will at first sight shew, how many years purchase an annuity to continue any number of years under 61 is worth, to be sold for present money, compound interest being computed at any of the said rates. So if you desire to know how many years purchase an annuity issuing out of lands for 21 years, to begin presently, is worth, if it were to be sold for ready money, when the current rate of interest is 6 per cent. Seek in the first column of Table IV. for 21 years, and carry your eye from thence equidistant to the head-line of the Table, till you come under 6, which (as before has been said) signifies 6 per cent. So in the fourth column you will find 11.76408, whereof you need only consider 11.76, which shews that the said annuity is worth 11 years purchase, (or 11 times one year's rent whatever it be) and 76 parts of one year's purchase divided into 100 parts, or $11\frac{3}{4}$ years purchase and little more. The same annuity, when money was at 8 per cent. was worth 10 years purchase, and about $\frac{1}{10}$ part of a year's purchase more, as the number in the column of 8 per cent. right against 21 years will discover.

721. As it would be too operose to make Tables to all times and rates, the use of these may be rendered more extensive, by following the methods, delivered in the following Auxiliary Propositions.

722. PROB. 13. auxiliary to PROB. 1. and 8.

To find the Amount, or present Worth of 1l. for a number of years greater than 60; at any rate of Compound Interest, specified in the Tables.

Example. What will 1l. amount to, in 100 years, allowing compound interest at 5l. per cent.?

Find in the left-hand column of Table I. two or more numbers (suppose 51 and 49) whose sum will make (100) the given number of years. Then multiply (12,04077 and 10,92133) the numbers corresponding to them, in the column under (5) the rate; so shall (131,50125) the product, be the number required.

The reason of this process will be evident from Art. 414.

The present worth will be found in the same manner by taking the numbers from Table III.

723. Table V. auxiliary to Table I. and Table III.

	2 per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.
1 Quarter	1,004963	1,007417	1,008637	1,009853	1,011065
2 Quarters	1,009950	1,014889	1,017349	1,019804	1,022252
3 Quarters	1,014963	1,022416	1,026137	1,029852	1,033564
	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.	
1 Quarter	1,012272	1,014674	1,017058	1,019426	
2 Quarters	1,024695	1,029563	1,034408	1,039230	
3 Quarters	1,037270	1,044671	1,052053	1,059419	

724. The foregoing Table contains the amounts of £1. for 1, 2, and 3 quarters of a year at the several rates therein mentioned, being severally three mean proportionals between £1. and the amount of £1. in 1 year at those rates: They are found by the method directed in Art. 550.

725. PROB. 14. auxiliary to PROB. 1. and 8.

To find the Amount or present Worth of £1. for Years and Quarters of a Year, at any Rate of Compound Interest specified in the Tables.

Example. What is the amount and present worth of £1. for $14\frac{1}{4}$ years, at 5*l.* per cent. compound interest?

Find by Table I. the amount (1,979932), or by Table III. the present worth (.505068) of £1. for (14) the number of years given at (5) the given rate. Find also by Table V. the amount of £1. for (3) the number of quarters given (viz. 1,037270): Then if the amount be required, multiply; or if the present worth be required, divide the former by the latter; and the product (2,053724) will be the amount, or the quotient (.487499) will be the present worth required?

726. PROB. 15. auxiliary to PROB. 4. and 9.

To find the amount or present Worth of an Annuity of £1. to continue a number of Years greater than 60, at any Rate of Compound Interest specified in the Tables.

Example. What is the present worth of an annuity of £1. to continue 100 years, allowing 5*l.* percent. compound interest?

From the given years (100) take 60 the extent of the foregoing Tables; and with the remainder (40) entering

Table IV. Find (17,15909) the present worth of an annuity, to continue that number of years at the given rate: Multiply this by (.053536) the present worth of 1*l.* due at the end of 60 years found in Table III. and to (.91883) the product add (18,92929) the present worth of an annuity of 1*l.* to continue 60 years, found in Table IV. so shall the sum (19,84812) be the present worth required.

727. The reason of this process depends on Art. 564. For if the sum of 40 terms of the Geometrical Progression in Table III. beginning at the 61st term, be added to the sum of the first 60 terms; the total will be the sum of 100 terms, the number required.

If the amount be required proceed in the same manner with the numbers in Table II. and I.

728. Table VI, auxiliary to Tables II. and IV.

Payments	2 per Cent.	3 per Cent.	3½ per Cet.	4 per Cent.	4½ per Cet.
Half yearly	1,004975	1,007445	1,008675	1,009902	1,011126
Quarterly	1,007469	1,011181	1,013031	1,014877	1,016720

Payments	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
Half yearly	1,012348	1,014781	1,017204	1,019870
Quarterly	1,018559	1,022227	1,025880	1,029519

The numbers in the above Table are computed by a Theorem, given by the justly celebrated Mr. *De Moivre*, in his Treatise of *Annuities on Lives*, which the Reader who understands Algebra may consult; but as it cannot easily be explained without that art, the manner of its construction must be here omitted.

729. PROB. 16. auxiliary to PROB. 4. and 9.

*The Amount or present Worth of an Annuity of 1*l.* payable by yearly payments, for any number of years, at any rate per cent. specified in the Tables, being given; to find what the Amount or present Worth of the same Annuity would be, if it were payable by equal half-yearly or quarterly Payments.*

Example. What is the present worth of an annuity of 1*l.* per annum, payable by equal (half-yearly) payments, and to continue 41 years, allowing compound interest at 5 per cent.?

Multiply (17,29437) the present worth of the annuity payable by yearly payments found in Table IV. by (1,012348) the number which, in Table VI. stands on a line with the word (half-yearly) and under (5) the rate; and the product (17,507918) will be the number required.

730. PROB. 17. auxiliary to PROB. 2, 3, 5, 6, 10, and 11.

When the Time or Rate cannot be exactly obtained by the Tables, to approach nearer to the truth therein.

Example. At what rate per cent. compound interest, will an annuity of 1*l.* to continue 8 years, be now worth 6*l.* 10*s.* $\frac{3}{4}$

Find in Table IV. on a line with (8) the time, the two nearest numbers to (6,5) the given present worth; which in this case will be 6,59589, the present worth of such an annuity at $4\frac{1}{2}$ per cent. and 6,46321, the present worth of the same at $5\frac{1}{2}$ per cent. the first exceeding the given number (6,5) by ,09589, and the latter being ,03679 less than it. And the difference of the said tabular present worths 6,59589, and 6,46321 being 0,13268; use the following proportion.

As (0,13268) the difference between the tabular present worths,

To (5) the difference between the tabular rates *viz.* (5— $4\frac{1}{2}$)

So is (,09589) the difference between the tabular present worth at $4\frac{1}{2}$ per cent. and the given present worth,

To (0,3613) the difference between the tabular rate $4\frac{1}{2}$ and the required rate.

Then to the tabular rate $4\frac{5}{5}$

Add the result of the proportion 0,3613

The sum is the rate required nearly 4,8613 or 4*l.* 17*s.* $2\frac{3}{4}d.$

The same manner of proceeding will do in any other case, *mutatis mutandis.*

731. If any questions should arise, in compound interest, at different rates from those in these Tables, the method of their solution will not be difficult to a Reader, who understands the management of Geometrical Progressions, as taught in *Chap. 35.* and knows how to use Logarithms.

But, if that be wanting, or if such questions frequently occur, it will be best to make Tables for that rate, in the manner of the above.

732. PROBLEM 18. *To find the present worth of the Reversion of an Annuity; having given the Annuity, Time of commencing, Duration, and Rate.*

Example. Having a lease, for 7 years to come, of some lands, by which I clear 32*l.* per annum, and being desirous to add 21 years to the same, in order to make my whole time therein 28 years; what fine ought I to pay to my landlord, allowing 6*l.* per cent. compound interest?

Find per Table IV. the present worth of an annuity of 1*l.* for (28) the whole duration of } 13,406 16
the intended annuity or lease, viz. _____ }

Also find by the same the present worth of an annuity of 1*l.* for (7) the time before the reversionary lease commences, viz. } 5,582 38
_____ }

Then shall the difference of these present worths be the present value of an annuity of 1*l.* for } 7,823 78
21 years to commence at the end of 7 years }

Which being multiplied by (32) the given annuity, will produce the fine required } 250,361 6

733. *Bishops, Deans and Chapters, Heads and Fellows of Colleges, &c.* follow certain customs, which have been used from time immemorial, in ascertaining the fines for the renewing of leases, held under them.

734. The most general of these customs is, to take one year's improved rent, as a fine, for renewing, that is adding, 7 years, to 14 yet to come, of an old lease of twenty-one years; which is allowing the tenant about 11,564 2*l.* per cent. compound interest for his money.

735. The following Tables are calculated to suit this custom, upon the same principles as Tables III. and IV. and are to be used just in the same manner.

T A B L E

TABLE VII.

Shewing the present worth of £1. due at the end of the years therein specified, at 11,5642 £ per cent. compound interest,

Years.

1	0, 89634
2	0, 80343
3	0, 72015
4	0, 64551
5	0, 57860
6	0, 51863
7	0, 46484
8	0, 41668
9	0, 37349
10	0, 33477
11	0, 30007
12	0, 26897
13	0, 24109
14	0, 21610
15	0, 19370
16	0, 17362
17	0, 15562
18	0, 13949
19	0, 12503
20	0, 11207
21	0, 10046

TABLE VIII.

Shewing the present worth of an annuity of £1. to continue any number of years therein specified, at 11,5642 £ per cent. compound interest.

Years.

1	0, 89634
2	1, 69978
3	2, 41993
4	3, 06544
5	3, 64404
6	4, 16266
7	4, 62752
8	5, 04420
9	5, 41769
10	5, 75240
11	6, 05253
12	6, 32150
13	6, 56259
14	6, 77869
15	6, 97239
16	7, 14001
17	7, 30163
18	7, 44112
19	7, 56616
20	7, 67823
21	7, 79869

736. The Use of Table VIII.

Ques. 1. If a College-tenant has 7 years to come or unspent in a lease of lands for 21 years, at 1*l.* yearly rent, and desires to have 14 years renewed or added to those 7 years, and so to take a new lease for 21 years to begin presently, what must he pay for a fine? *Answ.* 3*l.* 3*s.* 0*d.*

The Rule for finding out the answer of the question proposed, and such like, is this, *viz.*

From 7.77869 (being the number which answers to 21 years in Table VIII.) always subtract the tabular number which belongs to the number of years to come or unspent in the old lease; so the remainder will shew what fine must be paid for the years to be renewed or added, to make those unspent years in the old lease to be 21 years compleat again, at 1*l.* yearly rent.

So to solve the question proposed.

From the present worth of 1*l.* yearly rent for 21 years which is _____ } 7.77869

Subtract the present worth of the same rent for 7 years, (that were unspent in the old lease) _____ } 4.62752

And there will remain the fine sought, to wit, — 3.15117

That is to say, 3.15117*l.* or 3*l.* 3*s.* 0*d.* must be paid as a fine, for renewing or adding 14 years to 7 years, that were unspent in the old lease, the yearly rent being 1*l.* Also the said 3.15117 shews, that such a renewal is worth 3 years purchase, and $\frac{1}{58}$ parts of a year's purchase (whatever the rent be.)

Ques. 2. If a tenant that has 17 years yet to come in a lease of lands held of a College for 21 years at 50*l.* yearly rent, be desirous to renew 4 years, and so make those 17 years to be 21 years compleat again at the same rent, what must he give for a fine? *Answ.* 23*l.* 17*s.* 2*d.* 1*f.* For according to the Rule before given,

From the present worth of 1*l.* yearly rent for 21 years _____ } 7.77869

Subtract the present worth of the same rent for 17 years, (that were unspent in the old lease) _____ } 7.30163

And there will remain _____ 0.47706

Which multiplied by the rent _____ 50

The product will be the fine sought, to wit, 23*l.* 17*s.* 0*d.* 3*f.* } 23.85300

737. From Table VIII. by working as in the above Examples may be deduced the following Table IX. to shew without any operation the fine to be paid on a lease whose improved rent is 1*l. per annum*, or rather how many years purchase should be given for renewing, at any time before the expiration of the lease.

Years to be added.	TABLE IX. Shewing what fine ought to be paid for adding any number of years to a col- lege lease in or- der to make the same 21 years compleat,
1	0, 10046
2	0, 21253
3	0, 33756
4	0, 47706
5	0, 63268
6	0, 80630
7	1, 00000
8	1, 21600
9	1, 45719
10	1, 72616
11	2, 02623
12	2, 36100
13	2, 73449
14	3, 15117
15	3, 61603
16	4, 13465
17	4, 71325
18	5, 35876
19	6, 07891
20	6, 88234

The use of this Table will appear by comparing it with the two Examples in Art. 736. In the first of them, 14 years were to be added; and the answer 3,15117 will be found in this Table against the number 14. In the second, 4 years were to be renewed, and the fine to be paid for a rent of 1*l. per annum*, viz. 0,47706 will be found in this Table against the number 4.

738. Questions to exercise the preceding Tables.

- If the lease of an house be worth 153*l.* fine, and 16*l.* yearly rent, payable yearly for 21 years, and the lessee be desirous to bring

bring down the fine to $50l.$ and so to pay the more rent; the question is, what rent the tenant shall pay, accounting compound interest at the rate of 8 per cent. per annum? Answ. $26l. 5s. 8d.$

First, find the difference between the fines, which is $103l.$ Then by Prob. 12. seek what annuity or rent to continue 21 years, $103l.$ ready money will purchase at 8 per cent. so you will find $19l. 5s. 8d.$ which being added to the old rent $16l.$ gives $26l. 5s. 8d.$ which the tenant must pay, to the end that the fine may be diminished to $50l.$

2. There is a lease of certain lands to be let for 14 years for $250l.$ fine, and $44l.$ rent per annum, payable yearly; but the tenant is desirous to pay less rent, viz. 20 pounds per annum, and to give a greater fine; the question is, what fine ought to be paid to bring down the rent to $20l.$ per annum, accounting compound interest at the rate of 6 per cent. per annum? Answ. $473l. 1s. 7d.$

First find the difference between the rents, which is 24 pounds per annum. Then by Prob. 9. seek what an annuity or rent $24l.$ per annum, to continue 14 years, is worth in ready money at 6 per cent. per annum, so you will find $223l. 1s. 7d.$ which being added to the first fine 250 pounds, gives $473l. 1s. 7d.$ which the tenant must pay, to the end that the rent may be brought down to $20l.$ per annum.

3. What annuity to continue 13 years will $1999l.$ due 9 years hence purchase, compound interest being allowed on both sides at 8 per cent per annum? Answ. $126l. 10s. 4\frac{1}{2}d.$

By Prob. 8. the present worth of $1999l.$ due at the end of 9 years is $1000l.$

And by Prob. 12. $1000l.$ will purchase an annuity of $126l. 10s. 4\frac{1}{2}d.$ to continue 13 years.

C H A P. XLII.

*A Demonstration of the DOUBLE RULE of
FELLOWSHIP.*

739. THE Double Rule of Fellowship (commonly called the Rule of Fellowship with Time) presupposes two things, viz. 1. That the particular stocks of merchants in company, have continued unequal spaces of time, in the common stock. 2. That at the end of their partnership, the total gain or loss is to be divided among them, in such manner, that their shares may have such proportion between themselves, as those sums of interest-money have one to another, which at any rate per cent. (simple interest only being computed) might be gained by the particular stocks, within the respective times of their continuance in the common stock: Now for the effecting of such a proportional partition, the said Double Rule of Fellowship gives this direction, viz. divide the total gain or loss into such parts, which have the same proportion one to the other, as is between the products arising from the Multiplication of each particular stock by its correspondent time.

For example, suppose two merchants *A* and *B*, to be partners in traffick, for a certain time first agreed on between them, and that *A* permits his stock of 100*l.* to be employed in their joint traffick three months, and *B* his stock of 50*l.* eight months; I say, (according to the said Rule of Fellowship with Time) whatever the total gain or loss be, that part of it which belongs to *A* must have such proportion to the gain or loss of *B*, as 100 \times 3 (or 300) has to 50 \times 8 (or 400.) The truth of this Rule, taking the two premised suppositions for granted, may be thus demonstrated.

1. Supposing 100*l.* (the stock of *A*) to gain in three months any certain sum of money, as two pounds; I seek how much 50*l.* (the stock of *B*) will gain in the same time, and at the

said rate: So find $\frac{2 \times 50}{100}$ *l.* For,

$$\text{As } 100 : 2 :: 50 : \frac{2 \times 50}{100} \text{ l.}$$

2. Having

2. Having found what 50*l.* will gain in three months, I seek how much the said 50*l.* will gain in eight months at the same rate; and so I find $\frac{2 \times 50 \times 8}{100 \times 3}$ *l.*

$$\text{For as } 3 : 8 :: \frac{2 \times 50}{100} : \frac{2 \times 50 \times 8}{100 \times 3}$$

3. Thus it appears, that if 100*l.* in three months gains 2*l.* then 50*l.* in eight months will at the same rate gain $\frac{2 \times 50 \times 8}{100 \times 3}$; so that the proportion of the gain of *A* to the gain of *B*, is

$$\text{As } 2 \text{ is to } \frac{2 \times 50 \times 8}{100 \times 3} \text{ l.}$$

4. If both the terms (to wit, the Antecedent and Consequent) of the said proportion be severally multiplied by the said Denominator 100×3 , the products will be in the same proportion with the terms multiplied, viz. the gain of *A* will be to the gain of *B*,

$$\text{As } 2 \times 100 \times 3 \text{ is to } 2 \times 50 \times 8.$$

5. Lastly, because 2 (the supposititious gain first assumed) is a Multiplier as well in the Antecedent as in the Consequent of the last-mentioned proportion, it may be expunged out of both; and so the gain of *A*, will be to the gain of *B* in this proportion, (which was to be proved) to wit,

$$\text{As } 100 \times 3 \text{ is to } 50 \times 8.$$

C H A P. XLIII.

A Collection of pleasant and subtil Questions, to exercise all the Parts of VULGAR ARITHMETIC.

740. QUEST. 1. If a wedge of gold weighing $17\frac{1}{3}$ lb. of Troy weight be worth 679 $\frac{1}{3}$ l. Sterling, what is the value of $1\frac{1}{3}$ grain of that gold ? Answ. 2d.

$$1. 17\frac{1}{3} \text{ (or } \frac{50}{3}) \text{ of } \frac{1}{24} \text{ of } \frac{1}{20} \text{ of } \frac{1}{2} = \frac{1}{480}.$$

$$2. \frac{1\frac{1}{3}}{7} : \frac{4\frac{1}{3}}{7} :: \frac{1}{480} : \frac{1}{240}.$$

Quest. 2. A man dying gave to his eldest son $\frac{2}{3}$ of $\frac{1}{2}$ of his estate, to his second son $\frac{1}{3}$ of $\frac{1}{2}$ of his estate, and when they had counted their portions, the one had 40l. more than the other ; the remainder of the estate was given to the wife, and younger children : The question is, what was the portion of the eldest son, also of the second, and how much did belong to the wife and younger children ? Answ. the eldest son's portion 100l. the second son's portion 60l. and 440l. for the wife and younger children.

The Fractions being reduced, it will be manifest, that the eldest son had $\frac{1}{3}$, and the second $\frac{1}{15}$; also the difference of the said Fractions $\frac{4}{15}$, then say,

As $\frac{1}{3} : 40 :: 1 : 600$, the estate.

Now $\frac{1}{3}$ of 600 = 200, the eldest
And $\frac{1}{15}$ of 600 = 40, the second } son's share

Their sum 160,

Therefore there remains 440, for the wife, and younger children.

Quest. 3. A young man receives 66 $\frac{2}{3}$ l. which was $\frac{2}{3}$ of $\frac{1}{2}$ of his eldest brother's portion, and $3\frac{1}{2}$ times his eldest brother's portion was $1\frac{1}{2}$ times his father's estate ; the question is, what was the father's estate ? Answ. 560l.

$\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$
 As $\frac{1}{3} : 66\frac{2}{3} :: 1 : 200$ = the elder brother's portion.
 $200 \times 3\frac{1}{3} = 700$ = ditto $\times 3\frac{1}{3}$.

As $1\frac{1}{3} : 700 :: 1 : 560$ = the father's estate.

Quest. 4. If *A* can finish a work in 20 days, and *B* in 30 days; in what time will the work be finished by *A* and *B* working together? Answ. 12 days.

First find what quantity of the work will be done by each workman in one and the same time; then it will be, as the sum of those quantities is in proportion to the said time, so is $\frac{1}{2}$ or the whole work to the time in which such work will be finished by both workmen working together.

Days. Work. Days. Work.

30.	:	1	::	20	:	$\frac{1}{3}$
						Add 1

Sum $1\frac{2}{3}$

Hence it appears that *A* and *B* working together 20 days, will finish that work once, together with $\frac{2}{3}$ of the same work; therefore say again by the Rule of Three.

Work. Days. Work. Days.

$1\frac{2}{3}$:	20	::	1	:	12
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Quest. 5. A Gentleman a Chaise did buy,

An Horse and Harness too;

They cost the sum of threescore Pounds,

(Upon my word 'tis true;)

The Harness came to half of th' Horse,

The Horse twice of the Chaise:

And if you find the Price of them,

Take them, and go your ways.

Answer { Chaise ----- 154
 { Horse ----- 30.
 { Harness ----- 15.

Quest. 6. A cistern in a certain conduit is supplied with water by one pipe of such bigness, that if the cock *A* at the end of the pipe be set open, the cistern will be filled in $\frac{1}{2}$ an hour: But at the bottom of the cistern two other cocks *B* and *C* are placed, whose capacities are such, that by the cock *B* set open alone, (all the rest being stopt) the cistern supposed to be full, will be emptied in $1\frac{2}{3}$ hour: Also by the cock *C* set open alone, the cistern will be emptied in $2\frac{1}{3}$ hour: Now because more water will be infused by the cock *A*, than can

be expelled by both the cocks *B* and *C* in one and the same time; the question is, to find in what time the cistern will be filled, if all the said three cocks be set open at once? Answ. $1\frac{2}{5}$ hour.

After the manner of the fourth question of this Chapter, find how many times the cistern will be emptied in one and the same space of time, by the cocks *B* and *C* running together; also how much of the cistern will be filled by *A* in the same time; then will the difference shew how much of the cistern is gained by the filling cock in the said time: Lastly, as the cisterns or parts gained are in proportion to the correspondent time; so is the whole cistern, to the time wherein it will be gained or filled.

Hou.	Cist.	Hou.	Cist.
I. $2\frac{1}{3}$: 1	:: $1\frac{3}{7}$: $\frac{10}{49}$ { Add 1 } $\frac{10}{49}$ { Sum $1\frac{10}{49}$ } $\left. \begin{array}{l} \text{C} \\ \text{B} \\ \text{B and C} \end{array} \right\}$
Hou.	Cist.	Hou.	Cist.
II. $\frac{1}{2}$: 1	:: $1\frac{1}{3}$: $(2\frac{1}{7} \text{ filled by A})$
			$\left. \begin{array}{l} \text{in } 1\frac{1}{7} \text{ Hou.} \\ 1\frac{1}{2} \text{ gained by A} \end{array} \right\}$
III. $1\frac{1}{4}$: $1\frac{1}{3}$:: 1 : $1\frac{2}{5}$	

Ques. 7. Suppose a dog, a wolf, and a lion, were to devour a sheep, and that the dog could eat up the sheep in an hour, the wolf in $\frac{3}{4}$ hour, and the lion in $\frac{1}{2}$ hour: Now if the lion begin to eat $\frac{1}{4}$ hour before the other two, and afterwards, all three eat together, the question is, in what time the sheep would be devoured? Answ. $1\frac{1}{4}$ hour.

Hou. Sb. Hou. Sb.
I. If $\frac{1}{2} : 1 :: \frac{1}{3} : \frac{1}{4}$
Thus it appears that $\frac{1}{3}$ of the sheep would be eaten by the lion, before the dog and wolf began to eat.

2. Proceed according to the fourth question, so will you find the remaining $\frac{2}{3}$ to be eaten by them all in $\frac{3}{4}$ hour, which added to $\frac{1}{4}$ gives $1\frac{1}{4}$ hour, in which time the sheep would be devoured.

Ques. 8. If $120\frac{1}{3}l.$ is to be distributed among three persons, *A*, *B*, *C*, in such sort, that as often as *A* takes 5, *B* shall take 4; and as often as *B* takes 3, *C* shall take 2; what will be the share of each of them?

Answ. *A* $51\frac{1}{3}l.$ *B* $41\frac{2}{3}l.$ *C* $27\frac{1}{3}l.$

Find three numbers which may express the proportions of their shares, by the Rule of Three, or (to avoid Fractions) thus,

$$\begin{array}{r} 5 \dots \dots \dots 4 \\ 3 \dots \dots \dots 2 \\ \hline 15 : 12 : 8 \end{array}$$

Thus found

$$\begin{array}{rcl} 5 \times 3 & = & 15 \\ 3 \times 4 & = & 12 \\ 4 \times 2 & = & 8 \end{array}$$

$$35 : 120\frac{1}{3} :: \left\{ \begin{array}{l} 15 : 5\frac{1}{3} \\ 12 : 4\frac{1}{3} \\ 8 : 2\frac{1}{3} \end{array} \right.$$

Ques^t. 9. A governor of a certain garrison, being desirous to know how much money the port or passage of the garrison did amount to in certain months, made choice of a loyal servant, giving him orders to receive of every coachman passing with a coach 4*d.* of every horseman 2*d.* and of every footman $\frac{1}{2}$ *d.* Now, at the year's end, the servant making his account to the governor, gives him 94*l.* 15*s.* 10*d.* and lets him know, that as often as 5 passed with coaches, 9 passed on horseback; and as often as 6 passed on horseback, 10 passed on foot; the question is, how many coaches, horse-men, and footmen passed? Answ. 2500 coaches, 4500 horse-men, 7500 footmen.

Find three proportional numbers after the manner of the 8th question, which will be 5, 9, 15, then proceed as follows:

$$\begin{array}{r} d. \\ 5 \text{ Coaches} — 20 \\ 9 \text{ Horsemen} — 18 \\ 15 \text{ Footmen} — 7\frac{1}{2} \end{array}$$

$$\text{If } 45\frac{1}{2} : 22750 :: \left\{ \begin{array}{l} 5 : 2500 \\ 9 : 4500 \\ 15 : 7500 \end{array} \right.$$

Ques^t. 10. A factor would exchange 780*l.* Sterling for double ducats, dollars, and French crowns, the ducats at 7*s.* 6*d.* the piece, the dollars at 4*s.* 4*d.* and the French crowns at 6*s.* the piece, to be in such proportion, that $\frac{1}{5}$ of the number of ducats may be equal to $\frac{1}{3}$ of the number of dollars; and $\frac{1}{4}$ of the dollars equal to $\frac{1}{6}$ of the crowns; the question is, how many

many pieces of every coin he shall receive for his 780 pounds?
Ans. 600 ducats, 900 dollars, 1200 crowns.

Find three proportional numbers (after the manner of the eighth question) which will be 6, 4, 3:

$$\begin{array}{c} \frac{1}{2} : \dots : \dots : \frac{1}{2} \\ \frac{1}{2} : \dots : \dots : \frac{1}{15} \\ \hline \frac{1}{2} : \frac{1}{2} : \frac{1}{15} \\ 6 : 4 : 3 \end{array}$$

Thus it appears, that six times the number of ducats must be equal to four times the number of dollars, also equal unto three times the number of crowns. Then make choice of three numbers to answer those proportions, such as are these, 2, 3, 4, (for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which numbers proceed as follows:

$$\begin{array}{l} 2 \text{ ducats} - \frac{1}{2} \\ 3 \text{ dollars} - \frac{1}{15} \\ 4 \text{ crowns} - 1\frac{1}{2} \end{array}$$

$$\text{say } \frac{1}{2} : 225 : 780 :: \left\{ \begin{array}{l} \frac{1}{2} : 225 \\ \frac{1}{15} : 195 \\ 1\frac{1}{2} : 360 \end{array} \right.$$

$$\frac{1}{2} : 1 :: 225 : 600 \text{ ducats.}$$

$$\frac{1}{15} : 1 :: 195 : 900 \text{ dollars.}$$

$$\frac{1}{2} : 1 :: 360 : 1200 \text{ crowns.}$$

Ques. II. Twenty knights, 30 merchants, 24 lawyers, and 24 citizens, spent at a dinner 64 pounds; which sum was divided among them in such manner, that 4 knights paid as much as 5 merchants, 10 merchants as much as 16 lawyers, and 8 lawyers as much as 12 citizens; the question is, to know the sum of money paid by all the knights, also by the merchants, lawyers, and citizens?

Answer. The 20 knights paid 20 pounds, the 30 merchants 24 pounds, the 24 lawyers 12 pounds, and the 24 citizens 8 pounds.

Find four numbers to express the proportions of their payments by the Rule of Three, or (to avoid Fractions) in manner following; so will the proportional numbers be 4, 5, 8, 12; viz. 4 knights paid as much as 5 merchants, or 8 lawyers, or 12 citizens.

$$\begin{array}{rcl} 4 & \dots & 5 \\ 10 & \dots & 16 \\ 8 & \dots & 12 \end{array}$$

$$\begin{array}{cccc} 320 & : & 400 & : 640 & : 960 \\ 4 & & 5 & & 8 & & 12 \end{array}$$

Thus found

$$\begin{aligned} 4 \times 10 \times 8 &= 320 \\ 10 \times 8 \times 5 &= 400 \\ 8 \times 5 \times 16 &= 640 \\ 5 \times 16 \times 12 &= 960 \end{aligned}$$

Then presupposing that a knight is to pay 4s. proceed as follows, viz.

$$\begin{array}{l} 20 \text{ knights} \dots 4 \\ 30 \text{ merchants} \dots 4\frac{1}{2} \\ 24 \text{ lawyers} \dots 2\frac{1}{2} \\ 24 \text{ citizens} \dots 1\frac{1}{2} \end{array}$$

$$\text{say, if } 12\frac{1}{2} : 64 :: \left\{ \begin{array}{l} 4 : 20 \\ 4\frac{1}{2} : 24 \\ 2\frac{1}{2} : 12 \\ 1\frac{1}{2} : 8 \end{array} \right.$$

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Ques. 12. A certain man with his wife did usually drink out a vessel of beer in 12 days, and the husband found by often experience, that his wife being absent, he drank it out in 20 days; the question is, in how many days the wife alone could drink it out? Answ. 30 days.

Note. It is to be supposed that the husband, in 12 of the 20 days in which he drank alone, did drink as much as in the 12 days, wherein he drank with his wife; hence it follows, that in the remaining 8 of the said 20 days, he drank as much as his wife did in 12 days. Therefore by the Rule of Three say, if 8 gives 12, what 20? Answ. 30. View the following form of the work.

$$\begin{array}{r} \text{From } 20 \\ \text{Subtract } 12 \end{array}$$

$$\text{Then if } 8 : 12 :: 20 : 30$$

Ques.

Quest. 13. A person dying, left his widow 1780*l.* and 1250*l.* to each of his four children : He had been 25*½* years in trade, and had cleared (at an average) 126*l.* a year. What had he to begin with ? Answ. 3567*l.*

$$\begin{array}{rcl} & & 1. \\ \text{For he left to his wife} & = & 1780 \\ \text{to his children } 1250 \times 4 & = & 5000 \\ & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Therefore he left in all} & = & 6780 \\ \text{But he gained } 126 \times \frac{5}{2} = 63 \times 51 & = & 3213 \\ & & \hline \end{array}$$

$$\text{Therefore he began with} \quad \hline \quad 3567$$

Quest. 14. Two travellers, *A* and *B*, perform a journey to one and the same place, in this manner, *viz.* *A* travels 14 miles every day, and had travelled 8 days before *B* began ; upon the ninth day *B* sets forward, and travels 22 miles every day ; the question is, to find in what time *B* shall overtake *A* ? Answ. at the end of 14 days.

1. Find how many miles *A* had travelled before *B* set forward ? Answ. 112 miles.

$$\begin{array}{cccc} \text{Day} & \text{Miles} & \text{Days} & \text{Miles} \\ 1 & : & 14 & :: 8 : 112 \end{array}$$

2. Seek how many miles *B* gains of *A* in a day ? Answ. 8 miles : For,

$$22 - 14 = 8$$

$$\begin{array}{cccc} \text{Miles} & \text{Day} & \text{Miles} & \text{Days} \\ 8 & : & 1 & :: 112 : 14 \end{array}$$

Quest. 15. There is an island which is 36 miles in compass ; Now, if at the same time, and from the same place, two footmen, *A* and *B*, set forward to travel round about the said island, and follow one another in such manner, that *A* travels every day 9 miles, and *B* 7 miles ; the question is to find in what space of time they will meet again ; also how many miles, and how many times about the island each footman will then have travelled ?

Answ. They will meet at the end of 18 days from their first parting ; and then *A* will have travelled 162 miles (or 4*½* times the compass of the island;) and *B* will have travelled 126 miles (or 3*½* the compass of the island.)

Miles.			
From	9		
Subtract	7		
	<hr/>		
	2	Day.	Miles.
		1	:: 36
mult. 18			: 18
by 9		mult. 18	
	<hr/>	by 7	
36) 162 (4 $\frac{1}{2}$		<hr/>	36) 126 (3 $\frac{3}{4}$

Ques. 16. Two footmen *A* and *B*, depart at the same time from *London* towards *York*, travelling at this rate, viz. *A* goes 8 miles every day, *B* goes 1 mile the first day, 2 miles the second Day, 3 miles the third day, and in that progression he goes forward, travelling in every following day 1 mile more than in the preceding Day; the question is to know in how many days *B* will overtake *A*? Answ. in 15 days.

To resolve this, and such like questions, double 8 (the number of miles which *A* travels daily) which make 16, from which subtract 1, the remainder is 15, the number of days sought.

Ques. 17. If *Exeter* be distant from *London* 140 miles, and that at the same time one footman *A* departed from *London* towards *Exeter*, travelling every day 8 miles; another *B* from *Exeter* towards *London*, travelling every day 6 miles, the question is, in how many days they'll meet one another, and how many miles each footman will have then travelled? Answ. they'll meet at the end of 10 days, and then *A* will have travelled 80 miles, and *B* 60 miles.

$$\text{Add } \left\{ \begin{array}{l} 8 \text{ miles travelled daily by } A, \\ 6 \text{ miles travelled daily by } B, \end{array} \right.$$

Sum 14 miles which *A* and *B* together did travel daily.

M. Da. Miles. Da.

14 : 1 :: 140 : 10 in which time *A* and *B* will meet each other.

10 × 8 = 80 miles travelled by *A*.

10 × 6 = 60 miles travelled by *B*.

Ques. 18. A certain footman *A* sets out from *London* towards *Lincoln*, and at the same time another footman *B* departs from *Lincoln* towards *London*; also *A* travels every day $2\frac{1}{2}$ miles more than *B*. Now, supposing those two cities to be 100 miles distant one from the other, and that those two footmen do meet one another at the end of 8 days after the beginning of their journeys; the question is, how many miles each will have

have then travelled, as also how many miles each travelled daily? Answ. A 60 miles, B 40 miles: Also A travelled $7\frac{1}{2}$ miles every day, and B 5 miles.

Days. Miles. Days. Miles.

1 : $2\frac{1}{2}$:: 8 : 20

Hence it appears, that at the time of their meeting, A had travelled 20 miles more than B, which 20 miles being subtracted from 100 miles leave 80 miles, whereof the half is 40 miles which B had travelled, therefore A had travelled 60 miles.

Now to find how many miles each travelled daily, say,

Days. Miles. Days. Miles.

8 : 40 :: 1 : 5

Miles.

Therefore { $\frac{A}{B}$ } travelled { $\frac{7\frac{1}{2}}{5}$ } daily.

Quest. 19. There is an island which is $134\frac{1}{2}$ miles in compass; now at the same time, and from the same place, two footmen A and B begin a journey round about the said island, but they travel towards contrary parts, at this rate, viz. A travels 11 miles in every 2 days, and B 17 miles in 3 days; the question is to find in what space of time A and B will meet one another; and how many miles each will then have travelled?

Answ. they'll meet at the end of 12 days, and then A will have travelled 66 miles, and B 68 miles.

After the manner of the fourth question of this Chapter, the time sought will be found 12 days.

Days. Miles. Days. Miles.

2 : 11 :: 3 : $16\frac{1}{2}$

Add 17

Days. Miles. Days.

$3\frac{1}{2}$: 3 :: $134\frac{1}{2}$: 12

The miles travelled by each will be found in this manner.

Days. Miles. Days.

2 : 11 :: 12 : 66 miles travelled by A.

3 : 17 :: 12 : 68 miles travelled by B.

Quest. 20. If a clock has two indices (or hands,) one of which (to wit A) is carried twice round the whole circumference of the dial in one day; and the other (B) once in 30 days, and that both at once shewing the same point begin to be moved; the question is, in what time they will be again conjoined? Answ. $\frac{10}{11}$ day.

Day. Circum. Days. Circum.

$$1 : 2 :: 30 : 60$$

Subtract. 1

59

Hence it appears, that in 30 days *A* will have run through 60 circumferences, and *B* one circumference only in the same time; so that *A* gains of *B* 59 circumferences in 30 days, say therefore

Circum. Days. Circum. Day.

$$59 : 30 :: 1 : \frac{1}{3}$$

Quest. 21. If 6 lb. of sugar be equal in value to 7 lb. of raisins; 5 lb. of raisins to 2 lb. of almonds; 3 lb. of almonds to 5 lb. of currans; 2 lb. of currans to 18 d. how many pence are the value of 3 lb. of sugar? Answ. 21d.

$$\begin{aligned} 1. \text{ } \frac{6}{\$} \text{ S.} &= 7 \text{ R.} \\ 2. \text{ } \frac{5}{\$} \text{ R.} &= \frac{2}{\$} \text{ A. } 1. \\ 3. \text{ } \frac{3}{\$} \text{ A.} &= \frac{5}{\$} \text{ C. } 1. \\ 4. \text{ } \frac{2}{\$} \text{ C.} &= \frac{18}{\$} \text{ d. } 3. \\ 5. \text{ } \frac{3}{\$} \text{ d.} &= \frac{3}{\$} \text{ S. } 1. \end{aligned}$$

Answer $7 \times 3 = 21$.

Quest. 22. If 3 dozen pair of gloves be equal in value to 2 pieces of ribbon; 3 pieces of ribbon to 7 dozen of points; 6 dozen of points to 2 yards of Flanders lace; and 3 yards of Flanders lace to 8 shillings; how many dozen pair of gloves may be bought for 28 shillings? Answer 2 dozen pair of gloves.

$$\begin{aligned} 1. \text{ } \frac{3}{\$} \text{ G.} &= \frac{2}{\$} \text{ R. } 1. \\ 2. \text{ } \frac{3}{\$} \text{ R.} &= \frac{7}{\$} \text{ P. } 1. \\ 3. \text{ } \frac{6}{\$} \text{ P.} &= \frac{2}{\$} \text{ L. } 1. \\ 4. \text{ } \frac{3}{\$} \text{ L.} &= \frac{8}{\$} \text{ S. } \frac{7}{\$} \text{ G. } 3: 1. \\ 5. \text{ } \frac{7}{\$} \text{ G. } 4: \frac{7}{\$} \text{ S.} &= ? \text{ G. } \end{aligned}$$

Answer 2.

Quest. 23. Suppose a greyhound to be coursing a hare, in such sort, that the hare takes five leaps for every four leaps of the greyhound, and that the hare is one hundred of her own leaps distant from the greyhound; now if three of the greyhound's leaps be equal to four of the hare's; the question is, to know how many leaps the greyhound must take before he obtain his prey? Answ. 1200 leaps.

$$1. \text{ If } 3 : 4 :: 4 : ?$$

Thus

Thus it appears, that 4 of the greyhound's leaps are equal to $5\frac{1}{2}$ of the hare's leaps; and because by the question the greyhound takes 4 leaps for every 5 of the hare's, therefore the greyhound in every four of his leaps, gains $\frac{1}{2}$ of one of the hare's leaps; therefore say by the Rule of Three,

$$2. \text{ If } \frac{1}{2} : 4 :: 100 : 1200$$

Ques. 24. Bought 7 tuns of wine at 17*l.* per hogshead, which I sell again at 1*s.* per pint; what is the whole gain, and how much *per cent.*? Answ. the whole gain 229*l.* 12*s.* the gain *per cent.* 48*l.* 4*s.* $8\frac{1}{2}$ *d.*

$$\text{As } 1 \text{ bd.} : 17 \text{ l.} :: 7 \times 4 \text{ bds.} : 476 \text{ l.}$$

$$1 \text{ p.} : \frac{1}{20} :: 28 \times 63 \times 8 \text{ p.} : 705 \text{ l. } 12 \text{ s.}$$

$$\text{Therefore the whole gain} = 229 \text{ l. } 12 \text{ s.}$$

$$\text{As } 476 \text{ l. } 229 \text{ l.} :: 100 \text{ l.} : 48,235 \text{ l.}$$

Ques. 25. If 30*s.* be the hire of 8 men for 3 days; how many days must 20 men work for 15*l.*? Answ. 12 days.

Conditional terms $\frac{1}{2} \text{ l. } 8 \text{ Men. } 3 \text{ Days.}$

Corresponding 15 20 A.

$$\text{Therefore } A = \frac{8 \times 3 \times 15 \times 2}{3 \times 20} = 12.$$

Ques. 26. If 10 bushels of oats be enough for 18 horses 20 days; how many bushels will serve 60 horses, 30 days? Answ. 50 bushels.

B. H. Days.

Conditional terms 10 18 20

Corresponding A 60 30

$$\text{Therefore } A = \frac{60 \times 30 \times 10}{18 \times 20} = 50$$

Ques. 27. A man dies and leaves a legacy of 900*l.* to be disposed of among four of his relations, *viz.* A, B, C, and D, which legacy is to be disposed of in this order: B is to have twice as much as A, and C thrice as much as B, and D is to have as much, and $\frac{1}{3}$ as much as C; what must each person have?

Suppose A has 1

Then B . . . 2 Then $18 : 900 :: \left\{ \begin{array}{l} 1 : 50 \\ 2 : 100 \end{array} \right.$

..... C . . . 6 Or $1 : 50 :: \left\{ \begin{array}{l} 2 : 100 \\ 6 : 300 \end{array} \right.$

..... D . . . 9 $\left\{ \begin{array}{l} 2 : 100 \\ 6 : 300 \\ 9 : 450 \end{array} \right.$

$$\overline{18} \qquad \qquad \qquad \overline{900}$$

Ques. 28. A merchant would bestow 220*l.* in cloves, mace, and nutmegs, the cloves being at 5*s.* the pound, the mace at

3 s. the pound, and the nutmegs at 6s. the pound; now he would have of every sort an equal quantity, the question is, how many pounds he may have of each sort? Answ. 200 lb.

5.

5

11

6

$$22 : 1 :: 4400 : 200$$

The Proof.

lb.	s.	l.
200	at 5	amounts to — 50
200	at 11	amounts to — 110
200	at 6	amounts to — 60
		— 220

Quest. 29. A factor is to receive a sum of money, and is offered dollars at 4s. 4d. which are worth but 4s. 3d. or French crowns at 6s. 1½ d. which are worth but 6s. the question is by which coin he shall sustain the least loss? Answ. by the dollars.

$$d. \quad d. \quad d. \quad d.$$

$$52 : 1 :: 73\frac{1}{2} : 1\frac{4}{5}\frac{3}{4}$$

That is, in receiving the dollars every 6s. 1½ d. loses 1½ d., but in receiving the crowns 6s. 1½ d. loses 1½ d. which is a greater loss than 1½ d.

Quest. 30. A Manchester man buys yarn for 6s. the bundle, which not proving so good as he expected, he was willing to put it off again, so as to lose but 6 per cent. by it; the question is, at what price the bundle is to be sold for? Answ. 5s. 7½ d.

$$d. \quad d.$$

$$\text{For as } 100 : (100 - 6) = 94 :: 72 : 67,68$$

Quest. 31. *A, B, and C, put in 360l. and gained 270l. of which as oft as A took 3l. B took up 5l. and, as oft as B took 5l. C took up 7l. what did each gain and put in?*

$$\begin{array}{l} A \\ B \\ C \end{array} \begin{array}{l} 3 \\ 5 \\ 7 \end{array}$$

$$\begin{array}{l} \text{As } 15 : 270 :: \left\{ \begin{array}{l} 3 : 54 \\ 5 : 90 \end{array} \right\} \text{gain,} \\ \text{Or } 1 : 18 :: \left\{ \begin{array}{l} 3 : 54 \\ 5 : 90 \\ 7 : 126 \end{array} \right\} \end{array}$$

15

270

As

$$\begin{array}{l} \text{As } 270 : 360 :: \left\{ \begin{array}{l} 54 : 72 \\ 90 : 120 \end{array} \right\} \text{ sum put in.} \\ \text{Or } 3 : 4 :: \left\{ \begin{array}{l} 126 : 168 \\ \hline 360 \end{array} \right. \end{array}$$

Ques^t. 32. Five merchants, viz. *A*, *B*, *C*, *D*, and *E*, have gained 2025*l.* which they divide in such sort, that $\frac{1}{2}$ of the share of *A* is equal severally to $\frac{1}{4}$ of the share of *B*, $\frac{1}{3}$ of *C*, $\frac{1}{5}$ of *D*, $\frac{1}{6}$ of *E*; the question is, what was the share of every merchant? Answ. *A* 162*l.* *B* 324*l.* *C* 405*l.* *D* 486*l.* *E* 648*l.*

Divide a number at pleasure into several parts, which may be in such proportion as the shares required, and proceed according to the subsequent operation.

$$\begin{array}{r} A 2 \\ B 4 \\ C 5 \\ D 6 \\ E 8 \\ \hline \end{array} \quad \begin{array}{l} . \\ 2 : (162 \text{ for } A, \text{ whereof } \frac{1}{2} \text{ is } 81) \\ 4 : (324 \text{ for } B, \text{ whereof } \frac{1}{4} \text{ is } 81) \\ 5 : (405 \text{ for } C, \text{ whereof } \frac{1}{5} \text{ is } 81) \\ 6 : (486 \text{ for } D, \text{ whereof } \frac{1}{6} \text{ is } 81) \\ 8 : (648 \text{ for } E, \text{ whereof } \frac{1}{8} \text{ is } 81) \\ \hline 2025 \end{array}$$

Ques^t. 33. Two merchants *A* and *B* are in company, the sum of their stocks is 300*l.* the money of *A* continuing in company 9 months, the money of *B* 11 months, they gain 200*l.* which they divide equally; the question is to know how much each merchant did put in? Answ. *A* 165*l.* *B* 135*l.*

Divide 300 into two such parts which may be in proportion, as 11 to 9, so will the greater part be the stock of *A*, and the lesser the stock of *B*, which stocks being multiplied by their respective times, the products will be equal,

$$\begin{array}{r} 11 \\ 9 \\ \hline \end{array} \quad \begin{array}{l} 11 : 165 \text{ for } A. \\ \left\{ \begin{array}{l} 11 : 165 \\ 9 : 135 \text{ for } B. \end{array} \right. \end{array}$$

Ques^t. 34. Two merchants viz. *A* and *B*, are in company, *A* put in 325*l.* more than *B*, and the stock of *A* continued in

in company $7\frac{1}{2}$ months, B put in a certain sum which is unknown, and it continued in company $10\frac{1}{2}$ months: After that time they divided the gain equally; the question is, what each merchant put in? Answ. B 75*l.* and A 107*5l.*

Divide the product of the difference of their stocks multiplied by the time of A , by the difference of their times, so will the quotient be the stock of B , which added to 325*l.* gives the stock of A .

$$325 \times 7\frac{1}{2} = 2437\frac{1}{2}$$

$$\begin{array}{r} 3\frac{1}{2}) 2437\frac{1}{2} \\ \text{Add} \quad 325 \\ \hline \end{array} \quad (750 \text{ stock of } B.$$

$$1075 \text{ stock of } A.$$

Quest. 35. Two merchants company, A put in 20*l.* and B put in 80 ducats, they gained 67*l.* 10*s.* of which A took up 30*l.* what is the value of a ducat? Answ. 6*s.* 3*d.*

First, find a stock for B , equivalent to A 's stock, thus,

$$\text{As } 30 : 20 :: (67,5 - 30) = 37,5 : 25.$$

$$\text{Then as } 80 : 25 :: 1 : \frac{25}{80} = \frac{1}{8} = 6s. 3d.$$

Quest. 36. A , B , and C , company, and put in together 3860*l.* A 's money was in 3 months, B 's money was in 5 months, and C 's money was in 7 months; they gained 234*l.* which was so divided as the $\frac{1}{2}$ of A 's gain was equal to $\frac{1}{3}$ of B 's gain, and $\frac{1}{3}$ of B 's gain was equal to $\frac{1}{4}$ of C 's gain; what did each merchant gain and put in?

Suppose A 's gain was 4*l.* then must B have 6*l.* and C 8*l.* according to the tenor of the question, which numbers added together make 18. Then say,

$$\begin{aligned} \text{As } 18 : 234 &:: \left\{ \begin{matrix} 4 \\ 6 \\ 8 \end{matrix} \right\} : \left\{ \begin{matrix} 52 \\ 78 \\ 104 \end{matrix} \right\} \text{ } A's \text{ gain,} \\ \text{Or } 1 : 13 &:: \left\{ \begin{matrix} 4 \\ 6 \\ 8 \end{matrix} \right\} : \left\{ \begin{matrix} 52 \\ 78 \\ 104 \end{matrix} \right\} \text{ } B's \text{ gain.} \\ &\qquad\qquad\qquad C's \text{ gain.} \end{aligned}$$

234

Now suppose A put in 140*l.* then the product of A 's stock and time will be 3×140 .

$$\text{Then as } 52 : 3 \times 140 :: \left\{ \begin{matrix} 78 \\ 104 \end{matrix} \right\} : \left(\frac{3 \times 140 \times 78}{52} = \right) 636$$

$$\qquad\qquad\qquad \left(\frac{3 \times 140 \times 104}{52} = \right) 840$$

Therefore

Therefore if A put in B put in C put in

B put in $\frac{6}{5}^{\text{th}}$	=	140
And C put in $\frac{8}{7}^{\text{th}}$	=	126
		120
		386

Then as $386 : 3860 :: \begin{cases} 140 : 1400 \\ 126 : 1260 \\ 120 : 1200 \end{cases}$ A 's stock.
Or $1 : 10 :: \begin{cases} 1400 : A \\ 1260 : B \\ 1200 : C \end{cases}$.

Quest. 37. A hath $\frac{1}{2}$ of a ship, $B \frac{1}{3}$, $C \frac{1}{5}$, $D \frac{1}{15}$, the master clears 120*l.* how much must each owner have?

Answer $\begin{cases} A \text{ must have } 60 : 0 \\ B : \dots : 30 : 0 \\ C : \dots : 7 : 10 \\ D : \dots : 22 : 10 \end{cases}$

Quest. 38. There are 7 chests of drawers, in each of which there are 18 drawers, and in each of these there are 6 divisions; in each of which there are 16*l.* 6*s.* 8*d.* how much money is there in the whole? Answ. 12348*l.*

$$\text{For } \frac{7 \times 6 \times 18 \times 7}{49 \times 2 \times 18 \times 7} = 12348.$$

Quest. 39. How many men must be employed to reap 420 acres, in 17 days; if there were required 34 men to reap 54 acres, in 9 days? Answ. 140 men.

$$\text{For } \frac{34 \times 9 \times 420}{54 \times 17} = 140.$$

Quest. 40. If 1000*lb.* of beef, or pork, serve 250 seamen 7 days, how many pounds of the same will serve 550 seamen 9 weeks? Answ. 19800*lb.*

$$\text{For } \frac{550 \times 9 \times 7 \times 1000}{250 \times 7} = 19800.$$

Quest. 41. If a footman travel 240 miles in 12 days, when the days are 12 hours long, how many days may he travel 720 miles in, of 16 hours long? Answ. 27 days.

$$\text{For } \frac{12 \times 12 \times 720}{16 \times 240} = 27.$$

Quest. 42. A carrier received 4*s.* 2*d.* shillings for the carriage of 3*C.* weight 150 miles, I demand how much he ought to receive for the carriage of 7*C.* 3*qrs.* 4*lb.* 50 miles, at that rate? Answ. 36*s.* 4*d.*

	L.	C.	M.
Conditional terms	$\frac{4}{5}$	3	150
Corresponding	A.	$\frac{1}{14}\frac{1}{2}$	50

$$\text{Therefore } A = \frac{872 \times 50 \times 42}{112 \times 20 \times 3 \times 150} = \frac{109}{20 \times 3}$$

Ques^t. 43. An ancient lady, being demanded how old she was, to avoid a direct answer, said, I have 9 children, and there are 3 years between the birth of each of them; the eldest was born when I was 19 years old, which is exactly the age of the youngest; how old was the lady? Ans^w. 62 years.

Ques^t. 44. There are 100 stones which lie three feet or one yard upon the ground one from the other, and there is one employed to gather up the stones one by one, and bring them to a basket which standeth 3 feet from the first stone; how many yards of ground must he go backwards and forwards in all, before he hath brought the last stone to the basket?

The answer to this question may be obtained by Art. 536.
Thus,

To fetch the first stone he goes 2 yards.

To fetch the 100th . . . 200 ditto.

Sum of the greatest and least terms 202

Multiply by 100 the number of terms.

Divide by 2) 20200

Answer 10100 yards.

Ques^t. 45. Lent 109 guineas at 4 per cent. which, by the 18th of August 1740, was raised by the interest to as many moidores, wanting 2s. 6d. on what day did the bond bear date? Answer July 7, 1733.

109 moidores = 147 : 3 : 0

deduct 2 : 6

remains	147	:	0	:	6
109 guineas	114	:	9	:	0

interest 32 : 11 : 6

I. D. t.

Conditional terms 100 . 365 . 4

Corresponding 114,45 . A . 32,575

$$\text{Therefore } A = \frac{100 \times 365 \times 32,575}{4 \times 114,45} = 2597 \text{ days.}$$

Ques^t.

Quest. 46. If 3481 soldiers are to be placed in a square battle, how many are to be set in rank or in file? Answ. 59. (for the square root of 3481 is 59.)

Quest. 47. If 100*l.* being put forth for interest at a certain rate, will at the end of two years be augmented to 112 $\frac{36}{100}$ *l.* (compound interest, or interest upon interest being computed) what principal and interest will be due at the first year's end?

Answ. 106*l.* (composed of 100*l.* principal, and 6*l.* interest) which 106 is a mean geometrically proportional between 100 and 112,36.

$$\text{That is } 100 \times 112,36 = 106 \times 106.$$

Quest. 48. If 100*l.* being put forth for interest at a certain rate, will at the end of 3 years be augmented to 115,7625*l.* (compound interest being computed,) what principal and interest will be due at the first year's end?

Answ. 105*l.* (composed of 100*l.* principal, and 5*l.* interest) which 105 is the first of two mean proportional numbers between 100 and 115,7625*l.*

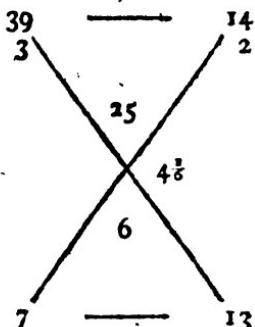
Quest. 49. *Vitruvius* (in lib. 9. cap. 3.) reports, that king *Hiero* having given orders for the making of a crown of pure gold, was informed that the workman had detained part of the gold; the king being much displeased at the deceit, recommended the examination of the busines to the famous *Archimedes of Syracuse*, who without defacing the crown discover'd the cheat in this manner, viz. experience telling him that a quantity of gold would possess less room, or space than the same quantity of silver, and consequently that a mixt mass of gold and silver of the same quantity would take up some mean space between the two former; he made a mass of pure gold of the same weight with the crown, likewise another mass of silver of the same weight; then having put the crown, as also the other two masses severally into a vessel filled up to the brim with water, he diligently reserv'd the water flowing over into another vessel, and from those three several quantites of water so expelled, he found out the quantity of gold and of silver in the crown: But since *Vitruvius* has not deliver'd the practical operation, I shall shew the same after the manner of *Cardanus*, *Gemma Frisius*, and other Arithmeticicians.

Let us therefore suppose the weight of the crown, as also of the two several masses, to have been 5*lb.* Suppose also that by putting the mass of gold into the vessel, 3*lb.* of water was expelled; by putting in the crown 3*lb.* and by putting in the mass of silver 4*lb.* The question therefore is to know how

much

much gold and how much silver the crown was composed of. This may be resolved by the Rule of False, after this manner : Suppose 3 lb. of gold to be in the crown, then there remained 2 lb. of silver; now say by the Rule of Three, if 5 lb. of gold expel 3 lb. of water, how much 3 lb. of gold ? Answ. $1\frac{1}{2}$ lb. Also if 5 lb. of silver expel $4\frac{1}{2}$ lb. of water, how much 2 lb. of silver ? Answ. $1\frac{1}{2}$ lb. of water; add therefore the water of the silver and of the gold together, to wit, $1\frac{1}{2}$ and $1\frac{1}{2}$, so there will arise $3\frac{1}{2}$ lb. of water: This ought to have been $3\frac{1}{2}$ lb. (for so much overflow'd by putting in the crown;) but it is too much by $\frac{1}{8}$; therefore $\frac{1}{8}$ is to be noted with +, for the error of the first position 3 lb. Again, feign another quantity of gold to have been in the crown, to wit, 2 lb. therefore there remain'd 3 lb. of silver; then say, if 5 lb. of gold expel 3 lb. of water, how much 2 lb. of gold ? Answ. $1\frac{1}{2}$ lb. of water: Also if 5 lb. of silver expel $4\frac{1}{2}$ lb. of water, how much 3 lb. of silver ? Answ. $2\frac{1}{2}$ lb. then add $1\frac{1}{2}$ lb. unto $2\frac{1}{2}$, the sum will be $3\frac{1}{2}$ lb. of water. This ought to have been $3\frac{1}{2}$ lb. but it is too much by $\frac{1}{8}$. Therefore $\frac{1}{8}$ is to be noted with + for the error of the second position 2 lb. Here because the errors are Fractions having a common Denominator, I take their Numerators, 7 and 13 instead of the errors, then multiplying cross-wise, to wit, 3 by 13 the product is 39; also 2 by 7, the product is 14, which subtracted from the former product 39, (because the errors are alike,) leaves 25 for a Dividend; also the difference between the errors 7 and 13 is 6 for a Divisor: Lastly, dividing 25 by 6 the quotient is $4\frac{1}{6}$; so much gold, therefore, was in the crown, and consequently (because the weight of the crown was 5 lb.) there was $\frac{1}{8}$ of silver, which may be proved thus : Say, if 5 lb. of gold expel 3 lb. of water, how much $4\frac{1}{2}$ lb. of gold ? Answ. $2\frac{1}{2}$ lb. of water: Again, if 5 lb. of silver expel $4\frac{1}{2}$ lb. of water, how much $\frac{1}{8}$ of silver ? Answ. $\frac{1}{2}$ lb. of water, which being added to $2\frac{1}{2}$ lb. the sum is $3\frac{1}{2}$ lb. of water, to wit, as much as flowed over when the crown was put into the vessel.

Here note, that in making a trial of this nature, there is no necessity that the mass of gold or of silver be of the same weight with the crown, or whatsoever thing is to be examined, but of what notable part of the weight you please.



C H A P. XLIV.

Of SPORTS and PASTIMES.

741. PROBL. I.

To discover a number which any one shall have in his mind, without requiring him to reveal any part of that or any number whatsoever.

AFTER any one has thought upon a number at pleasure; bid him double it, and to that double bid him add any such even number as you please to assign; then from the sum of that addition let him reject one half, and reserve the other half: Lastly, from this half bid him subtract the number which he first thought upon; then may you boldly tell him what number remains in his mind after that subtraction is made, for it will always be half the number which you assigned him to add.

For Example: Suppose he thought upon 6, the double thereof is 12, to which bid him add some even number at your pleasure, suppose 4, so will the sum be 16, whereof the half is 8, from which if he subtract 6, (the number first thought on) the remainder is 2, (to wit, half the number 4, which was by you assigned to be added;) which remainder you discover, notwithstanding all the operation was performed in his mind; without his making known any number whatsoever. Note, That the adding of an even number, as aforesaid, is not of necessity, but only to avoid a fraction that will arise, by taking the half of an odd number.

742. The Reason of the Rule.

If to the double of any number (which number for distinction sake I call the first) a second number be added, the half of the sum must necessarily consist of the said first number, and half the second; therefore if from the said half sum the first number be subtracted, the remainder must of necessity be half the second number which was added.

743. PROBL. II.

Two numbers, the one even and the other odd, being proposed unto two persons, to the end they may (out of your sight) severally chuse one of those numbers; to discover which of these numbers each person had chosen.

Suppose you have propounded to *Peter* and *John*, two numbers, the one even and the other odd, as 10 and 9; and that each of those persons is to chuse one of the said numbers unknown to you. Now to discover which number each person made choice of, you must take two numbers, the one even and the other odd, as 2 and 3; then bid *Peter* multiply that number which he has chosen by 2; and cause *John* to multiply that number which he has pitched upon by 3; that done, bid them add the two products together, and let them make known the sum to you, or else demand of them whether the said sum be even or odd, or by any other way more secret endeavour to discover it, by bidding them take the half of the said sum, for by knowing whether the said sum be even or odd, you obtain the principal end to be aimed at; because if the said sum be an even number, then infallibly he that multiplied his number by your odd number, (to wit, by 3) did chuse the even number (to wit, 10); but if the said sum happen to be an odd number, then he whom you caused to multiply his number by your odd number, (to wit 3) did infallibly chuse the odd number (to wit, 9).

For Example: If *Peter* had made choice of 10, and *John* 9, suppose you required *Peter* to multiply his number 10 by 2, and *John* to multiply his number 9 by 3; the products will be 20 and 27, whereof the sum is 47, which being an odd number, you may thence conclude that *John*, whom you caused to multiply his number by 3, chose the odd number 9, and therefore *Peter* took 10. But if you had ordered *John* to multiply his number 9 by 2, and *Peter* to multiply his number 10 by 3, the products would have been 18 and 30, whereof the sum is 48, which is an even number; from whence you may infer, that he that multiplied his number by 3 pitched upon the even number, and therefore *Peter* chose 10, and *John* 9.

The reason of the said Rule will appear from Note 1. and 2. to Art. 64. and Note 1. and 3. to Art. 47.

744. P R O B L. III.

A certain number of distinct things being propounded, to dispose them in such an order, that casting away always the ninth, or the tenth, or any other that shall be assigned, to a certain number, those remaining may be such as were first intended to be left.

This Problem is usually proposed in this manner, *viz.* Fifteen *Christians* and fifteen *Turks* being at sea in one and the same ship in a terrible storm, and the pilot declaring a necessity of casting the one half of those persons into the sea, that the rest might be saved; they all agreed, that the persons to be cast away should be set out by lot after this manner, *viz.* the thirty persons should be placed in a round form like a ring, and then beginning to count at one of the passengers, and proceeding circularly, every ninth person should be cast into the sea, until of the thirty persons there remained only fifteen. The question is, how those thirty persons ought to be placed, that the lot might infallibly fall upon the fifteen *Turks*, and not upon any of the fifteen *Christians*? For the more easy rememb'ring of the Rule to resolve this question, I shall presuppose the five vowels, *a*, *e*, *i*, *o*, *u*, to signify five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, and (*u*) five; then will the Rule itself be briefly comprehended in these two following verses:

*From Numbers, Aid, and Art,
Never will Fame depart.*

In which verses you are principally to observe the vowels, with their correspondent numbers before assigned; and then beginning with the *Christians*, the vowel *o* (in *from*) signifies, that four *Christians* are to be placed together; next unto them the vowel *u* (in *Num.*) imports that five *Turks* are to be placed together; in like manner *e* (in *bers*) denotes two *Christians*; *a* (in *Aid*) one *Turk*; *i* (in *Aid*) three *Christians*; *a* (in *and*) one *Turk*; *a* (in *Art*) one *Christian*; *e* (in *ne*) two *Turks*; *e* (in *ver*) two *Christians*; *i* (in *will*) three *Turks*; *a* (in *Fame*) one *Christian*; *e* (in *Fame*) two *Turks*; *e* (in *de*) two *Christians*; *a* (in *part*) one *Turk*.

745. The invention of the said Rule, and such like, depends upon the subsequent process, *viz.* If the number of persons be thirty, let thirty figures or cyphers be placed circularly, or else in a right line as you see,

oooooooooooooooooooooooooooooooo

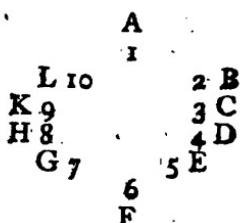
That done, begin to count from the first, and mark the ninth (or what other shall be assigned) by putting a point or cross over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to do the same, beginning again when you shall be at the end (if the cyphers are placed in a right line,) and passing over those, which you had already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen; for then all the cyphers marked shall be those which must be cast away, and the others those that are to remain. Hence it is evident, that if you observe how those cyphers marked, are disposed among those which are not marked, you will easily make a Rule for any number whatsoever.

By this invention (as some conjecture) the famous historian *Josephus the Jew*, preserved his life very subtilly in the cave, to which himself and forty of his countrymen had fled from the furious and conquering *Romans* at the siege of *Jotapata*: For his said countrymen having most wickedly resolved to kill one another, rather than yield to their enemies, he at length (when no arguments that he could use would dissuade them from so horrid an act) prevailed with them to execute their tragical design by lot; and so by the help of the aforesaid artifice, as we may suppose, himself with one other person only remaining alive, after the rest were inhumanly murdered, they agreed to put an end to the lot, and thereby save their lives. This story you may see at large in the fourteenth chapter of the third Book of the History of *Josephus*, of the Wars of the *Jews*.

746, P R O B L . IV.

Many numbers which proceed from 1 or unity, in a progression, according to the natural order of numbers, (such as these, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a ring; to discover which of those numbers any one has thought upon.

Let any multitude of numbers in the aforesaid progression, suppose these ten, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be mark'd upon ten ivory-counters (or for want thereof upon ten small pieces of paper) which may be represented by these ten letters, A, B, C, D, E, F, G, H, K, L, viz. suppose 1 to be writ upon the counter A, 2 upon B, 3 upon C, &c. Then having placed those counters circularly as you see (with their blank faces uppermost, and the figures underneath, that the



subtly of the sport may the better be concealed) let any one think upon any number of units which does not exceed 10; that done, bid him touch one of those counters at pleasure, and to the number on the back-side of the counter touched (which you cannot be ignorant of, having noted well the place of I or A) add secretly in your mind, the just number of all the counters, and reserve the sum; then bid him imagine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, until he has made up the aforesaid sum, which you reserved, so will his computation infallibly end on the counter upon which the number thought of is marked.

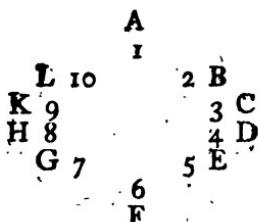
For example, suppose he thought 7 or G, and that he touched B, to wit, 2; add to 2 the number of all the counters, to wit, 10, so the sum will be 12; then bid him count to 12, beginning at B and going backward, and esteeming B to be the number thought, to wit, 7; so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H; and lastly, 12 upon the counter G, which being turned up will shew the number thought.

747. The reason of this Rule is not difficult to be apprehended, two principles being presupposed, the one is this; to wit, many counters or things whatsoever being disposed orderly one after the other, in one continued line, whether it be right or circular; if you value or name the first counter to be some number of units at pleasure, and continue to count forward according to the natural order of numbers, until another number be named which falls upon the last counter: Or if you imagine or name the last counter, to be the same number of units as before you put upon the first, and continue to count backwards to the first counter; I say, that the same number will be named at the end of both these computations: For example, in these 9 letters, A. B. C. D. E. F. G. H. K. if the letter A be esteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you esteem

K to be 4, and count backwards from K to A, the letter A will likewise fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principle is this, to wit, many counters being disposed in a round form like a ring; if you esteem any one of those counters to be some number at pleasure, and then from that counter if you count circularly, until you end upon the counter where you began, the number last named will be equal to the sum of the number of all the counters, and of the number which you put upon the first counter; for example, if D be one of ten letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progression of



numbers till you end with D, where you began; the number 17, which is composed of 10 and 7, will necessarily fall upon D; for 9 (which is the number of letters in the circumference besides D) being added to 7 (which was first put upon D) makes 16, to which 1 being added, (because D ends as well as begins the circumference) the sum is 17.

Now these two principles being presupposed, it will not be difficult to apprehend the reason of the aforesaid Rule in all cases that can happen; for imagine that one has thought upon 7, or the counter G, then that counter which he shall touch must either be the same counter G, or some other that precedes or follows G.

First therefore, supposing the counter or number touched to be the same with the number thought, the truth of the Rule will be then evident; for by the Rule given, he will begin to count from the same G to 17, putting 7 upon G, therefore by the second presupposition the number 17 will fall upon G.

Secondly,

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10; then according to the Rule adding 10 (the numbers of all the counters placed circularly) to 10 or L, the (counter touched) bid him count backwards to 20 by beginning at L, and esteem L to be 7. Now, because by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the first presupposed principle, if we esteem L to be 7 and count backwards, the number 10 will infallibly fall upon G, and then the number 20 shall also fall upon the same G by the second presupposed principle.

Lastly, imagine he touched some number or counter which precedes 7 the number sought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having first imagined B to be the number thought 7; and going backwards to A, L, K, &c. Now because by proceeding to count at B, which is 2, and beginning to count forward to C, D, &c. the number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards F, E, &c. the number 7 will fall upon B (by the first presupposed principle); therefore when one assumes B to be 7, and counts towards A, L, &c. to any assigned number, it is in effect as much as when one imagines G to be 2, and counts towards F, E, &c. unto the said assigned number, for each of those computations will end in the same point; but it is manifest (by the second presupposed principle) that esteeming G to be 2, and counting towards F, E, D, &c. round the whole circumference; the number 12 will fall upon the same G. And because G being supposed to be 2, and counting on the same coast as before, the number 7 falls upon B; therefore if the computation be continued on the same coast from B 7, to 12, the number 12 will fall upon the same G. So that the practice of this sport in all its cases is demonstrated.

Note. That to the number of the counter touched you may not only add the number of all the counters once (as the Rule directs) but twice, thrice or more times: For example, B being touched, you may cause him to count to 12, or to 22; or to 32, 42, &c. the reason whereof is evident from the second presupposed principle.

748. P R O B L. V.

Many numbers being shewed by pairs, to wit, two by two, unto any person, that he may think upon any one of those pairs at pleasure, to discover the pair that was thought upon.

Let 20 numbers, suppose these, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, be writ upon ivory-counters (or for want thereof upon small pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, &c. Then dispose them into pairs as you see, viz. suppose 1 and 2 to be one pair, 3 and 4 to be another pair, &c. and

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

of these pairs let any one think upon which pair he pleases. That done, you are to distribute the said 20 numbers into ranks, in form of a long square, until there be 5 numbers in length, and 4 in breadth, after this manner, viz. lay the three first numbers, 1, 2, and 3, in a rank (as you see in the second figure) from A towards B; then place 4 under 1, and 5 after 3, (in the said rank AB.) Again, place 6 under 4, and 7 after 5, (in the said rank AB.) Then place 8 under 6, also 9, 10, 11, on the right-hand of 4, in the rank CD. Again, place

place 12 under 9, and 13 on the right-hand of 11, in the rank CD, and 14 under 12. Moreover, place 15, 16, 17, on the right-hand of 12, in the rank EF. Lastly, place 18, 19, 20, on the right-hand of 14, in the rank GH, so will all the numbers be ranked as you see in the Table. That done, you are to demand of him that thought upon two numbers as aforesaid, in what rank or ranks the said numbers happen to be found, *viz.*

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

in which of the ranks AB, CD, EF, GH, or in which two of the said ranks: Now if he answer that the two numbers he first thought upon are in the first rank AB, then 1 and 2 will be the numbers thought or kept in mind; if in the second CD, then 9 and 10 shall be the numbers thought of; if in the third rank EF, then 15 and 16 will be the numbers thought: If they are in the fourth rank GH, then 19 and 20 shall be the numbers thought; but if he say that the numbers thought are in different ranks, then you are heedfully to mark the said numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which may be called the keys of the sport, in regard they serve not only to discover the two numbers thought, when they are both in one and the same rank (as aforesaid;) but even when they are in two different ranks: For in this latter case, as soon as it hath been declared to you in which two ranks the two numbers thought are placed, you are to take the key of the highest of those two ranks, and descending in a down-right line from the first number of that key to the lower of the said two ranks, you'll there find one of the two numbers thought, and upon the right-hand of the second number of the said key, at the same distance sideways, from the second number of the key, (as one of the numbers thought was distant from the first number of the key,) you will find the other number thought.

For example, suppose the two numbers thought are 7 and 8, and it is declared to you, that they are in the first and fourth ranks; take then the key of the highest of those two ranks;

to

to wit, of the first, which is 1 and 2, and descending down-right from 1 to the fourth rank, you'll there find 8 one of the numbers thought: Then seek sideways on the right-hand of 2 (the second number of the key) a number as far separated from 2, as 8 is distant from 1, and you'll find 7 the other number thought.

Again, suppose he says that the numbers thought are in the second and third ranks: Take then the key of the second rank which is 9 and 10, and descending down-right from 9 to the third rank, you shall there find 12, which is one of the numbers thought; then seek sideways on the right-hand of 10, (the second number of the key) a number as far distant from 10, as 12 is from 9, and you'll find 11, which is the other number thought.

The reason of this will be apparent, from a serious consideration of the placing of the numbers according to the Rules before given: For it is thereby evident, that of the first numbers coupled two by two, there can never be found more than one pair in one and the same rank; and of all the other pairs, one number is always found in one rank, and the other number in another rank.

Note also, that this sport may be practised with divers persons at once, and not only with 20 numbers, but with any such multitude of numbers as is produced by the multiplication of any two numbers that differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42, which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the numbers in ranks according to the directions before given: And for the more easy comprehending of that order, I have, in the following Table, ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illustration than can be expressed by many words,

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

749. PROBL. VI.

Three jealous husbands with their wives, being ready to pass by night over a river, do find at the river side a boat which can carry but two persons at once, and for want of a waterman they are necessitated to row themselves over the river at several times : The question is, how those six persons shall pass 2 by 2, so that none of the 3 wives may be found in the company of 1 or of 2 men, unless her husband be present?

They must pass in this manner, viz. first two women pass, then one of them brings back the boat and repasses with the third woman ; that done, one of the three women brings back the boat, and sitting down upon the ground with her husband, permits the other two men to pass over to find their wives : Then one of the said men with his wife brings back the boat, and placing her upon the ground, he takes the other man, and repasses with him : Lastly, the woman who is found with the three men enters the boat, and at twice goes to fetch over the other two women,

750. PROBL. VII.

A Country-man having a fox, a goose, and a peck of corn, in his journey came to a river, where it so happened that he could carry but one over at a time. Now, as no two were to be together that might destroy each other ; so he was at his wit's end how to dispose of them ; for, says he, tho' the corn can't eat the goose, nor the goose eat the fox, yet the fox can eat the goose, and the goose eat the corn : The question is, how he must carry them over ?

First he must carry over the goose, leaving the fox and corn, (for the fox will not eat the corn) then, returning back, he may carry over the fox, bringing the goose back again ; then leaving the goose, he may carry over the corn ; lastly, he must return to fetch the goose.

751. PROBL. VIII.

Two merry companions are to have equal shares of eight gallons of wine, which are in a vessel containing exactly eight gallons : now to make this equal partition they have only two other empty vessels, of which one contains five gallons, and the other three ;

the question is, how they shall exactly divide the wine by the help of those three vessels?

First, from the vessel which contains 8 gallons, and is full of wine, let 5 gallons be poured into the empty vessel of 5, and from this vessel so filled, let three be poured into the empty vessel of three, so there will remain 2 gallons within the vessel of 5. Then let the three gallons that are within the vessel of 3 be poured into the vessel of 8, which will now have 6 gallons within it; that done, let the two gallons which are in the vessel of 5, be put into the empty vessel of 3: Then of the six gallons of wine that are within the vessel of 8, fill again the 5, and from those 5 pour out one gallon into the vessel of 3, which wanted only one gallon to fill it; so there will remain exactly 4 gallons within the vessel of 5, and 4 gallons within the other two vessels. This question may be resolved in another way, but I leave that as an exercise to the sagacity of the ingenious reader.

Now though at first sight it may be thought by some, that the three last mentioned Problems cannot be resolved by any certain Rule, but only by many trials; yet by infallible argumentation and discourse, the solution of those questions may be found out, or else the impossibility of them, if by chance they should be propounded impossible; as the most ingenious *Gasper Bachet* has manifested in a little book in the French Tongue, intitled, *Problemes plaisans & delectables qui se font par les nombres*, from which book I have extracted the contents of this chapter.

C H A P. XLV.

Of M E A S U R I N G.

752. THE various kinds of measuring, are three, viz.
I. Lineal, called (by workmen) *Ranning Measure*,
in that it respects length only, the parts of which are

12	Inches,	} 1 Foot,
3	Feet,	
16½	Feet,	

753. Superficial, or Square Measure, in that it respects Length and Breadth, the parts are, viz.

144	Inches,	} 1 Foot, ½ a Foot, ¼ of a Foot, ⅓ of a Foot, ⅔ a quarter of a Foot, 1 Rod, ½ a Rod.
72	Inches,	
36	Inches,	
48	Inches,	
18	Inches,	
272½	Feet,	
136½	Feet,	
1296, or 9	Feet,	
Inches		1 Yard.

Note, That in dividing by $272\frac{1}{2}$, which reduces Superficial Feet to Square Rods, workmen take no notice of the $\frac{1}{4}$ of the foot, dividing only by 272, which gives the content a little too much.

754. Solid, or Cube Measure, respecting Length, Breadth, and Thickness, whose parts are,

Inches.	
1728,	} 1 Foot, ⅔ of a Foot, ⅔ of a Foot, ⅔ of a Foot,
1296,	
864,	
432, &c.	
27 Feet,	

755. Of SUPERFICIAL MEASURE.

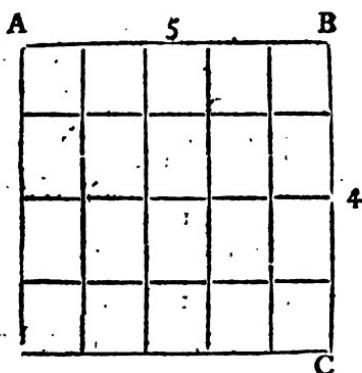
A General Rule.

Multiply the Length by the Breadth, and the Product is the Superficial Content.

Demonstration.

Demonstration.

For if a line AB, being divided into any number of equal parts, be moved constantly at right angles with another line, BC, divided into any other number of such equal parts, as are in AB; it will, when advanced to any part of BC, describe thereby as many equal squares as there are parts in it: And if moved on farther, to the next part, it will describe as many more squares, and so the same number of squares will be as often described, as there are parts in the line, BC; and consequently there must be as many squares in the surface, made by these lines, as there are units in the product of the parts of the two sides.



756. Now the content of any thing may be found three several ways, viz.

1. By Decimals,
2. By Aliquot Parts.
3. By cross Multiplication.

All which I shall promiscuously use in the following Examples. But the latter of the three is used by most workmen, except glaziers, whose foot being decimaly divided, they cast up the content of their work by Decimals.

757. *Cross Multiplication*, is performed as in the following Example.

Let it be required to multiply 47 feet 8 inches by 9 feet 4 inches: See the operation.

F.

$$\begin{array}{r}
 F. \quad I. \\
 47 : 8 \\
 9 : 4 \\
 \hline
 423 \\
 6 : 0 \\
 15 : 8 \\
 2 : 8 \\
 \hline
 444 : 10 : 8
 \end{array}$$

Here the feet are multiplied by the feet, then (cross-wise) *viz.* the 9 feet in the Multiplier by the 8 inches in the Multiplicand, gives 72 inches or 6 feet, and the 47 feet in the Multiplicand being multiplied by 4 inches in the Multiplier, give 15 foot 8 inches. Lastly, the inches in both the factors, being multiplied one into the other, make 32 parts, or 2 inches, 8 parts, the total of which, *viz.* 444 f. 10 in. 8 par. is the product of 47 foot 8 inches, by 9 foot 4 inches.

758. The Content of the Work, is given either by the Foot, the Yard, the Square, or the Rod, *viz.*

759. By the Foot.

Glazing and Masonry.

Dimensions are taken in feet and inches, the Content given per Foot square.

In Glazing.

Example. How many Feet of Glazing are there in that pane of glass which is 4 feet 5, and 2 feet 42.

$$\begin{array}{r}
 2,42 \\
 4,5 \\
 \hline
 1210 \\
 968 \\
 \hline
 10,890 \qquad \text{Answer, 10 f. ,890}
 \end{array}$$

In Masonry.

Example. Suppose a yard pav'd with free-stone, *viz.* length 22 feet 4 inches, breadth 19 feet 7 inches; how many square feet?

$$\begin{array}{r}
 22 : 4 \\
 19 : 7 \\
 \hline
 198 \\
 22 \\
 6 : 4 \\
 12 : 10 \\
 \hline
 2 : 4 \\
 \hline
 437 : 4 : 4 \text{ Facit } 437 - 4 \cdot 4
 \end{array} \quad \text{Feet. In. Part.}$$

360. By the yard.

Painting, Joinery, Plastering, &c.

Dimensions are taken in feet and inches; but the content given in square yards, found by dividing the superficial feet by 9.

In Painting.

Example. The height of a room being 12 feet 4 inches, and 84 feet 11 inches about, how many square yards?

$$\begin{array}{r}
 84 . 11 \text{ about} \\
 12 . 4 \text{ high} \\
 \hline
 1019 . 0 = 84 : 11 \times 12 \\
 28 . 3 . 8 = 84 : 11 \times \frac{1}{4} \\
 \hline
 9) 1047 . 3 . 8 \quad \text{Yds. Feet. Inch. Part.} \\
 116 : 3 . 3 . 8 \text{ Answ. } 116 . 3 . 3 . 8
 \end{array}$$

Joinery.

Example. How many yards of wainscot does that room take up, whose height is 12 feet 3 inches, and compasses 104 feet 6 inches?

Feet. Inch.

104 . 6

12 . 3

$$\begin{array}{r}
 1254 . 0 = 104 . 6 \times 12 \\
 26 . 1 . 6 = 104 . 6 \times \frac{1}{4} \\
 \hline
 9) 1280 . 1 . 6 \quad \text{Yds. Feet. Inch. Part.} \\
 142 . 2 . 1 . 6 \text{ Answ. } 142 . 2 . 1 . 6
 \end{array}$$

Plastering.

Plaistering.

Example. How many yards in a ceiling 47 feet 4 inches, 7 parts long, and 18 feet broad?

$$\begin{array}{r} 47 \cdot 4 \cdot 7 \text{ long} \\ 18 \text{ broad} = 6 \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 284 \cdot 3 \cdot 6 \\ 3 \\ \hline \end{array}$$

$$9) \begin{array}{r} 852 \cdot 10 \cdot 6 \\ \hline \end{array}$$

$$94 \cdot 6 \cdot 10 \cdot 6 \quad \text{Yar. Ft. In. Part.}$$

$$\text{Facit } 94 \cdot 6 \cdot 10 \cdot 6$$

Under this head you may further note, that if you multiply the $\frac{1}{2}$ part of the length, by the $\frac{1}{2}$ of the breadth, the product will be the content in square yards, it being the same to take $\frac{1}{2}$ of lineal, as $\frac{1}{2}$ of superficial measure.

761. By the Square.

Partitioning, Flooring, Roofing, Tiling.

Dimensions are taken in feet and inches, but the content or value is by the square of 10 feet, done by cutting off two figures of the superficial feet towards the right-hand,

Partitioning.

Example. How many squares are contained in that work which is 199 feet 10 inches in length, and 10 feet 7 inches high?

$$\begin{array}{r} 199 \cdot 10 \\ 10 \cdot 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1998 \cdot 4 = 10 \\ 66 \cdot 7 \cdot 4 = \frac{1}{2} \\ 49 \cdot 11 \cdot 6 = \frac{1}{4} \\ \hline \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \times 199 : 10$$

$$21,14 \cdot 10 \cdot 10 \quad \text{Squar. Feet. Inch. Parts.}$$

$$\text{Answ. } 21 \cdot 14 \cdot 10 \cdot 10$$

From which sometimes the content of doors and other vacancies are deducted.

Flooring.

Example. A floor being 49 feet, 7 inches, 4 parts long, and 26 feet 6 inches broad, how many squares?

$$\begin{array}{r}
 49 : 7 : 4 \\
 26 : 6 : 0 \\
 \hline
 1289 : 10 : 8 \\
 24 : 9 : 8 \\
 \hline
 \end{array}$$

Answ. 13,14. 8. 4

Roofing.

Example. Suppose an house 18 feet 4 inches in front, and 37 feet 10 inches in depth, how many squares in the roof?

$$\begin{array}{r}
 37 : 10 \\
 3 \\
 \hline
 113 : 6 \\
 6 \\
 \hline
 \frac{3}{3}) 681 : 0 \\
 12 : 7 : 4 \\
 \hline
 \frac{2}{2}) 693 : 7 : 4 \\
 346 : 9 : 8 \text{ added} \\
 \hline
 \end{array}$$

Squares. Feet. Inch.

10. 40. 5 10,40. 5. 0

Here note, after you have multiply'd the depth by the front, and so found the content of the ground-plot in feet, half of those being added, gives the content of a pitch-roof, which reduce into squares.

762. By the rod, as in

Brick-work.

Dimensions are taken in feet and inches, and the content, given per the square of a rod of $16\frac{1}{2}$ foot.

The content of this work is found in the same manner as all the other, to wit, by multiplying the length by the breadth, and only differs from the former in two things.

763. When you have so found the superficial content in feet, divide it, by 272, the square feet in one rod, the quote is rods, the content being always so given.

764. That when 'tis more or less than a brick and $\frac{1}{2}$ thick, it must be reduced to that thicknes, being always measured by the square rod of a brick and $\frac{1}{2}$. To do which observe,

Rule.

Rule. Multiply the product of the length and height of the wall by as many half bricks as it contains in thickness, which product divided by 3, gives the content of the wall 1 brick and $\frac{1}{2}$ thick, which superficial feet divide by 272, the quote is square rods reduced to 1 brick and $\frac{1}{2}$; or more short by the following Table.

Brick.

For	$\left\{ \begin{array}{l} 1 \text{ subtract } \frac{1}{2} \\ 2 \text{ add } \frac{1}{2} \\ 3 \text{ multiply by } 2 \\ 4\frac{1}{2} \text{ multiply by } 3 \\ 6 \text{ multiply by } 4 \end{array} \right\}$	It will reduce the said thickness to 1 brick and $\frac{1}{2}$.
-----	--	--

Example 1. Suppose a wall of a garden 231 feet about, and 13 feet 4 inches high, and 1 brick $\frac{1}{2}$ thick, how many rods?

$$\begin{array}{r}
 213 \\
 13 \cdot 4 \\
 \hline
 2769 \cdot 0 \\
 71 \cdot 0 \\
 \hline
 272) 2840 \cdot 0 \quad (10 \cdot 120 \\
 \quad \quad \quad 120
 \end{array}
 \qquad \text{Rods, Feet.}$$

Example 2. How many rods of brick-work in that wall which is 40 feet 7 inches long, and 11 feet high, being one brick thick?

Feet. Inches.

$$\begin{array}{r}
 40 \cdot 7 \\
 11 \cdot 0 \\
 \hline
 3) 446 \cdot 5 \\
 148 \cdot 9 \cdot 8 \\
 \hline
 272) 297 \cdot 7 \cdot 4
 \end{array}$$

Rods. Ft. Inch. Parts.

1 . 25 . 7 . 4 Answer 1 . 25 . 7 . 4 .

Example 3. If a wall be 254 feet about, and 12 feet 7 inches high, and 3 bricks thick, how many rods?

254		
12 . 7		
<hr/>		
3048 . 0		
127 . 0		
21 . 2		
<hr/>		
3196 . 2		
<hr/>		
272) 6392 . 4	(23 : 136 : 4	Rods. Feet. Inches.

765. Practical Questions in the Mensuration of Superficial Figures.

Ques. 1. If the side of a square Superficies be 3 foot, what is the area or content of that Superficies? Or (which is the same thing) how many Squares, every one of which is a foot square, are contained in that Superficies?

Answe. 9 square feet, which content is found out by multiplying the given side 3 by itself, *viz.* 3 multiplied by 3 produces 9.

In like manner, if the side of a square pavement of stone be 15.7 feet, the superficial content of that pavement will be 246.49 feet, that is, 246 feet and an half very near, (for 15.7 multiplied by itself, produces 246.49.)

Likewise a square piece of wainscot, whose side is 3.24 yards, will be found to contain 10.4976 yards, or 10 yards and an half almost; for 3.24, multiplied by itself, to wit, by 3.24, will produce 10.4976.

Also if the side of a square piece of land be 37.25 perches, the content in square perches (neglecting the Fraction in the Product) will be found 1387, which being reduced will give 8 acres, 2 rods, and 27 perches for the content of that square piece of land.

Ques. 2. If a long square be 8 feet in length, and 5 feet in breadth, what is the superficial content?

Answe. 40 feet; which content is found out by multiplying the length by the breadth, *viz.* 8 multiplied by 5 produces 40. So if one of the lights of a glass-window supposed to be in form of a long square, has for its length 3.06 feet, and breadth 1.47 feet, the content of that glass will be 4.4982 feet, or 4 feet and an half almost, for 3.06 multiplied by 1.47 produces 4.4982.

In like manner, if there be a piece of wainscot, plastering, or any other superficies in form of a long square, which is in length 6.325 yards, and in breadth 3.214 yards; the superficial content will be found 20.32 + yards; that is, 20 yards, one quarter of a yard, and somewhat more, for 6.325 multiplied by 3.214 produces 20.32 +.

Likewise a piece of Tiling in form of a long square, whose length is 18.5 feet, and breadth 11.7 feet, will be found to contain 216.45 square feet, which may be reduced to 2.1645 squares of tiling, by allowing (according to custom) 100 square feet to one square of tiling.

Also if a piece of land in form of a long square be 48.75 perches in length, and 36.25 in breadth, the area or content in perches will be found 1767.18 +, which 1767 perches being reduced, will give 11 acres and 7 perches for the content of that piece of ground.

Ques^t. 3. If it be required to set forth in a meadow one acre of grass to lie in the figure of a long square, and that the length of it be limited or agreed to be 20 perches, what must the breadth be?

Answ. 8 perches, which breadth is found out by dividing 160 (the number of square perches contained in an acre) by the given length 20. If two acres were required, then 320 (to wit, twice 160) must be divided by the given side, whether it be the length or breadth; so if 7.25 perches be prescribed for the breadth of two acres, the length must be 44.13 + perches.

In like manner, if the breadth of a board be 1.32 foot, and it be demanded how far one ought to measure along the side of it to have a superficial foot, or a foot square of that board; divide 1 by the given breadth, so you'll find in the quotient this decimal fraction .757 +, which represents three quarters of a foot, or nine inches and somewhat more, and so much in length ought to be measured along the side of that board to make a superficial foot. Likewise if the breadth of a board be given in inches, then 144 (the number of square inches contained in a superficial foot square) being divided by the given breadth, the quotient will shew how many inches ought to be measured along the side of that board to make a superficial foot; so the breadth of a board being 9 inches, the length forward to make a superficial foot will be found 16 inches.

Ques^t. 4. If a piece of arras-hangings, in the form of a long square, has for its length 6 $\frac{1}{4}$ yards English, and breadth 4 yards; how many square ell^s, or sticks Flemish are contained in that

C c 3 piece,

piece, when the length of a *Flemish* ell is equal to $\frac{1}{4}$ yard *English*? Answ. $44\frac{1}{2}$ square ells or sticks *Flemish*.

Forasmuch as by supposition, a *Flemish* ell in length, has such proportion to an *English* yard in length, as 3 to 4, and consequently the square of the one to the square of the other, as 9 to 16: Therefore in a direct proportion, as 9 is to 16; so in any given number of square yards *English*, to a number of square ells *Flemish*, which will take up equal space with the said square ells *English*. Also in a direct proportion, as 16 is to 9, so is any given number of square ells *Flemish*, to a number of square yards *English*, which will take up an equal space with the said *Flemish* ells: Therefore to resolve the aforesaid question, first find the number of square yards *English*, contained in the said piece of arras, by multiplying the length and breadth in yards mutually one by the other, then proceed according to the aforesaid proportion; so the work will stand thus:

$$\begin{aligned} \text{I. } 6\frac{1}{2} \times 4 &= 25 \text{ square yards English.} \\ \text{II. } 9 : 16 &:: 25 : 44\frac{1}{2} \text{ square ells Flemish.} \end{aligned}$$

Quest. 5. If a piece of tapestry, in the form of a long square, be in length $15\frac{1}{4}$ ells *Flemish*, and in breadth $4\frac{1}{2}$ ells *Flemish*, how many square yards *English* are contained in that piece, when 4 ells *Flemish* in length are equal to 3 yards *English*? Answ. $37\frac{1}{4}$ square yards *English*.

$$\begin{aligned} \text{I. } 15\frac{1}{4} \times 4\frac{1}{2} &= 66\frac{1}{2}. \\ \text{II. } 16 : 9 &:: 66\frac{1}{2} : 37\frac{1}{4}. \end{aligned}$$

766. If the three sides of a piece of land that lies in form of a Triangle be 15 perches, 14 perches, and 13 perches, what is the area or number of square perches contained in that Triangle?

Answ. 84 perches, or half an acre and four perches, which content is found out by this Rule, viz.

From half the sum of the three sides of any plane Triangle, subtract each of the three sides severally, and note the three remainders; then multiply the said half sum and those three remainders one into the other: (according to the Rule of continual Multiplication;) that done, extract the square root of the last product, so shall such square root be the area or content of the Triangle.

Perches.

	Perches.
The three sides of a Triangle	15 14 13
The sum of the three sides	<u>42</u>
The half of that sum	21
The three remainders found out by subtracting every side from the half sum	6 7
The product arising from the continual Multiplication of the four last numbers	7056
The square root of which product is the content required, to wit,	84

Another Example.

	Perches.
The three sides of a Triangle	120, 5
	112, 6
	90, 3
The sum of the three sides	<u>323, 4</u>
The half of that sum	161, 7
The three remainders found by subtracting each side from the half sum	41, 2 49, 1 71, 4
The product arising from the continual Multiplication of the four last numbers	23355380,1096
The square root of that product	4832,74+

Wherefore I conclude, that the content of a plain Triangle, whose three sides are 120.5 perches, 112.6 perches, and 90.3 perches, is 4832.74 + perches, which reduced, give 30 acres and 32 perches, (the fraction of a perch being neglected.)

Now since every irregular piece of ground may be divided into Triangles; for a four-sided field will be divided into two Triangles by one imaginary straight line leading overthwart from corner to corner, called a Diagonal Line; a five-sided field into three Triangles by two Diagonals; a six-sided ground into four Triangles by three Diagonals, &c. the Rule before given will be of excellent use to find out the contents of large fields, especially if the land be of a dear value; as also when any controversy arises by reason of the different admeasure-

ments of surveyors of land: For if the sides of those Triangles be measured in the field, and their lengths be agreed on, all artists to whom the reason of the Rule before given is known, will agree in one and the same content. But yet this way of measuring presupposes that there is no obstacle, as water, wood, or other impediment, to hinder the measuring of the sides of those Triangles into which the field is divided as aforesaid.

767. If the diameter of a circle be 28.25, what is the circumference?

Answ. 88.749892: For as 1 is to 3.1415927, so is the diameter to the circumference: Therefore always multiplying the diameter given by the said 3.1415927, the product will be the circumference required.

768. If the diameter of a circle be 28.25, what is the superficial content of that circle?

Answ. 626.7968: For as 1 is in proportion to .7853982, so is the square of the diameter to the superficial content. Therefore multiplying the said decimal fraction .7853982 by the square of the given diameter (which square is the product of the multiplication of the diameter by itself,) the product shall be the superficial content required.

769. If the diameter of a circle be 28.25, what is the side of a square which may be inscribed within the same circle?

Answ. 19.97577: For the square root of half the square of the diameter, or the square root of the double of the square of the semi-diameter, will be the side of the inscribed square sought. Otherwise, as 1 is to .707166, so is the diameter to the side required. Therefore if you multiply (always) the said .707166 by the diameter given, the product will be the side of the inscribed square required.

770. If the circumference of a circle be 88.75, what is the diameter?

Answ. 28.25: For as 1 is to .3183099, so is the circumference to the diameter. Therefore if .3183099 be multiplied by the given circumference, the product shall be the diameter required.

771. If the circumference of a circle be 88.75, what is the superficial content of that circle?

Answ. 626.797: For as 1 is to .0795775, so is the square of the circumference to the superficial content. If therefore .0795775 be always multiplied by the square of the given circumference, the product will be the superficial content sought.

772. If the circumference of a circle be 88.75, what is the side of a square that may be inscribed within the same circle?

Answ. 19.97566 : For as 1 is to .225078, so is the circumference to the side required. Therefore if 225078 be always multiplied by the circumference given, the product will be the side of the inscribed square sought.

773. If the superficial content of a circle be 626.8, what is the diameter?

Answ. 28.25 : For as 1 is to 1.2732395, so is the content to the square of the diameter. Therefore multiplying 1.2732395 by the given content, the square root of that product shall be the diameter required.

774. If the superficial content of a circle be 626.8, what is the circumference?

Answ. 88.75 : For as 1 is to 12.566371, so is the content to the square of the circumference. If therefore 12.566371 be always multiplied by the given content, the square root of the product will be the circumference required.

775. If the superficial content of a circle be 626.8, what is the side of a square equal to the same circle?

Answ. 25.03598 : For the square root of the given content is the side of the square required.

776. Of Solid Measure, which respects Length, Breadth, and Thickness.

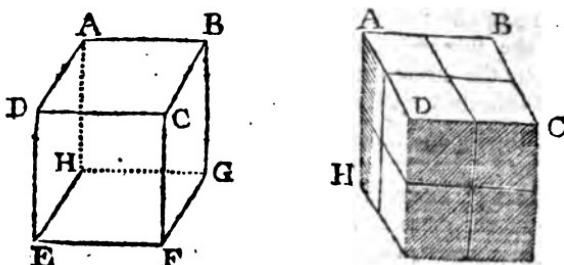
Rule. Multiply the Length by the Breadth, and that product by the Depth.

Otherwise, Multiply the area of the base by the Height, the product gives the solid content, which if in feet, divide by 27, the solid feet in one yard, which reduces them to solid yards, if in inches, by 1728, gives solid feet, &c.

Demonstration.

For a surface ABCD being imagined to move, suppose upwards or downwards, so that its points A, B, C, D, describe the lines AH, BG, CF, DE, and consequently each line therein describes other surfaces, &c. there will be generated by this motion, a magnitude called a solid, having three dimensions, viz. Length, Breadth, and Thickness: Therefore that surface ABCD, being divided into little square area's, and moving the height AH, divided by a like measure for length, may be conceived to produce as many small cubes, as is the number of the little area's, in the surface ABCD, multiplied by the number of the divisions in the height AH.

Examp.



Examp. If the side of a cube be 12 inches, how many cubical inches are contained in that cube?

Answ. 1728. What a cube is, may be well represented by a dye, which is a little cube itself, being a rectangular or square solid, that has an equal length, breadth and depth, and is comprehended under six equal squares: Now if the side of one of those equal squares (which is also the side of the cube,) be 12 inches, the superficial content of that square will be 144 square inches; for 12 multiplied by 12 produces 144, which multiplied by the depth 12 inches, produces 1728 cubical inches, and such is the solid content of that cube, whose side is 12 inches: So that by one foot of timber or stone, in whatsoever kind of solid it be found, is understood a cube, containing 1728 cubical or dye-square inches, and consequently half a foot solid contains 864 cubic inches, and a quarter of a foot solid contains 432 cubic inches.

In like manner, if the side of a cube of stone be 2.53 feet, the solid content of that cube will be found 16.194 + feet; for 2.53 being multiplied by itself, produces 6.4009 superficial feet, which product being multiplied by the said 2.53, will produce 16.194 + solid feet.

Also if the side of a cube of stone or wood be 6 inches, or .5 foot, the solid content will be found 216 cubic inches, or .125 parts of a foot solid; (for 6 multiplied cubically produces 216; likewise .5 multiplied cubically produces .125;) whence it may be inferr'd, that 8 little cubes of stone or wood, every one of which is half a foot, or 6 inches square, are contained in a foot of stone or timber; for 8 times 216 produces 1728 (being the number of cubic inches contained in a foot solid;) likewise 8 times .125 produces 1, (to wit, one entire foot solid.)

777. If the breadth of a squared piece of timber, supposed to be straight and terminated at both ends by two equal squares, be 1.55 feet, the depth also 1.55 feet, and the length 17.33 feet, how many cubic feet are contained in that piece of timber?

Answ. 41.635 feet; that is, 41 feet and an half, and about half a quarter of a foot: Which solid content is found out by this Rule, viz. multiply the breadth 1.55 by the depth 1.55, the product will be 2.4025 superficial feet, which is the content of the base, (that is, the area of either of the two equal squares at the ends of the piece: Lastly, multiplying the said base 2.4025 by the length 17.33, the product will be 41.635 +, which is the solid content required.

In like manner, if the breadth of a squared piece of timber, supposed to be straight and terminated at both ends by two equal long squares (which are called the bases) be 2.34 feet, the depth 1.61 feet, and the length 17.58 feet, the solid content will be 66.23 + feet; for (as before) multiplying the breadth by the depth, and that product by the length, the last product will be the solid content required.

778. If the breadth, as also the depth of a squared piece of timber having equal square bases, be 1.55 feet, how far ought one to measure along the length of that piece of timber to make a foot solid?

Answ. .4162 parts of a foot, or 5 inches very near; which decimal is thus discovered, viz. first find the superficial content of the base, which will be 2.4025 (for 1.55 multiplied by 1.55 produces 2.4025:) Then dividing 1 (to wit, 1 solid foot) by the base 2.4025, the quotient will be .4162, or 5 inches almost, and so far ought to be measured along the length of the piece to make a foot solid. In like manner, if the breadth be 2.34 feet, and the depth 1.61 feet, the length forward along the piece to make one solid foot, will be found .2654 parts of a foot, or three inches and almost $\frac{1}{3}$ part of an inch.

779. If a straight squared piece of timber be terminated by unequal bases, whereof one contains 1.92 superficial feet, the other .85 feet, and the length of that piece of timber be 17.4 feet, what is the solid content, or how many cubical feet are contained in that piece of timber?

Answ. 23.474 + feet; (found out by one of Mr. Oughtred's Rules for measuring a segment of a pyramid in Problem 21. Chap. 19. of his *Clavis Mathematica*.) The Rule is this:

Multiply the greater base by the less, and extract the square root of that product; then multiply the sum of the two bases and that square root by one third part of the length of the solid proposed, so will the last product be the solid content required.

Example.

The greater base	—	—	—	1 .92
The lesser base	—	—	—	0 .85
The product of the Multiplication of those two bases	—	—	—	1 .6320
The square root of that product	—	—	—	.1 .2775
The sum of that square root and the two bases	—	—	—	4 .0475
One third part of the length is	—	—	—	5 .8
The product of the Multiplication of the two last numbers is the solid content required	—	—	—	23 .4755

780. A pyramid is a solid comprehended under plain surfaces, and from a triangular, quadrangular, or any multangular base, diminishes equally less and less, till it finish in a point at the top; now if the superficial content of the base of a pyramid be 5.756 feet, and the height of it 14.25 feet, (which height is the length of the perpendicular line that falls from the top of the pyramid to the base;) what is the solid content of that pyramid?

Answ. 27.341 + feet; for if the area of the base of a pyramid be multiplied by one third part of the height thereof, the product will be the solid content of the pyramid; therefore $5.756 \times 4.75 = 27.341$ feet = the solidity of the pyramid proposed.

Note. If a pyramid be cut into two segments by a plane parallel to the base, one of those segments will be a pyramid, and the other will have two unequal bases; for the measuring of which latter segment, a rule has been already given in Art. 777. the area of each base being known.

781. A Cone is a Solid, which has a circle for its base, from whence it grows, equally less and less (like a round steeple of a church) till it end in a point at the top; now if the area of the base of a Cone be 5.756 feet, and the height of it be 14.25 feet, what is the solid content of that Cone?

Answ. 27.341 feet; for if the area of the base of a Cone be multiplied by one third part of the height thereof, the product shall be the solid content of the Cone.

Note. If a Cone be cut into two segments by a plane parallel to the base, one of those segments will be a Cone, and the other segment will have two unequal bases which are circles; the

the solidity of which latter segment may be found out by the rule before given in Art. 777. the area of each base (or circle) being known.

782. A Cylinder is a solid, which may be well represented by a stone-roll, such as are used in gardens for the rolling of walks. Now if the circumference of a Cylinder be 4.57 feet, and the length 3.25 feet, what is the solid content of that Cylinder?

Answ. 5.4 + feet, thus found out: First, by the help of the given circumference 4.57, find out the superficial content of that circle, (being the base of the Cylinder;) which content (by Art. 769.) will be found 1.661967 foot; then multiplying the said 1.661967 by the given length 3.25, the product will be 5.401393, which is the solid content required.

783. If the Base of a Cylinder be 1.6619 feet, how much in length of that cylinder will make a foot solid?

Anw. .6017 parts of a foot; for 1 (to wit, 1 solid foot) being divided by the base 1.6619, gives in the quotient the decimal .6017 for the length required.

784. A globe is a perfect round body contained under one surface; in the middle of the globe there is a point called the center, from whence all straight lines drawn to the outside are of equal length, and called semi-diameters, the double of any one of which is equal to the diameter of the globe; now if the diameter of a globe of stone be 1.75 foot, how many feet solid are contained in that globe?

Answ. 2.80616 feet; for as 1 is to .5235988, so is the cube of the diameter to the solid content of the globe: Therefore multiplying the cube of the diameter by the said decimal .5235988, the product will be the solid content required: So the diameter 1.75 being first multiplied by itself, the product will be 3.0625, which multiplied by the said 1.75, gives in the product 5.359375, to wit, the cube of the diameter, which being multiplied by .5235988, the product thence arising will be 2.80616, which is the solidity of the globe propounded.

785. What is the diameter of a globe of stone, which contains 4 cubical or solid feet?

Answ. 1.96 + foot; for as 1 is to 1.9098593, so is 4 (the solid content given) to a fourth proportional, to wit, 7.639437, whose cubic root is 1.96949, the diameter required.

786. In Timber and Stone. The Content is given by the Solid Foot.

Examp. 1. In a stone { Length 5 : 9 } how many solid
Breadth 3 : 11 } feet?
Depth 2 : 8 }

$$\begin{array}{r}
 3 \cdot 11 \\
 5 \cdot 9 \\
 \hline
 19 \cdot 07 \\
 1 \cdot 11 \cdot 6 \\
 0 \cdot 11 \cdot 9 \\
 \hline
 22 \cdot 6 \cdot 3 \\
 2 \cdot 8 \cdot 0 \\
 \hline
 45 \cdot 0 \cdot 6 \\
 15 \cdot 3 \cdot 1 \cdot 6 \text{ Answ. } 60 : 00 : 8 \\
 3 \cdot 9 \cdot 0 \cdot 6 \\
 \hline
 60 \cdot 0 \cdot 8 \cdot 0
 \end{array}$$

787. In Digging. The Content is given by the Solid Yard.

Examp. 2. A vault digged 9 feet deep, $4\frac{1}{2}$ feet long, and 3 feet 9 inches broad, how many solid yards?

$$\begin{array}{r}
 4 \cdot 6 \text{ Length} \\
 9 \cdot 0 \text{ Depth} \\
 \hline
 40 \cdot 6 \\
 3 \cdot 9 \text{ Breadth} \\
 \hline
 121 \cdot 6 \\
 20 \cdot 3 \\
 10 \cdot 1 \cdot 6
 \end{array}$$

27) 151 . 10 . 6 Yards. Feet. Inches. Parts.
Answ. 5 . 16 . 10 . 6

788. If a bullet of brass of .8 inches diameter weigh 72 pounds, what shall a bullet of brass weigh whose diameter is 4 inches?

Since like solids are in triple proportion to their homologous sides, diameters, lines, &c. it holds.

As the cube of the diameter given,

To the weight thereof :

So is the cube of the other diameter,

To the weight thereof.

That is, As $8 \times 8 \times 8 : 72 :: 4 \times 4 \times 4 : 9$.

$$\text{For } \frac{72 \times 4 \times 4 \times 4}{8 \times 8 \times 8} = \frac{72 \times 1 \times 1 \times 1}{2 \times 2 \times 2} = \frac{72}{8} = 9.$$

789. If a ship of 200 tun be 44 foot long at the keel, of what length shall the keel be of a ship of 220 tun?

As $100 : 44 \times 44 \times 44 :: 220 : 44 \times 44 \times 44 \times \frac{22}{100}$.

That is, $44 \times 44 \times 44 \times \frac{22}{100}$ is the cube the length required.

$$\text{And } \sqrt[3]{44 \times 44 \times 44 \times \frac{22}{100}} = 44 \times \sqrt[3]{2,2} = \text{the length.}$$

$$\text{But } \sqrt[3]{2,2} = 1,30059; \text{ therefore } 44 \sqrt[3]{2,2} = 57,22596.$$

790. The side of the cube being given to find the side of that cube that shall be double, treble, quadruple, &c. in quantity to the given cube.

Multiply the cube-root of 2, 3, 4, &c. by the side of the given cube, and the product will be the side of the cube required, &c.

Examp. There is a cubical vessel, whose side is 12 inches, and it is required to find the side of that vessel which shall contain 3 times as much.

The cube-root of 3 is (per Table in Art. 482.) 1,442250

Multiply by 12

Answer 17,30000

If you would find a side that shall contain $\frac{1}{2}$ as much, $\frac{3}{2}$ as much, $\frac{4}{3}$ as much, &c. then divide the side of the given cube by the cube-root of 2, 3, 4, &c.

791. The concave diameter of two guns, being known, together with the quantity of gunpowder sufficient to charge one, to find what will be sufficient to charge the other.

Rule. The capacities are one to another, as the cubes of their diameters.

Examp. If 45 pounds of gunpowder be sufficient to charge a gun, whose concave diameter is $1\frac{1}{2}$ inch, how much gunpowder will suffice to charge a gun whose concave diameter is 7 inches?

As $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} : 45 :: 7 \times 7 \times 7 : 45 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$.

That is $\frac{5 \times 14 \times 14 \times 14}{3} = 4573\frac{1}{2}$.

792. Questions relating to Solids.

1. If a piece of timber 48 feet long, 6 inches broad, and 14 inches thick, cost 10*l.* what is the value of five such pieces which are 6 feet long, 4 inches broad, and 7 inches thick?

	Pieces.	Feet.	Br.	Thickness.	Len.
Conditional terms	1	48	6	14	10.
Corresponding	5	6	4	7	A.

$$\text{Therefore } A = \frac{5 \times 6 \times 4 \times 7 \times 10}{48 \times 6 \times 14} = \frac{25}{72} = 24 \text{ i.e. } 8d.$$

2. If 300 men, in 15 days, dig a trench 5600 feet long, 6 deep, and 12 wide, how long must that trench be, whose depth is 8 feet, width 14 feet, dug by 2700 men in 25 days.

	Men.	Days.	Len.	Dep.	Wid.
Conditional terms	300	15	5600	6	12.
Corresponding	2700	25	A.	8	14

$$\text{Therefore } A = \frac{2700 \times 25 \times 5600 \times 6 \times 12}{300 \times 15 \times 8 \times 14} = 54000.$$

3. If 248 men, in $5\frac{1}{2}$ days, dig a trench $23\frac{1}{4}$ yards long, $2\frac{1}{2}$ deep, and $3\frac{2}{3}$ wide; in how many days will 24 men dig a trench $33\frac{1}{4}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{2}{3}$ wide?

	Men.	Days.	Len.	Dep.	Wid.
Conditional terms	248	$\frac{11}{2}$	$\frac{9}{4}$	$\frac{7}{3}$	$\frac{11}{3}$
Corresponding	24	A.	$\frac{13}{4}$	$\frac{7}{2}$	$\frac{11}{3}$

$$A = \frac{248 \times 11 \times 135 \times 7 \times 28 \times 4 \times 3 \times 3}{2 \times 4 \times 2 \times 5 \times 24 \times 93 \times 7 \times 11} = 189.$$

4. If 24 men, working 189 days, 14 hours each day, dig a trench $33\frac{1}{4}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{2}{3}$ wide; how many hours per day must 217 men work to dig a trench $23\frac{1}{4}$ yards long, $2\frac{1}{2}$ deep, and $3\frac{2}{3}$ wide in $5\frac{1}{2}$ days?

	Men.	Days.	Hours.	Len.	Dep.	Wid.
Conditional terms	24	189	14	$\frac{13}{4}$	$\frac{7}{2}$	$\frac{28}{3}$
Corresponding	217	$\frac{11}{2}$	A.	$\frac{9}{4}$	$\frac{7}{3}$	$\frac{11}{3}$

$$A = \frac{24 \times 189 \times 14 \times 93 \times 7 \times 11 \times 2 \times 4 \times 2 \times 5}{4 \times 3 \times 3 \times 217 \times 11 \times 135 \times 7 \times 28} = 16.$$

So find the side of a cube that shall be equal in solidity to any given solid, as a Globe, Cylinder, Prism, Cone, or such like.

The Cube-root of the solid content of any solid body given, is the side of a Cube of equal solidity. So if the content of a Globe were found to be 15625 solid inches, seek the Cube-root of 15625, which is 25, which is the side of a Cube of equal capacity.



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